Out

In & of the Conformal Window

Based on work with: A. Deuzeman, M.P. Lombardo, K. Miura, T. Nunes da Silva (lattice) A. Barranco, J. Russo (AdS/CFT)
Does conformal symmetry play a role well above the EWSB scale?
The Conformal Window
✓ quark gluon plasma (QGP): high T - low Nf
✓ preconformal regime (T=0, low T - high Nf)
✓ conformal regime (T=0)
Walking?

- $N_f < N_f^c$
- $N_f \leq N_f^c$
- $N_f^{AF} > N_f > N_f^c$
- $N_f > N_f^{AF}$

IRFP

or

UVFP
Strong coupling dynamics

Preconformal dynamics

AS

IRFP dynamics

\[\frac{1}{g^2}\]

\[N_f\]

[Deuzeman, Lombardo, Nunes EP '12]
Preconformal Dynamics
Preconformal
\[ k_{SB} \propto k_0 \theta (N_f^{cr} - N_f) |N_f^{cr} - N_f|^{-1/\Theta} \exp \left( -\frac{\pi}{2\epsilon \sqrt{\alpha |N_f^{cr} - N_f|}} \right) \]

power-law
(due to running coupling)

exponential-law
(Miransky-KBT scaling)

\[ \beta(g^2) = -\Theta (g^2 - g_x^2) + \ldots \quad \Theta < 0 \]
From a IR scale to a UV scale

\[
\frac{T_c}{\Lambda_L} \cdot a(\beta_c)\Lambda_L = \frac{1}{N_t}
\]

**UV** → \( \frac{\Lambda_{\text{ref}}}{\Lambda_L} = \exp \left[ \frac{\beta_L^{\text{ref}}}{4N_c b_0} \right] \)

**IR** → \( \Lambda_{\text{ref}} = \exp \left[ \frac{\beta_L^{\text{ref}}}{4N_c b_0} \right] \)

Very rough extrapolation

\( N_f^c = 11(2) \) for \( \beta_L^{\text{ref}} = 2 \)

\( 1.1 < 1/|\theta| < 2.5 \)

\( N_f^c = 9(1) \) for \( \beta_L^{\text{ref}} = 4.0 \)
Inside the Conformal Window
Conformal PT

[Deuzeman, Lombardo, Nunes EP '12]
The Spectrum
Strong coupling dynamics and bulk transitions
Strong coupling dynamics

$S$

$A S$

$N_f$

$1/g^2$
The bulk transition(s)
Symanzik improvement @ strong coupling

Gauge action:

\[ S_G = \beta_0 \text{Re}(1 - U(1 \times 1)) + \beta_1 \text{Re}(1 - U(2 \times 1)) \]

\[ \beta_0 = \frac{5}{3} \beta, \quad \beta_1 = -\frac{1}{12} \beta \]

\[ \beta = \frac{6}{g^2} \]

nearest neighbor  next-to-nearest neighbor

Fermion action:

\[ S_F = \alpha^4 \sum_{x, \mu} \eta_\mu(x) \bar{\chi}(x) \frac{1}{2a} \{ c_1 [U_\mu(x) \chi(x + \mu) - U_\mu^\dagger(x - \mu) \chi(x - \mu)] \]
\[ + c_2 [U_\mu(x) U_\mu(x + \mu) U_\mu(x + 2\mu) \chi(x + 3\mu) \]
\[ - U_\mu^\dagger(x - \mu) U_\mu^\dagger(x - 2\mu) U_\mu^\dagger(x - 3\mu) \chi(x - 3\mu) \} \}

\[ + a^4 m \sum_x \bar{\chi}(x) \chi(x) \]  

\text{Naik term}  
3rd-nn
We know that:

Hermiticity of the Transfer matrix is lost (complex energy eigenvalues)
When and how does it manifest?

Luscher, Weisz ’84

A solvable model: (1d) Ising chain with n-n-n interactions (ANNNI models)

Arisue, Fujiwara ’84
This case:

Naik term modifies the free fermion propagator

\[
S_F(p)^{-1} = \sum_\mu i\gamma_\mu \left( \frac{9}{8} \sin p_\mu - \frac{1}{24} \sin 3p_\mu \right)
\]

modified by strong coupling interactions → ghosts

Baryon number density

\[
n(\mu) = \frac{d}{d\mu} \log Z(\mu) = n_1(\mu) + n_3(\mu)
\]

\[
n(\mu = 0) = 0 \quad \text{in two ways:} \quad \begin{cases} 
    n_1 = n_3 = 0 \\
    n_1 = -n_3 \neq 0
\end{cases}
\]

oscillatory component allowed in Goldstone channel

forward-backward asymmetry allowed

*Plausibly related to $S_4$ ($T=S_4^2$) investigated by Cheng, Hasenfratz, Schaich '12*
Signatures


\[ S_G = \beta_0 \text{Re}(1 - U(1 \times 1)) + \beta_1 \text{Re}(1 - U(2 \times 1)) \]

\[ \beta_0 = \frac{5}{3} \beta, \quad \beta_1 = -\frac{1}{12} \beta \]

\[ \chi_\pi = \frac{\langle \bar{\psi} \psi \rangle}{m} = \int C_\pi(t) \]

Oscillation \implies \text{discontinuity}

\[ \chi_\text{conn} \]

\[ \chi_\pi - \chi_\text{conn} \]
Degeneracy and chiral symmetry
The asymmetry

\[ A \sim C' \left( 1 - (-1)^t \right) \left( e^{-mt} - e^{-m(T-t)} \right) \]

\( C' \approx 1027 \)

\( m \approx 0.62 \)
Remarks

Hermiticity loss of the transfer matrix (complex eigenvalues) is a general property of Symanzik improved gauge theories.

We have found an example where the Naik improvement of the staggered fermion action generates a new phase of the system signalled by a discontinuity of the chiral susceptibility (change of mass slope of the chiral condensate).

The same theoretical analysis is potentially useful for the lattice formulation of strongly coupled systems such as graphene.
AdS/CFT
Disappearance of the CW
Which scenario is realized?

SQCD: duality guarantees that the (electric) theory is infinitely strongly coupled below the CW

FP pair annihilation
see Kaplan et al ‘09

see Kaplan et al ‘09
SQCD and QCD $\beta$-functions

A conformal window for SQCD exists in the region $3/2 \, N_c < N_f < 3N_c$  

$\beta_{g} = -\frac{g^3}{16\pi^2} \frac{3N_c - N_f (1 - \gamma_0)}{1 - \frac{g^2 N_c}{8\pi^2}}$  

SQCD: Seiberg '95

QCD?: Large N limit  

$\beta(g_c) = -\beta_0 \infty \, g_c^3 + \frac{\beta_i}{4} g_c^3 \left( \frac{\partial \log Z}{\partial \log \Lambda} + c_F \frac{g_c^2}{16\pi^2} \right) + c_F \frac{g_c^3}{16\pi^2} \left( 1 + \gamma(g_c^2)/2 \right)$  

Reproduces 2-loop beta in the (perturbative) Veneziano limit  

YM: Bochicchio '08  

Caveat: $\exists$ IRFP also for $N_f=0$ - $g^*$ is RG scheme dependent  

see also Brodsky, Schrock '08
AdS/CFT

UV brane/branch | IR brane/branch

$z=0$ | $z\rightarrow 0$

CFT* | CFT

AdS$_5$ gravity theory

warp factor $\sim 1/z$

(increasing compositeness)

“IR/UV correspondence” $z \rightarrow 0$ IR gravity

$z \rightarrow 0$ UV field theory
An example of FP merging in “modified” SQCD

Large \( N_f, N_c \): \( N_f/N_c \) fixed - SUGRA backgrounds

\[ \Downarrow \]

SQCD + quartic operators

\[ N_f < 2N_c \] \( \text{UV limit: } \beta \to \beta_{\text{NSVZ}}(\gamma_0 = -1/2) \)

\[ N_f = 2N_c \] \( \text{UVFP at strong coupling} \)

\[ N_f > 2N_c \] Seiberg dual (\( N_c \to N_f-N_c, N_f-2N_c \) flips sign)

Maldacena, Nunez '04
Casero Nunez Paredes '08
Conte Gaillard Ramallo '11

Barranco EP Russo '11
Conformal symmetry might play a role in particle physics at or well above the EWSB scale.

Large-Nf QCD is an instructive theory playground

✓ The conformal window opens at around $N_f \sim 12$
✓ The spectrum and the physics of phase transitions provide distinctive signatures of (pre)conformality
✓ A preliminary study shows a change of trend of $T_c$ for $N_f > 6$

Symanzik Improvement in strongly coupled systems can generate new phases. The same considerations apply to non-abelian gauge theories in the conformal window as well as systems such as graphene.

AdS/CFT is in its infancy, but useful and insightful tool, when trying to make connection with SQCD or QCD.
The bulk transition(s)

The bulk transition(s) does not increase with $m$ decreasing.

$N_t$ dependence only for $N_t \leq 12$
Chiral susceptibilities
mass dependence
Bare quark masses span a range 0.01 to 0.07 at various $\beta$ for $N_f=12$
Bare quark masses span a range 0.025 to 0.15 at various $\beta$ for $N_f=16$

Damgaard, Heller, Krasnitz, Olesen 1997
Fodor, Holland, Kuti, Nogradi, Schroeder 2011
Zoom in at Nf=12

Data cover the same dynamical region
This is compatible with a negative $\beta$ function
For a fixed $m_\pi/m_\rho$ the inverted behavior with $\beta_L$ is compatible with a positive $\beta$ function
Pseudo Goldstone mass and chiral condensate

$\langle \bar{\psi} \psi \rangle^2$

$M_{\pi}^2$

curvature opposite to FV corrections

exact chiral symmetry anomalous dimensions $\neq 0$

broken chiral symmetry

Kocic Kogut Lombardo 1993
$N_f=12$: lattice data

Exact chiral symmetry with non zero anomalous dimensions