Saturn's North Polar Hexagon

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Credit: NASA/JPL-Caltech/SSI,
http://www.ciclops.org/view/7437/Saturn_Rev_175_Raw_Preview_3
Overview

- Discovery
  - Voyager, Pic-du-Midi and HST
  - Initial measurements
- Early Theory
  - Godfrey's four hypotheses
  - The Rossby Wave model
- Recent Studies
  - Cassini images
  - Numerical and experimental results
  - A new theory
Discovery
1988, Godfrey polar projected Voyager Images
- Discovered hexagonal structure at ~77N and an impinging vortex
- Structure rotated at $(6.3 \pm 8) \times 10^{-8}$ rad/s relative to planetary rotation
  - Using edge as baseline
  - Stationary within uncertainties!
- Later measured rotation more precisely using vortex tracking:
  - $(-8.13 \pm 0.52) \times 10^{-9}$ rad/s
- All velocities quoted in remainder of talk relative to the Saturnian Radio rotation
  - Presumed to be rotation rate of interior
  - Still somewhat uncertain
Confirmation by Pic-du-Midi and HST

Vortex visible, hexagon faint

Hexagon rotation:
-1.17 E−8 rad/s
-1.15 E−8 rad/s

Both associated vortex with hexagon and measured rotation rate by tracking the vortex feature

Pic-du-Midi Observatory in the French Pyrenees


Hubble Images
Velocity Measurements

• Godfrey used cloud tracking methods to measure jet velocities
  • This is different from the overall structural rotation
  • Zonal velocity flow at center: 100 m/s
• Profiles important for theories
Early Theories
Why stationary? What is it?

- Godfrey suggested four hypothetical causes
  1) Coincidence, stationary rate good as any other (unlikely, unsatisfying)
  2) Forcing, from above or below, at the SR period
  3) Patterns composing hexagon actually auroras
     - Godfrey remarked unlikely
     - Also refuted by Cassini imaging
  4) Hexagonal structure causes SR rotation rate
     - Assuming hexagonal features are clouds, unlikely to affect magnetic field (Gierasch)
The Forcing Hypothesis

- Forcing from above by auroras
  - Atmospheric depth (within troposphere), seasonal independence, and lack of southern hexagon makes this unlikely

- Forcing from below
  - Suggested by Gierasch
  - Interior thermal convection
  - Difficult to observe

Saturn’s UV aurora lighting up the poles
A Mathematical Approach

- **Rossby Waves:**
  - Low frequency, large scale planetary waves
  - Used in studies of Earth's Atmosphere
- Characterized by
  - Westward drift relative to mean background flow
- Properties exhibited by
  - Hexagon

Isobars on Earth. Solid contours indicate height of 500 hPa surface (60 m intervals)

- Under the **geostrophic approximation,** horizontal flow velocity run parallel to lines of constant pressure. This is why atmospheric flows can be represented by isobar charts.

\[
\epsilon = \frac{\text{Relative Acceleration}}{\text{Coriolis Effect}} \\
\approx \frac{O(U^2/L)}{O(2\Omega U)} \\
\approx \frac{U}{2\Omega L}
\]
Definitions

- Velocity field, \( u \)
  - Vector assigned to every point in fluid

- Vorticity ("spinniness")
  - Definition: \( \omega = \nabla \times u \)
  - Vorticity: Velocity Field as Current: Magnetic Field

- Relative vorticity
  - Curl of velocity field as measured in rotating frame

- Planetary vorticity: \( 2\Omega \)

- Absolute vorticity
  - Sum of planetary and relative vorticity: \( \omega_a = \omega + 2\Omega \)
Theory Applied to Hexagon

• Two important parameters:
  1) Gradient of surface-normal component of the planetary vorticity in the β-plane approximation: \( \beta = 2 \Omega \cos \theta / a \)
  2) Negative curvature of mean zonal flow, or the gradient of the relative vorticity \( (\omega = \nabla \times \mathbf{u}) : -u_{yy} \)

Velocity profile \( U \) modeled by Gaussian

Within ± 1/e distance from peak \( -u_{yy} \gg \beta \)
- Ignore planetary vorticity gradient
- Assume barotropic wave

Planetary vorticity dependent on latitude: \( f = 2 \Omega \sin \theta \)

Dispersion relation:
\[
c = U - \langle -U_{yy} \rangle_c (r/n)^2
\]
where \( r \) is radius of the latitude circle.

For hexagon,
\[
r = 1.4 \times 10^7 \text{ m}, \quad U = 100 \text{ m/s}, \quad \langle -U_{yy} \rangle_c \approx 2.2 \times 10^{-11} \text{ m}^{-1} \text{s}^{-1}
\]
gives, \( c \approx 0 \)
Vertical Structure and Perturbation Term

- Caveats in theory so far
  - Neglected vertical structure
    - No dependence on z-component (barotropic)
  - Neglected meridional confinement
- Can place constraints on vertical structure
  - Solve a quasi-geostrophic potential vorticity equation of streamfunction:
    \[ \Psi, \ u = -\frac{\partial \Psi}{\partial y}, \ v = \frac{\partial \Psi}{\partial x} \]
  - Assume vertical structure separable from horizontal structure
- Solution to equation includes perturbing term, assumed to be the impinging vortex
  - Meridional confinement satisfied with inclusion of perturbation term
  - Solution wave is vertically trapped
    - Forced by heating from below or other surface flows (from Earth's atmospheric studies)
Pre-Cassini Summary

- Hexagonal structure and an impinging vortex discovered by Godfrey in 1988 from Voyager images
  - Later confirmed by Pic-Du-Midi and HST
- Rotation rate measured
  - Virtually stationary wrt SR rotation
  - Jets within have velocities greater than 100 m/s
- Four hypotheses offered by Godfrey
  - The 2nd, forcing hypothesis is most realistic
  - Forcing from interior heat below posited by Gierasch
- Rossby wave model
  - Provides mathematical description of structure's rotation rate
  - Explains impinging vortex as perturbation term
    - Result of vertical constraints
    - Explains meridional confinement
Recent Studies
Cassini Observations

• North pole tilted away from sun. First visible light image in 2009 by Cassini Imaging Subsystem (ISS).
• Ephemeral polygonal waves seen in south, but none permanent.
• No evidence of impinging vortex!

Composite Infrared Spectrometer (CIRS) temperature map at mid-infrared wavelengths.

Cassini Visual-IR Mapping Spec (VIMS) polar projected images at 5.1 μm of hexagon.
Things to Note

- Hexagon exists deep into troposphere
- Unaffected by seasonal variations
  - Formation cannot be due to solar effects
- Disappearance of impinging vortex
  - Perturbative Rossby wave theory not accurate
- Deviation from expected potential vorticity conservation (from zonal velocity measurements)
- New theoretical and experimental approach needed
Rotational Polygons in the Laboratory

- Rotating polygon flows recreated in several experiments

- Two of note:
  - Jansson et. al.
    - Stable polygons driven by rotating bottom plate
  - Barbosa Aguiar et. al.
    - Differential disk
    - Differential ring
The Jansson Experiment

Rotating bottom plate separate from cylinder

Generated polygons with various sides
Results and Speculated Cause

- Polygons rotated much slower than rotating plate
- Number of sides increased with rotation frequency and decreased with fluid height
- Attributed cause to minute wobbling of bottom plate
  - Breaks azimuthal symmetry
The Barbosa Aguiar Experiment

The two setups used to force out a jet-like flow. Either a separate ring or disk was rotated at a different rate from the rest of the tank.

The bottom was either sloped or flat. A sloped bottom simulated a non-zero $\beta$ value.
Results

Hexagonal pattern formed from ring setup

Hexagonal pattern formed from disk setup

Flow Image Analysis

Velocity field of ring setup
Results Cont.

Violation of the Rayleigh-Kuo criterion, $\beta - u_{yy} < 0$

Necessary condition for barotropic instability

Velocity and vorticity gradient profile as measured by Voyager

Velocity and vorticity gradient profile as measured in the ring setup experiment with $\beta=0$. 
Barotropic Instability

- Qualitatively, due to horizontal shear (frictional) forces
  - Kinetic energy in the mean zonal flow converted to kinetic energy of arising eddies
- Rayleigh-Kuo violated in surrounding regions of hexagon
  - Not sufficient condition for barotropic instability
- Additional instability analysis using measured velocity profiles indicated that instability arises in north polar region but not south polar region, as is observed
- Numerical simulation conducted by separate group
  - Violation of Rayleigh-Kuo criterion
  - Speculated due to barotropic instability
- Caveat: vertical structure and detailed Saturnian atmosphere features not well known.
  - Cannot recreate precise conditions in laboratory
Summary and Conclusion

- Detailed images of Saturn's hexagon taken by Cassini
  - No sign of impinging vortex
  - Hexagon exists deep into troposphere
  - No such permanent feature in southern region
- Polygonal flows simulated in laboratory and numerically
  - Jansson et. al. demonstrated polygon formation caused by rotating bottom plate
  - Barbosa Aguiar et. al. attributed to polygonal formation to barotropic instability
    - Analogous velocity and vorticity gradient profiles in lab as on Saturn
  - Numerical simulation supported barotropic instability theory
- Barotropic instability model is the best we have of hexagon today
References


Backups
Pic-du-Midi and HST Rotation

![Graph 1: Voyager and Pic-du-Midi Longitudes vs. Time](image1)

![Graph 2: HST Data vs. Days from 9 November 1990](image2)
Cassini Velocity Profile

- Absolute vorticity tends to be conserved in 2D models of fluids
  - Sum of the planetary and relative vorticities: $\omega_a = \nabla \times u + 2\Omega$
  - Result of potential vorticity conservation in shallow water theory

- Potential vorticity: $\Pi = \frac{\omega_a}{\rho} \cdot \nabla \lambda$
  - Can be shown to be conserved under three conditions for 3D fluids
    - Barotropic fluid
    - Frictional forces negligible
    - Property $\lambda$ (e.g. density, potential temperature) is conserved

Mean zonal velocity profile measured by cloud tracking Cassini imagery. The dashed and dotted lines are model wind velocities assuming a constant absolute vorticity.

- Hexagon velocity structure doesn't agree with absolute vorticity conservation.
- Combining Cassini thermal measurements with velocity profile measurements gives a potential vorticity distribution that exhibits a step at hexagon’s latitude
Barotropic Fluid

Defining equation: \( \frac{\nabla \rho \times \nabla p}{\rho^2} = 0 \)

- Means surfaces of constant \( \rho \) coincide with surfaces of constant \( p \).
- Simplifies vorticity equation of motion
- Fluids that are NOT barotropic are called baroclinic fluids

Importance
- Assume atmospheric pressure depends only on \( z \) (altitude). i.e. \( p = p(z) \)
- In a baroclinic fluid, at a surface of constant pressure one could have density variations
- Since we're at surface of constant pressure, lighter element feels same pressure force as heavier element
- Vertical motion cannot be neglected for a baroclinic fluid
Barotropic Instability

- Qualitatively, due to horizontal shear (frictional) forces
  - Kinetic energy in the mean zonal flow converted to kinetic energy of arising eddies
- Rayleigh-Kuo criterion a necessary condition
- Arises from instability of dynamical fluid with respect to small fluctuations or disturbances
  - Fluctuations can then grow in amplitude
Saturn's Radio Rotation

- Saturnian Kilometric Radio Rotation
  - Radio frequency waves emitted from auroral regions
  - Cassini found to be different from previously measured

- Mechanism of generation uncertain
  - Due to electron interactions in magnetic field