COMPOSITE DARK MATTER EXCLUSIONS FROM THE LATTICE

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An LSD Production

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Graham Kribs
Sergey Syritsyn

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A slice of the Universe

![Pie chart showing percentages of dark energy, dark matter, and normal matter in the universe.]

- Dark Energy: 75%
- Dark Matter: 21%
- Normal Matter: 4%
A SLICE OF THE UNIVERSE

We Are Here
(QCD, EM, SM, etc.)
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A SLICE OF THE UNIVERSE

New Physics!!

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(QCD, EM, SM, etc.)
A SLICE OF THE UNIVERSE

New Physics!!

We Are Here
(QCD, EM, SM, etc.)

How do we know DM is there?

- Rotation Curves of Galaxies
- Gravitational Lensing
Gravity says it is there but...

...do we have any clue what it is?

- How does it interact with SM?
- How does it interact with itself?
- What is its spin or parity?
- Is it simultaneously matter and anti-matter?
- What is its Mass?

None of these questions have been answered to date...
Gravity says it is there but...

...do we have any clue what it is?

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- What is its spin or parity?
- Is it simultaneously matter and anti-matter?
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None of these questions have been answered to date...

...but there are several clues that can tell quite a bit
Three Primary Properties of dark matter

1. Candidate should be Stable
   - Explain why dark matter has survived to today
     ➡ Implies a new symmetry and/or charge

2. Candidate should be EW Charge Neutral
   - Explain why no visible evidence
     ➡ Implies lightest stable particle is chargeless

3. Candidate should explain observed relic density
   \[ \rho_D \sim 0.2 \rho_c \]

How can this come about?
One approach to DM theories:

Choose DM Mass
Choose DM Interactions

"WIMP Miracle"

Assume Interactions at/near EW Scale

$\rho_D \sim 0.2 \rho_c$

$M_D \sim \text{TeV}$
Observe a different relation:

\[ \rho_D \sim 5\rho_B \]
\[ M_D n_D \sim 5M_B n_B \]
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If DM density is thermal: Unjustified Accident
Observe a different relation:

\[ \rho_D \sim 5 \rho_B \]

\[ M_D n_D \sim 5 M_B n_B \]

If DM density is thermal: Unjustified Accident

Natural if DM density is also tied to asymmetry

\[ n_D \sim n_B \quad \Rightarrow \quad M_D \sim 5 \text{ GeV} \]

\[ M_D \gg M_B \quad \Rightarrow \quad n_B \gg n_D \sim e^{-M_D/T_{sph}} \]

Sphaleron connection

Direct or Indirect coupling to EW
Thermal vs. asymmetric

However:

Asymmetric relic density suggests negligible thermal abundance

### Diagram

- **Small Thermal Abundance** → **Large Annihilation Rate** → **Strong Couplings**

Tricky to achieve for perturbative, elementary DM

**Strongly-coupled composite theories most interesting...**

...this is where the lattice can play significant role!
1. Candidate should be Stable
   ➔ Implies a new symmetry and/or charge
   Example: Baryons - Baryon Number
             Mesons - G-parity
             Y.Bai, R.J.Hill (2010)

2. Candidate should be EW Charge Neutral
   ➔ Implies lightest stable particle is chargeless
   Example: Can form neutral baryons

3. Candidate should explain observed relic density
   ➔ Asymmetry/sphalerons require charge couplings
   Example: Charged Constituents
Before asking any other question, what models are excluded already by DD?

- Lattice simulates “vectorlike” theories
  - Directly addresses question in this case

- Lattice input can be built into theories with “chiral” couplings (Higgs, etc.)
  - DM phenomenologists can build theories using lattice results
Direct Detection Limits

Two observational channels:

1. Spin-independent (coherent) - Very tight constraints
\[ \sigma \lesssim 10^{-45} \text{ cm}^2 \]

2. Spin-dependent - Much weaker constraints
\[ \sigma \lesssim 10^{-37} \text{ cm}^2 \]

Xenon100 - arXiv:1207.5988
Assume a Dirac particle with net Z-boson charge

\[ \sigma_{SI} \approx \frac{2}{\pi} G_F^2 m_N^2 \frac{\bar{N}^2}{A^2} \approx \frac{\bar{N}^2}{A^2} (3 \times 10^{-38} \text{ cm}^2) \]

Current spin-independent bounds: \[ \sigma \lesssim 10^{-45} \text{ cm}^2 \]

Excludes particles of this kind to masses greater than thousands of TeV

Neutralinos avoid this: Majorana \[ \rightarrow \] Spin-Dependent

This will plague composites with odd numbers of EW doublets!
Technicolor Implications?

Reminder: Want lightest stable particle to be chargeless

If even number of colors: Not Difficult

Example:
Nc (massless) EW doublets
All charge assignments cancel

If odd number of colors: Very Difficult

Nc (massless) EW doublets  →  Net Charge
(Nc-1) (massless) EW doublets
1 massive EW singlet  →  No Charge

Our work will focus on odd Nc “vectorlike” theories that do not mediate EW breaking
### How we might see it?

<table>
<thead>
<tr>
<th>Dim-5</th>
<th>Dim-6</th>
<th>Dim-7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$</td>
<td>$(\bar{\psi}\psi)\nu_\mu \partial_\nu F^{\mu\nu}$</td>
<td>$(\bar{\psi}\psi) F_{\mu\nu} F^{\mu\nu}$</td>
</tr>
</tbody>
</table>

**Magnetic Moment**

- Odd $N_c$
  - No baryon flavor sym.

- Odd $N_c$
  - Baryon flavor sym.

- Even $N_c$
  - No Baryon flavor sym.

- Even $N_c$
  - Baryon flavor sym.

**Charge Radius**

- Odd $N_c$
  - Baryon flavor sym.

- Even $N_c$
  - Baryon flavor sym.

**Polarizability**

- Odd $N_c$
  - Baryon flavor sym.

- Even $N_c$
  - Baryon flavor sym.
Cross-Section Calc.
\[ |\mathcal{M}|^2 = \frac{e^4}{Q^4} \mathcal{L}_A^{\mu\nu} \mathcal{L}_B^{\mu\nu} \]
\[ \mathcal{L}_X^{\mu\nu} = \frac{1}{N_X} \sum_{X,X'} \langle X | J_{em}^\mu | X' \rangle \langle X' | J_{em}^\nu | X \rangle \]
Cross-Section Calc.

\[ |\mathcal{M}|^2 = \frac{e^4}{Q^4} \mathcal{L}_A^{\mu\nu} \mathcal{L}_B^{\mu\nu} \]

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Spin-0:

\[ \mathcal{L}_X^{\mu\nu} = 4F^2(Q^2) \bar{p}^\mu \bar{p}^\nu \]

\[ \bar{p}^\mu = \frac{1}{2}(p' + p)^\mu \]
\[ |\mathcal{M}|^2 = \frac{e^4}{Q^4} \mathcal{L}^\mu_\nu^A \mathcal{L}^\mu_\nu^B \]

\[ \mathcal{L}^\mu_\nu_X = \frac{1}{N_X} \sum_{X,X'} \langle X|J^\mu_{em} X'\rangle \langle X'|J^\nu_{em} X\rangle \]

**Spin-0:** \[ \mathcal{L}^\mu_\nu_X = 4F^2(Q^2)\vec{p}^\mu \vec{p}^\nu \]

**Spin-1/2:** \[ \mathcal{L}^\mu_\nu_X = 4\vec{p}^\mu \vec{p}^\nu (F^2_{1X} + \frac{Q^2}{4M^2}F^2_{2X}) - (Q^2g^\mu_\nu + q^\mu q^\nu)(F^2_{1X} + F^2_{2X}) \]

\[ \vec{p}^\mu = \frac{1}{2}(p' + p)^\mu \]
\[ |\mathcal{M}|^2 = \frac{e^4}{Q^4} \mathcal{L}_A^{\mu\nu} \mathcal{L}_B^{\mu\nu} \]

\[ \mathcal{L}_X^{\mu\nu} = \frac{1}{N_X} \sum_{X,X'} \langle X | J_{em}^\mu | X' \rangle \langle X' | J_{em}^\nu | X \rangle \]

**Spin-0:**
\[ \mathcal{L}_X^{\mu\nu} = 4F^2(Q^2)\bar{p}^\mu \bar{p}^\nu \]
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**Spin-1/2:**
\[ \mathcal{L}_X^{\mu\nu} = 4\bar{p}^\mu \bar{p}^\nu (F_{1X}^2 + \frac{Q^2}{4M^2} F_{2X}^2) - (Q^2 g^{\mu\nu} + q^\mu q^\nu)(F_{1X} + F_{2X})^2 \]

**Large disgusting nucleus:**
\[ \mathcal{L}_X^{\mu\nu} = 4W_{2X}(Q^2, q \cdot p)\bar{p}^\mu \bar{p}^\nu - W_{1X}(Q^2, q \cdot p)(Q^2 g^{\mu\nu} + q^\mu q^\nu) \]
Cross-Section Calc.


\[
\frac{d\sigma^\text{SI}_{MM}}{dE_R} = \frac{\pi (2\kappa\alpha)^2 Z^2}{4(m_D + m_N)^2 E_R^{\text{max}}} \left( \frac{(m_D + m_N)^2}{m_D^2} \frac{E_R^{\text{max}}}{E_R} - \frac{2m_N}{m_D} - 1 \right) |F_c(E_R)|^2
\]

\[
\frac{d\sigma^\text{SI}_r}{dE_R} = \frac{16\pi (Z\alpha)^2 Z^2 m_N^2 m_D^2 r^4}{(m_D + m_N)^2 E_R^{\text{max}}} |F_c(E_R)|^2
\]

\[
E_R^{\text{max}} = \frac{2m_N m_D^2 v^2}{(m_D + m_N)^2}
\]

*Non-perturbative lattice input*
Cross-Section Calc.


\[ \frac{d\sigma_{MM}^{SI}}{dE_R} = \frac{\pi (2\kappa\alpha)^2 Z^2}{4(m_D + m_N)^2 E_R^{\text{max}}} \left( \frac{(m_D + m_N)^2}{m_D^2} \frac{E_R^{\text{max}}}{E_R} - \frac{2m_N}{m_D} - 1 \right) |F_c(E_R)|^2 \]


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\[ E_R^{\text{max}} = \frac{2m_N m_D^2 v^2}{(m_D + m_N)^2} \]

*Non-perturbative lattice input

\[ \sigma^{SI} = \int dv f(v) \int_{E_{\text{min}}^{N}}^{E_{\text{max}}^{N}} dE_R \frac{d\sigma^{SI}}{dE_R} \]

Xenon100:

\[ E_{\text{min}}^{Xe} = 6.6 \text{ keV} \]

\[ E_{\text{max}}^{Xe} = 30.5 \text{ keV} \]
Three-Point Operators

\[ \langle N(p')|\bar{q}(x)\gamma^\mu q(x)|N(p)\rangle = \bar{u}(p') \left[ \gamma^\mu F_1(q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2M_B} F_2(q^2) \right] u(p) e^{i\mathbf{q}\cdot\mathbf{x}} \]

\[ F_1(q^2) = 1 + \frac{1}{6}\langle r^2 \rangle q^2 + \cdots \]

\[ F_2(q^2) = \kappa + \frac{1}{6}\langle \tilde{r}^2 \rangle q^2 + \cdots \]

Isovector: \[ \langle N|\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d|N\rangle \]

Isoscalar: \[ \langle N|\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d|N\rangle \]

“Neutron”: \[ \langle N|\frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d|N\rangle \]
Three-Point Operators

Correlation Functions via path integral:

\[ C_\mathcal{O} = \langle \mathcal{O} \rangle = \int d[U] \mathcal{O} \det(D_{\text{lat}}(U))e^{-S_G(U)} \]

\[ C_{NN}(\tau, p) = \sum_x e^{-ip \cdot x} \langle \overline{N}(x, \tau)N(0) \rangle \]

\[ C_{N\mathcal{O}N}(\tau, \tau_0, p, p') = \sum_{x, y} e^{-ip' \cdot x + i(p' - p) \cdot y} \langle \overline{N}(x, \tau_0)\mathcal{O}(y, \tau)N(0) \rangle \]

Taking the appropriate ratio:

\[ R_\mathcal{O}(\tau, \tau_0, p, p') \equiv \tau_0 \stackrel{\tau}{\longrightarrow} 1 \langle n(p')|\mathcal{O}|n(p') \rangle + \mathcal{O}(e^{-\Delta \tau}) + \mathcal{O}(e^{-\Delta(\tau_0-\tau)}) \]
Three-Point Calculation

Propagator Contractions:

\[
\bar{q}_{i'}^{\alpha'}(y) q_i^{\alpha}(x) = S_{i' i}^{\alpha' \alpha}(y, x)
\]

\[C_{NN}(\tau, p)\]

\[t = 0 \quad t = \tau\]

1 Propagators \quad One measurements
Three-Point Calculation

$t = 0 \quad t = \tau \quad t = \tau_0$

2 Propagators $\Rightarrow$ One measurements
One time insertion

Need:

$\tau$ & $\tau_0$ Large $\Rightarrow$ Reduce Excited States
$\tau$ & $\tau_0$ Not too large $\Rightarrow$ Signal-to-noise reduction
Three-Point Calculation

\[ t = 0 \quad t = \tau \quad t = \tau_0 \]

2 Propagators \[ \rightarrow \]
One measurements
One time insertion

Isovector: (No disconnected diagrams)

\[ \bar{u}\gamma^\mu u \quad - \quad \bar{d}\gamma^\mu d \]
Three-Point Calculation

\[ t = 0 \quad \quad t = \tau \quad \quad t = \tau_0 \]

2 Propagators \rightarrow One measurements
One time insertion

Isoscalar: (Need disconnected diagrams)

\[ \bar{u} \gamma^\mu u \quad + \quad \bar{d} \gamma^\mu d \]

Omit in current calculation
Three-Point Calculation

\[ t = 0 \quad t = \tau \quad t = \tau_0 \]

2 Propagators \rightarrow One measurements
One time insertion

Transverse charge density:

(© courtesy of J. Wasem)
How do we define lattice spacing in physical units?

**Lattice QCD:**

Hadron Masses, HQ potentials, etc.

\[ a M_\Omega = \# \quad \Rightarrow \quad a \approx \frac{\#}{1670 \text{ MeV}} \]

**Technicolor:**

“Higgs” vev

\[ a f_\pi \xrightarrow{m_f \to 0} \# \quad \Rightarrow \quad a \approx \frac{\#}{246 \text{ GeV}} \]

**Dark Matter:**

Dark Matter Mass

\[ a M_B = \# \quad \Rightarrow \quad a \approx \frac{\#}{M_B} \]
Scale Setting

How do we define lattice spacing in physical units?

**Lattice QCD:**

Hadron Masses, HQ potentials, etc.

\[ aM_\Omega = \# \quad \rightarrow \quad a \approx \frac{\#}{1670 \text{ MeV}} \]

**Technicolor:**

“Higgs” vev

\[ af_\pi m_f \rightarrow 0 \quad \rightarrow \quad a \approx \frac{\#}{246 \text{ GeV}} \]

**Dark Matter:**

Dark Matter Mass

\[ aM_B = \# \quad \rightarrow \quad a \approx \frac{\#}{M_B} \]

Vary this value
Calculation Details

10 DWF Ensembles:
- $32^3 \times 64 \times 16$ lattices

$am_\rho \sim \frac{1}{5}$

2 flavor: $m_f = 0.010 - 0.030$

6 flavor: $m_f = 0.010 - 0.030$

Table 1: 2 Flavor

<table>
<thead>
<tr>
<th>$m_q$</th>
<th># Configs</th>
<th># Meas</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>564</td>
<td>1128</td>
</tr>
<tr>
<td>0.015</td>
<td>148</td>
<td>296</td>
</tr>
<tr>
<td>0.020</td>
<td>131</td>
<td>262</td>
</tr>
<tr>
<td>0.025</td>
<td>67</td>
<td>268</td>
</tr>
<tr>
<td>0.030</td>
<td>39</td>
<td>154</td>
</tr>
</tbody>
</table>

Table 1: 6 Flavor

<table>
<thead>
<tr>
<th>$m_q$</th>
<th># Configs</th>
<th># Meas</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>221</td>
<td>442</td>
</tr>
<tr>
<td>0.015</td>
<td>112</td>
<td>224</td>
</tr>
<tr>
<td>0.020</td>
<td>81</td>
<td>162</td>
</tr>
<tr>
<td>0.025</td>
<td>89</td>
<td>267</td>
</tr>
<tr>
<td>0.030</td>
<td>72</td>
<td>259</td>
</tr>
</tbody>
</table>
Baryon Mass

Red - 2 Flavor
Blue - 6 Flavor

Feynman-Hellmann:
\[ \sigma = M_q \frac{\partial M_B}{\partial M_q} \]
Axial Charge

Red - 2 Flavor
Blue - 6 Flavor

Preliminary
Magnetic Moment

Isovector

Red - 2 Flavor
Blue - 6 Flavor

“Neutron”

Isoscalar

\[ \mu = \frac{\kappa}{2M_B} \]
**Charge Radius**

**Isovector**

Red - 2 Flavor

Blue - 6 Flavor

“Neutron”

\[
\langle r^2 \rangle = \frac{1}{V} \int d^3r \, \rho(r)r^2
\]

**Isoscalar**

Preliminary
Exclusion Plots

Red - 2 Flavor
Blue - 6 Flavor
Dashed - Xenon100

arXiv: 1207.5988
Future Directions

1) Composites of different color

- Even colors most interesting

   Naturally avoids current constraints

Phenomenology:

- Baryon group theory (possible FFs)
- Chargeless Combinations
- Flavor Symmetric baryons?

Lattice:

- Focus on 2 and 4 color (4 color more natural)
- Efficient codes, map parameter space, etc.
Future Directions

2) Extracting polarizabilities

- Dominant effect for flavor sym. baryons
  
  **Operator insertion methods not ideal**

Background Field Method:

\[ E \sim M + \frac{1}{2} \alpha \mathcal{E}^2 + \cdots \]

**Small fields require large volumes**

Lattice:

- Integrate background field into Dslash inversion
- Dynamical background fields
3) Examine new hierarchy of masses

Larger fermion mass

Fewer light particles need to be explained
3) Composite DM with Scalar Constituents

Avoids bounds...
avoids simulating fermions...
Based purely on observational DM data:

- Composite dark matter is the most “natural”

Lattice can address place initial bounds on models
- Tight constraints on odd Nc theories
- Will explore charge radii and polarizabilities of even Nc theories

Lattice and phenomenology go hand-in-hand
- Build in perturbative “chiral” sector
- Define viable theories and study possible cosmological observations
This research was supported by the LLNL LDRD “Illuminating the Dark Universe with PetaFlops Supercomputing” 13-ERD-023 and by the LLNL Multiprogrammatic and Institutional Computing program through the Tier 1 Grand Challenge award that has provided us with the large amounts of necessary computing power.