Exploring for walking technicolor from QCD

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for the LatKMI collaboration

- Lattice meets experiment 2012 @ Boulder -

Oct. 27, 2012
LatKMI collaboration

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E. Rinaldi, K. Yamawaki, T. Yamazaki

A. Shibata
“Higgs boson”

- Higgs like particle fund at LHC
- $m_H = 126 \text{ GeV}$
- spin, parity, other properties are under investigation
- so far consistent with Standard Model Higgs ($J^{PC}_{\text{PC}}=0^{++}$) fundamental scalar
- but it could be different
- one of the possibilities
  - walking technicolor
    - “Higgs” = pNGB due to breaking of the approximate scale invariance
requirements for model

• nearly conformal: walking

• $\gamma_m \sim 1$

• input: $F = 246 / \sqrt{N}$ GeV
  • $N$: # weak doublet from new techni-sector

• could $m_H$ (0++) be made light: $\sim 126$ GeV
models being studied:

- SU(3)
  - fundamental: Nf=6, 8, 10, 12, 16
  - sextet: Nf=2
- SU(2)
  - adjoint: Nf=2
  - fundamental: Nf=8
- SU(4)
  - decuplet: Nf=2

![SU(N) Phase Diagram](image)

The four bands represent respectively fermions in the fundamental $\text{Fund}$, adjoint $\text{Adj}$, and two-index symmetric and antisymmetric $\text{2S}$, $\text{2A}$ representations. The upper limit of each band corresponds to the number of flavors where asymptotic freedom is lost, as obtained from one-loop perturbative computations. The lower limit of each band yields the number of flavors above which the theories develop an IR fixed point. The location of these lower limits relies upon assumptions about the nonperturbative dynamics of the theories. Lattice simulations can provide first-principle evidence in favor or against this picture and compute the critical exponents that characterize the fixed points. Figure courtesy of F. Sannino.

2. Tools

Numerical tools that were originally designed for investigating lattice QCD have been used in order to identify the existence of IRFPs. We describe briefly the main ideas, the observables that are used in the different approaches, and their expected behavior in the presence of an IRFP. For each case we try to emphasize the sources of systematic errors that need to be kept under control in order to draw robust conclusions from numerical data.
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  - fundamental: \(N_f=6, 8, 10, 12, 16\)
  - sextet: \(N_f=2\)
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  - adjoint: \(N_f=2\)
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  - adjoint: Nf=2
  - fundamental: Nf=8

- SU(4)
  - decuplet: Nf=2
SU(3) + N_f=12 [fundamental]

[LatKMI collab. PRD86 (2012) 054506]
Hadron spectrum:

$m_f$-response in mass deformed theory

- IR conformal phase:
  - coupling runs for $\mu < m_f$: like $n_f=0$ QCD with $\Lambda_{QCD} \sim m_f$
  - multi particle state: $M_H \propto m_f^{1/(1+\gamma_m^*)}$; $F_\pi \propto m_f^{1/(1+\gamma_m^*)}$ (criticality @ IRFP)
  - ratio of the masses, decay constant is constant as function of $m_f$

- $S\chi$ SB phase:
  - ChPT (but, large $N_f$, small $F$ $\Leftrightarrow$ real QCD)
    - hard to get to the chiral regime
  - at leading: $M_\pi^2 \propto m_f$; $F_\pi = F + c m_f$
Simulation

- HISQ (Highly Improved Staggered Quarks)
  - being used for state-of-the-art QCD calculations / MILC,..

- tree level Symanzik gauge

\[ \beta = \frac{6}{g^2} = 3.7, \quad V = L^3 \times T: \frac{L}{T} = \frac{3}{4}; \quad L = 18, 24, 30, \quad 0.04 \leq m_f \leq 0.2 \]

\[ \beta = \frac{6}{g^2} = 4.0, \quad V = L^3 \times T: \frac{L}{T} = \frac{3}{4}; \quad L = 18, 24, 30, \quad 0.05 \leq m_f \leq 0.24 \]

- \( N_f = 4 \) HISQ for the reference of \( S \chi SB \) for comparison

- using MILC code v7 with some modifications (non-rational HMC)
staggered flavor symmetry for $N_f=12$ HISQ

- comparing masses with different staggered operators for $\pi$ & $\rho$ for $\beta=3.7$

![Graph showing effective mass of both operators at $\beta=3.7$, $M$ vs. $m_f$. The graph includes data points for PS, PV, VT, and SC.]

- excellent staggered flavor symmetry, thanks to HISQ

---

a crude analysis: $F_\pi/M_\pi$ vs $M_\pi$

**N_f=12: HISQ**

**N_f=4: HISQ $\beta=3.7$**
a crude analysis: $F_\pi/M_\pi$ vs $M_\pi$

$N_f=12$: HISQ

$N_f=4$: HISQ $\beta=3.7$

- $\beta=3.7$: small mass: consistent with hyper-scaling
a crude analysis: $F_\pi/M_\pi$ vs $M_\pi$

$N_f=12$: HISQ

$N_f=4$: HISQ $\beta=3.7$

- $\beta=3.7$: small mass: consistent with hyper-scaling
- $\beta=4.0$: volume to small ? unlikely in the hyper-scaling region

In the following sections, further detailed study using these data are performed. From the observation here, we note that the only the smaller mass data would qualify the hyper-scaling test if there is any, if only a leading mass dependence is taken into account. Further, the difference of the constant is made possible due to a discretization effect.
a crude analysis: $M_ρ/M_π$ vs $M_π$

$N_f=12$: HISQ
a crude analysis: $M_\rho/M_\pi$ vs $M_\pi$

$N_f=12$: HISQ

flat: $\beta=3.7$

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a crude analysis: $M_\rho/M_\pi$ vs $M_\pi$

$N_f=12$: HISQ

- flat: $\beta=3.7$
- flat: $\beta=4.0$

- $\beta=3.7$ & 4.0: small mass (wider than $F_\pi$): consistent with hyper scaling (HS)
a crude analysis: $M_\rho/M_\pi$ vs $M_\pi$

$N_f=12$: HISQ

- $\beta=3.7$ & 4.0: small mass (wider than $F_\pi$): consistent with hyper scaling (HS)
- mass dependence at the tail is due to non-universal mass correction to HS
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one may attempt to perform a matching

assuming $(am)^2$ error is small
a crude analysis: $M_\rho/M_\pi$ vs $M_\pi$

$N_f=12$: HISQ

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$\Rightarrow a(\beta=3.7)/ (\beta=4.0) > 1$

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$N_f=12$: HISQ

- one may attempt to perform a matching
- assuming $(am)^2$ error is small
- $a(\beta=3.7) / (\beta=4.0) > 1$
- movement: correct direction in asymptotically free domain!

- $\beta=3.7 \ & \ 4.0$: small mass (wider than $F_\pi$): consistent with hyper scaling (HS)
- mass dependence at the tail is due to non-universal mass correction to HS
conformal (finite size) scaling

• Scaling dimension at IR fixed point [Wilson-Fisher]; Hyper Scaling [Miransky]

• mass dependence is described by anomalous dimensions at IRFP
  • quark mass anomalous dimension $\gamma^*$
  • operator anomalous dimension

• hadron mass and pion decay constant obey same scaling
  \[ M_H \propto m_f^{1+\gamma^*} \quad F_\pi \propto m_f^{1+\gamma^*} \]

• finite size scaling in a $L^4$ box (DeGrand; Del Debbio et al)
  • scaling variable: \[ x = Lm_f^{1+\gamma^*} \]
  \[ L \cdot M_H = f_H(x) \quad L \cdot F_\pi = f_F(x) \]
$N_f=12$ see if data align at some $\gamma$

$$x = Lm^\frac{1}{1+\gamma}$$
$N_f=4$ see if data align at some $\gamma$

\[ x = L m_f^{\frac{1}{1+\gamma}} \]
$N_f=4$ see if data align at some $\gamma$

$$M\pi L \propto m_f^{1/2} L$$

$$x = L m_f^{1+\gamma}$$

FIG. 7.

FIG. 8.
$N_f=4$ see if data align at some $\gamma$
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measure of the “alignment”
without resorting to a model
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- $\gamma$ of optimal alignment will minimize:

$$P_p(\gamma) = \frac{1}{N} \sum_K \sum_{j \notin K} \frac{|\xi_p^j - f_p^{(K)}(x_j)|^2}{\delta^2 \xi_p^j},$$

- $\xi_p = LM_p$ for $p=\pi, \rho$; $\xi_F = LF_\pi$

- $f_p(x)$: interpolation .... linear
  - (quadratic for a systematic error)

- if $\xi^j$ is away from $f(x_i)$ by $\delta \xi^j$ as average $\rightarrow P=1$

- optimal $\gamma$ from the minimum of $P$

- similar definition of the measure: DeGrand, Giedt & Weinberg
measure of the “alignment” without resorting to a model

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- similar definition of the measure: DeGrand, Giedt & Weinberg

- systematic error due to small $L$, large $m$ estimated by examining the $x$ and $L$ range dependence

\[ P(\gamma) \text{ for } M_\pi, F_\pi, M_\rho \text{ at } \beta = 3.7 \]
TABLE VII. Summary of the optimal values of $\gamma$. See the text for details.

<table>
<thead>
<tr>
<th>quantity</th>
<th>$\beta$</th>
<th>all</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_\pi$</td>
<td>3.7</td>
<td>0.434(4)</td>
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<td>$F_\pi$</td>
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<td>$M_\pi$</td>
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<td>0.425(9)</td>
<td>0.436(6)</td>
<td>0.437(4)</td>
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<td>$F_\pi$</td>
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<td>$M_\rho$</td>
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• $\beta=3.7$: smaller m: closer to $M_\pi$

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<tr>
<td>$M_\pi$</td>
<td>4</td>
<td>0.414(5)</td>
<td>0.420(7)</td>
<td>0.418(6)</td>
<td>0.411(5)</td>
<td>0.397(7)</td>
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<td>$F_\pi$</td>
<td>4</td>
<td>0.580(15)</td>
<td>0.552(21)</td>
<td>0.602(20)</td>
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<td>0.544(27)</td>
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- $\beta=3.7$: smaller $m$: closer to $M_\pi$
- $\beta=3.7$: larger $V$: closer to $M_\pi$
- $\beta=4.0$: not conclusive: possibly due to large $m \rightarrow$ take variation as sys. err.
summary of $\gamma$ obtained by minimizing $P(\gamma)$
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- $\gamma$: consistent with 2 $\sigma$ level except for $F_\pi$ at $\beta=4.0$
A summary of $\gamma$ obtained by minimizing $P(\gamma)$:

- $\gamma$: consistent with 2 $\sigma$ level except for $F_\pi$ at $\beta=4.0$

- Remember: $F_\pi$ at $\beta=4.0$ speculated to be out of the scaling region
summary of $\gamma$ obtained by minimizing $P(\gamma)$

- $\gamma$: consistent with 2 $\sigma$ level except for $F_\pi$ at $\beta=4.0$
- remember: $F_\pi$ at $\beta=4.0$ speculated to be out of the scaling region
- universal low energy behavior: good with $0.4<\gamma<0.5$
Conformal type global fit with finite volume correction

\[ \xi = L M_\pi, \ L F_\pi, \ L M_\rho \]

\[ \xi = c_0 + c_1 L m_f^{1/(1+\gamma)} \cdots \text{fit a}, \]

\[ \xi = c_0 + c_1 L m_f^{1/(1+\gamma)} + c_2 L m_f^\alpha \cdots \text{fit b}. \]

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( \alpha )</th>
<th>( \chi^2/\text{dof} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit a</td>
<td>0.449(3)</td>
<td>-</td>
</tr>
<tr>
<td>fit b-1</td>
<td>0.411(9)</td>
<td>( \frac{(3-2\gamma)}{(1+\gamma)} )</td>
</tr>
<tr>
<td>fit b-2</td>
<td>0.423(7)</td>
<td>[2]</td>
</tr>
</tbody>
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- simultaneous fit it with a leading mass dependent correction is not bad

- b-1: Ladder Schwinger-Dyson,  b-2: \( (am)^2 \) lattice artifact
  - [see, LatKMI PRD85(2012)074502]

- resulting \( \gamma \) is consistent with the model independent analysis

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ChPT fit (after infinite volume extrapolation)

\[ h(m_f) = c_0 + c_1 m_f + c_2 m_f^2 \]

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<th>fit range</th>
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<td>fit 1 : [0.04, 0.08]</td>
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<td>1.21(18)</td>
<td>-2.2(1.5)</td>
<td>0.29</td>
</tr>
<tr>
<td>fit 1 : [0.04, 0.1]</td>
<td>0.0162(30)</td>
<td>1.31(85)</td>
<td>-3.01(58)</td>
<td>0.37</td>
</tr>
<tr>
<td>fit 1 : [0.04, 0.12]</td>
<td>0.0231(18)</td>
<td>1.093(48)</td>
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<td>3.30</td>
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- 2nd order polynomial fit is reasonably good for small mass range & \( c_0 > 0 \)
ChPT fit (after infinite volume extrapolation)

$$h(m_f) = c_0 + c_1 m_f + c_2 m_f^2$$

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\[ h(m_f) = c_0 + c_1 m_f + c_2 m_f^2 \]

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<tr>
<td>fit 1: [0.04, 0.08]</td>
<td>-0.0057(91)</td>
<td>1.82(32)</td>
<td>15.2(2.6)</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td>[0]</td>
<td>1.62(3)</td>
<td>16.76(45)</td>
<td>0.88</td>
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ChPT fit (after infinite volume extrapolation)

$h(m_f) = c_0 + c_1 m_f + c_2 m_f^2$

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- consistent with $c_0=0$ for the smallest mass range
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- consistent with \( c_0 = 0 \) for the smallest mass range
- But: \( N_f[M_\pi/(4\pi F)]^2 \sim 40 \) at lightest point → difficult to tell real chiral behavior
\( N_f=12 \) Summary


- \( \beta=3.7, \ 4.0 \): consistent with being in the asymptotically free regime
- \( M_\pi, F_\pi, M_\rho \): consistent with the finite size hyper scaling for conformal theory
- resulting \( \gamma^* \) from different quantities, lattice spacings are consistent except
  - \( F_\pi \) at \( \beta=4.0 \) (\( m_f \) likely too heavy for universal mass dep. to dominate)
- careful continuum scaling required to get more accurate than \( 0.4<\gamma^*<0.5 \)
- real / remnant (approximate) conformal property definitely exists
- could not exclude \( S \chi SB \) with very small breaking scale
- even if \( S \chi SB, \gamma_m \) too small for walking theory of phenomenological interest
- \( N_f<12 \) should be examined for the quest of the walking technicolor theory
$SU(3) + N_f=8$ [fundamental]

examined with same setup / method
candidate of the walking technicolor?
[preliminary]

[LatKMI collab., Lattice2011/2012]
$M_\pi^2$ vs. mf

$N_f=8, \beta=3.8$
$f_\pi$ vs. $mf$

$N_f=8, \beta=3.8$
$M_\rho$ vs. mf

Nf=8, $\beta=3.8$
hyperscaling test $m_\pi$
hyperscaling test $m_\pi$

$r = 0.5$

good alignment
hyperscaling test $f_{\pi}$
hyperscaling test $f_\pi$

\[ \gamma = 0.0 \]
\[ \gamma = 0.25 \]
\[ \gamma = 0.5 \]
\[ \gamma = 0.75 \]
\[ \gamma = 1 \]
\[ \gamma = 1.25 \]

\[ \gamma = 1 \] good alignment
$P(\gamma)$ analysis

\begin{align*}
\text{quantity} & & \gamma \\
M_\pi & & 0.596(7) \\
f_\pi & & 0.917(11) \\
M_\rho & & 0.741(34)
\end{align*}

$N_f=8$
**$P(\gamma)$ analysis**

![Graphs showing $P(\gamma)$ for $N_f=8$ and $N_f=12$](image)

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As we cannot completely resolve these trends in the mass dependence, we regard these variations of $\gamma$ with respect to the change of the window as the systematic error on the central value of $\gamma$ obtained with "all" data. We put the asymmetric error for both $x$ and $L$ directions separately estimated by the maximum variations from the central value. The $21/\sigma$ is not shown because the error bars are smaller than the size of the symbols.
**$P(\gamma)$ analysis**

![Graph](image1.png)

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$N_f=8$

![Graph](image2.png)

$N_f=12$

2012年10月27日土曜日
$N_f=8$ [preliminary] Summary
N_f=8 [preliminary] Summary

- likely: f π ≠ 0, m ρ ≠ 0 for m_f → 0

- no common optimal γ → suggesting no exact conformality

- γ (expected to be approximate) larger than N_f=12, promising.
\( N_f=8 \) [preliminary] Summary

- likely: \( f_\pi \neq 0, \ m_\rho \neq 0 \) for \( m_f \to 0 \)

- no common optimal \( \gamma \to \) suggesting no exact conformality

- \( \gamma \) (expected to be approximate) larger than \( N_f=12 \), promising.

- candidate of walking?
N_f=8 [preliminary] Summary

• likely: f_π≠0, m_ρ≠0 for m_f→0

• no common optimal γ → suggesting no exact conformality

• γ (expected to be approximate) larger than N_f=12, promising.

• candidate of walking ?

• needs further study!
0++  glueball spectrum

[VERY preliminary]
0++ glueball
0++ glueball

- could a WTC model produce light 0++?
0++ glueball

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• promising results from a model in the conformal window: SU(2) + 2 adjs
0++ glueball

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  - Del Debbio et al [PRD82 (2010) 014510]: m_f≠0, 0++ glueball lighter than pion
0++ glueball

• could a WTC model produce light 0++?

• promising results from a model in the conformal window: SU(2) + 2 adjs
  • Del Debbio et al [PRD82 (2010) 014510]: mf≠0, 0++ glueball lighter than pion

• test for SU(3) nf=12 (consistent with conformal) underway...
SU(3) $N_f=12$, $0^{++}$ techni-glueball [preliminary]

- effective mass from variational method (e.g. E. Gregory et al arXiv:1208.1858)

- $0^{++}$ techni-glueball is righter than techni-pion @ $m_f=0.06$

- but...
SU(3) $N_f=12$, $0^{++}$ techni-glueball [preliminary]

$m_f=0.06$

Scalar glueball normalized correlator: $\beta=4.0$ $am_f=0.06$
SU(3) $N_f=12$, $0^{++}$ techni-glueball [preliminary]

$m_f=0.06$

Scalar glueball normalized correlator: $\beta=4.0$ $am_f=0.06$

Graph showing $C(t)/C(0)$ versus $t/a$ for $L=18$ and $L=24$. Another graph showing $M_\pi$ versus $L$ for different values of $0.05$, $0.06$, $0.08$, $0.1$, $0.12$, $0.16$, and $0.2$. The $y$-axis is labeled $M_\pi$ with values ranging from 0.001 to 1.0, and the $x$-axis is labeled $L$ with values ranging from 15 to 35.
SU(3) $N_f=12$, $0^{++}$ techni-glueball [preliminary]

$m_f=0.08$

Scalar glueball normalized correlator: $\beta=4.0$ am$_f=0.08$

- finite volume effect needs to be carefully studied...
Outlook

• continue for SU(3) $N_f=8, 12$

• underway / planned / wish list for both $N_f=12 / 8$

  • lighter mass

  • more hadrons

  • glueball: study of finite volume effects

  • isosinglet scalar

  • and more...
Thank you for your attention
ChPT inspired infinite volume limit \((\beta=3.7)\)

\[
M_\pi(L) - M_\pi = c_{M_\pi} \frac{e^{-LM_\pi}}{(LM_\pi)^{3/2}}
\]

- ChPT type finite volume effect \(\rightarrow\) chiral fit results not inconsistent with \(S\chi SB\)