8.325 Homework 6
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Due: Thur. May 1.

Problem 1) Peskin & Schroeder, Problem 19.1, page 686-687

Problem 2) Axial-Anomaly in Dimensional Regularization

Compute the axial anomaly for QED in four-dimensions from the triangle diagram using dimensional regularization (show all your steps i.e. not just those displayed in Peskin). Demonstrate that your result is equivalent to a matrix element of the operator equation

\[ \partial_{\mu} j^{\mu 5} = -\frac{e^2}{16\pi^2} F^{\alpha \beta} \tilde{F}_{\alpha \beta} \]  

which we discussed in two different ways in lecture.

Problem 3) Baryon and Lepton Number

Let \( B^{\mu} \) be the current for baryon number, and \( L^{\mu} \) be the current for lepton number. Show that \( B^{\mu} \) has an anomaly, but that \( B^{\mu} - L^{\mu} \) does not.

Problem 4) The decays \( \pi^0 \rightarrow \gamma \gamma \) and \( \eta \rightarrow \gamma \gamma \)

a) Compute the matrix element and the decay rate \( \Gamma_{\pi^0} \) for \( \pi^0 \rightarrow \gamma \gamma \) through the anomaly. (You may use results from lecture.) Using the experimental values for \( m_\pi \) and \( f_\pi \) compare your result with the experimental value for the decay rate in the PDG (http://pdg.lbl.gov/).

b) Consider \( \eta^0 \), the 8'th Goldstone boson of the spontaneous symmetry breaking \( SU(3)_L \times SU(3)_R \rightarrow SU(3)_V \) in QCD. Assume that the decay \( \eta^0 \rightarrow \gamma \gamma \) also proceeds through the axial anomaly and compute \( \Gamma_{\eta^0}/\Gamma_{\pi^0} \).