Projectile/Circular Motion

Announcements:

- Reminder these lectures are posted. Just click on the Notes and Reading Calendar from the main web page to find them.
- Reminder of the SmartPhysics Prelectures.
- Covering Material in Chap 4, Sec1-6 today.
- Fair amount of math this lecture!

http://www.colorado.edu/physics/phys1110/phys1110_sp12/
Q. The position vector of a particle moving with constant velocity is shown below at two different times, an earlier time $t_1$ and a later time $t_2$. Which arrow shows the direction of the velocity vector?

To get from position 1 to position 2 requires movement to the right. Alternatively:

$$
\vec{R}_2 - \vec{R}_1 = =
$$

(A) (B) (C) (D) (E) None of these
Projectile motion

This describes the motion of a body (bullet, basketball, motorcycle, etc.) in free fall after being launched.

The **only** acceleration is due to gravity and is always straight down.

Thus, the velocity in the horizontal direction is constant

so \( x = x_0 + v_{0x}t \) and \( v_x = v_{0x} \)

In the vertical direction there **is** acceleration from gravity

so with \( a = -g \) then \( y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \) and \( v_y = v_{0y} - gt \)

Also, \( y = y_0 + \frac{v_{0y} + v_y}{2}t \) and \( v_y^2 = v_{0y}^2 - 2g(y - y_0) \)
Two pitchers are standing side by side, and simultaneously throw baseballs A and B.

If the balls follow the parabolic trajectories shown,

A: A
B: B
C: both hit at the same time
D: need more information

Answer: Projectile A is in flight longer. The y-motion is independent of the x-motion. The flight times are controlled solely by the y motion. Projectile A has a larger y-component of initial velocity (you can see it from the initial slope, but also from the fact that A goes up higher) Think of this: you throw two coins up in the air. One goes higher. Which one is in the air longer?
Initial velocity in horizontal and vertical directions depends on angle $\alpha$ and speed $v_0$ of launch.

$$v_{0x} = v_0 \cos \alpha \quad \text{and} \quad v_{0y} = v_0 \sin \alpha$$

Solving projectile motion problems

Realize the horizontal and vertical motions are independent. Their only connection is through the time the projectile is in the air.
Solving a projectile motion problem

A basketball launched on a level surface travels 15 m and reaches a maximum height of 6.4 m. What is the initial velocity $\vec{v}_0 = (v_0, \alpha)$?

1. Draw a diagram
2. Figure out what we know
3. Figure out what we need:

To get $v_{0x}$ we need the flight time to use $x = x_0 + v_{0x}t$

Can get $v_{0y}$ from $v_y^2 = v_{0y}^2 - 2g(y - y_0)$ with $y = y_{\text{max}}$ and $v_y = 0$

$v_{0y} = \sqrt{2(9.8 \, \text{m/s}^2)(6.4 \, \text{m} - 0 \, \text{m})} = 11.2 \, \text{m/s}$

Can now use $y = y_0 + v_{0y}t - \frac{1}{2} gt^2$ or $v_y = v_{0y} - gt$ to get $t$
Projectile motion problem solved

Projectile follows path shown reaching a maximum height of 6.4 m and distance of 15 m. Find the initial velocity: \( \vec{v}_0 = (v_0, \alpha) \)

Get \( v_{0y} \) from \( v_y = v_{0y} - 2g(y - y_0) \)

\[
v_{0y} = \sqrt{2(9.8 \text{ m/s}^2)(6.4 \text{ m} - 0 \text{ m})} = 11.2 \text{ m/s}
\]

Use \( v_y = v_{0y} - gt \) to get \( t = \frac{v_{0y} - v_y}{g} = \frac{11.2 \text{ m/s}}{9.8 \text{ m/s}^2} = 1.15 \text{ s} \) (to highest point)

Use \( y = y_0 + v_{0y}t - \frac{1}{2} gt^2 \) to get \( 0 = 0 + v_{0y}t - \frac{1}{2} gt^2 \) so \( t = \frac{2v_{0y}}{g} = 2.3 \text{ s} \)

To get \( v_{0x} \) we use \( x = x_0 + v_{0x}t \) so \( v_{0x} = \frac{x - x_0}{t} = \frac{15 \text{ m}}{2.3 \text{ s}} = 6.5 \text{ m/s} \)

Therefore

\[
v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(6.5 \text{ m/s})^2 + (11.2 \text{ m/s})^2} = 13 \text{ m/s}
\]

\[
\alpha = \tan^{-1}\left(\frac{11.2 \text{ m/s}}{6.5 \text{ m/s}}\right) = 60^\circ
\]
Some projectile motion warnings

Following are true **only** when $\Delta y = 0$ !!!!!
(when the launch and landing points are at the same height)

- The time of flight is twice the time to reach the maximum height.
- The initial and final vertical velocities have the same magnitude and opposite directions.
- Maximum range is obtained with an angle of 45°
Q. A bullet is fired horizontally from a rifle on the Moon (no air) with initial speed $v_0$. Assuming a level and endless ground, which statement, if any, is **false**?

A. During the flight, the minimum speed of the bullet is $v_0$.
B. During its entire flight the acceleration is constant.
C. The flight time increases as $v_0$ increases.

**OR** D. All of the above statements are true.

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A. $v_x = v_{0x} = v_0$ and $v = \sqrt{v_x^2 + v_y^2}$ so $v \geq v_0$.

B. Acceleration is due to gravity and is constant downward.

C. Let $y_0$ be the gun height so the bullet lands at $y=0$. Since $v_{0y}=0$, 
   
   \[ 0 = y_0 - \frac{1}{2} g_{moon} t^2 \] 
   
   so 
   \[ t = \sqrt{\frac{2y_0}{g_{moon}}} \], independent of $v_0$. 

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Clicker question 3

Set frequency to BA
A helicopter takes off vertically, with a constant speed $v_0$. While rising, the pilot releases a cigarette butt out the open window. (Neglect air resistance!) The cigarette will

A: Initially rise with speed $v_0$, reduce its speed, come to rest, and then increase its speed as it goes down.

B: Not go up at all, but will immediately start to gain downwards speed as it drops to the ground.

C: Drop to the ground with a constant speed.

The butt begins with an upward speed of $v_0$. As soon as the pilot lets go, this is a projectile with initial velocity $+v_0$. It behaves just the same as any projectile released with an UPWARDS $v_0$. It goes up, slows and stops, then speeds up as it falls.
A rifle is accurately aimed at a rabid monkey hanging from the branch of a tree. The instant the gun is fired, the monkey releases the branch and starts falling. The monkey is well within the range of the rifle. The initial speed of the bullet is $v_0$. What happens?

The bullet finds its target, regardless of the value of $v_0$. (Assuming $v_0$ is large enough to reach the air below the monkey.)
Solution to monkey & hunter problem

We want to find out if the bullet and monkey every occupy the same place at the same time.

At what time does the bullet reach the x-position of the monkey?

From \( x = x_0 + v_{0x}t \) and \( v_{0x} = v_0 \cos \alpha \) : \( t = \frac{x - x_0}{v_0 \cos \alpha} = \frac{d}{v_0 \cos \alpha} \)

At that time, what are the y-positions of bullet and monkey?

Monkey starts at \( y_0 = d \tan \alpha \) with \( v_{0y} = 0 \) so \( y_{\text{monkey}} = d \tan \alpha - \frac{1}{2} gt^2 \)

Bullet has \( v_{0y} = v_0 \sin \alpha \) at \( y_0 = 0 \) so \( y_{\text{bullet}} = v_0 \sin \alpha t - \frac{1}{2} gt^2 \)

Bullet hits monkey if \( y_{\text{bullet}} = y_{\text{monkey}} \); that is, if \( v_0 \sin \alpha t = d \tan \alpha \)

But \( v_0 \sin \alpha t = v_0 \sin \alpha \frac{d}{v_0 \cos \alpha} = d \tan \alpha \) so it always hits!
Uniform circular motion

Particle moving in a circle at constant speed

Definitions

The period $T$ is the time to complete one revolution (units are seconds)

The frequency $f$ is the number of revolutions per time (units of Hz = s$^{-1}$)

The angular frequency $\omega$ is the number of radians per time (units of rad/s)

Some formulae: $f = \frac{1}{T}$, $f = \frac{v}{2\pi R}$, $\omega = \frac{v}{R}$, $\omega = 2\pi f$
Centripetal acceleration

A particle goes around a circle of radius $R$ with constant speed so $|v_1| = |v_2|$

Since $\vec{v}_1$ and $\vec{v}_2$ are perpendicular to their respective radials and have the same length as do the two radials, we identify two similar triangles so $\frac{|\Delta \vec{v}|}{v_1} = \frac{|\Delta \vec{s}|}{R}$ or $|\Delta \vec{v}| = \frac{v_1}{R} |\Delta \vec{s}|$

and average acceleration is $\langle a \rangle = \frac{|\Delta \vec{v}|}{\Delta t} = \frac{v_1}{R} \frac{|\Delta \vec{s}|}{\Delta t}$

Instantaneous acceleration:

$$a_{rad} = \lim_{\Delta t \to 0} \frac{v_1}{R} \frac{|\Delta \vec{s}|}{\Delta t} = \frac{v}{R} \frac{ds}{dt} = \frac{v^2}{R}$$
In order to maintain constant speed, the acceleration vector for uniform circular motion *must* always be perpendicular to the velocity vector, i.e. pointing to the center of the circle.
Q. An object is moving along a circular path and is **slowing down**, as shown. Which arrow best represents the object’s acceleration vector at point X?

Since \( \vec{a} = \frac{\Delta \vec{v}}{\Delta t} \) the acceleration vector points in the same direction as \( \Delta \vec{v} \)

\[
\Delta \vec{v} = \vec{v}_2 - \vec{v}_1 = \text{(Diagram showing the direction of } \Delta \vec{v})
\]
Summary

• Material is getting more difficult – go over the slides for this presentation and ask questions if you are confused.

• Remember the Prelecture assignments (SmartPhysics) for Wednesday.