Review

Announcements:

• All homework solutions are available on D2L
• Final exam is **Monday, Dec. 6** at **7:30pm** in this room.
• People requiring extra time will start at 6:15pm – HUMN 186
• Solutions to the final exam will be on D2L by Tuesday afternoon.
• There will not be class on Friday – setting aside time for you to stop by, correct scores, ask questions.
Special Relativity

Postulates of relativity:

1. All physical laws are the same in all inertial reference frames
2. The speed of light is the same in all inertial reference frames

Relativity of simultaneity: events which are simultaneous in one frame may not be in another; even the order may be switched.

Proper time $\Delta t_0$ between two events is the time measured in the frame in which both events occur at the same location.

Proper length $L_0$ of an object is the length measured in the rest frame of the object.

Time dilation (moving clocks run slower): $\Delta t = \gamma \Delta t_0$

Length contraction (moving objects are shorter in the direction they are traveling): $L = L_0 / \gamma$

http://www.colorado.edu/physics/phys2170/
Jimmy sees two lightning flashes and determines that one happens 10 ns before the other. Which statement is true:

A. Another inertial observer must find the order of events, the location of events, and the time difference to be the same.

B. Another inertial observer must find the order of events to be the same but the location and time difference may be different.

C. Another inertial observer must find the order of events and the locations to be the same but the time difference may be different.

D. Any other inertial observer might find the order of events, the locations, and the time difference to be different.

E. Say what?
Clicker question 1  
Set frequency to DA

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C. Another inertial observer must find the order of events and the locations to be the same but the time difference may be different.

D. Any other inertial observer might find the order of events, the locations, and the time difference to be different.

E. Say what?

In general, length, time, and order can be different for different inertial observers.

However, due to causality and finite speed of light, if the events are far enough apart it is impossible for the order to be reversed.
Lorentz transformations are used to transform coordinates (position and time) of an event from a rest frame \( S \) to a moving frame \( S' \).

\[
x' = \gamma (x - vt) \quad y' = y \quad z' = z \quad t' = \gamma \left( t - \frac{vx}{c^2} \right)
\]

The *inverse* Lorentz transformations go the other way (\( S' \) to \( S \)) and can be obtained by swapping primes and signs.

Velocity addition formula is also different than the Galilean velocity addition formula:

\[
u' = \frac{u - v}{1 - uv / c^2}
\]

Relativistic Doppler shift for observer and source approaching at \( \beta \).

\[
f_{\text{obs}} = f_{\text{source}} \sqrt{\frac{1 + \beta}{1 - \beta}}
\]
Clicker question 2

Two events take place 90 m apart with an intervening time interval of 0.60 µs in reference frame S. What is the speed of the reference frame $S'$ which measures the proper time between the two events?

A. 0  \quad x' = \gamma(x - vt)

B. 0.25c  \quad y' = y

C. 0.50c  \quad z' = z

D. 0.90c  \quad t' = \gamma(t - \frac{v}{c^2} x)

E. c

\[ x' = \gamma(x - vt) = 0 \]
Clicker question 2

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A. 0
B. 0.25c
C. 0.50c
D. 0.90c
E. c

Proper time $\Delta t_0$ between two events is the time measured in the frame in which both events occur at the same location.

So in the S’ frame, both events must occur at point 0. That is, $x' = 0$ when $x = 90$ m and $t = 0.60$ µs.

To get $x' = \gamma (x - vt) = 0$ requires $x - vt = 0$, i.e. $x = vt$.

Solving for $v$:

$$v = \frac{x}{t} = \frac{90 \text{ m}}{0.6 \text{ µs}} = 150 \text{ m/µs} = 0.5c$$
Conservation of momentum and energy continues to hold for isolated systems if we redefine momentum and energy as:

\[ p = \gamma_u m u \]
\[ E = \gamma_u m c^2 \]

for mass \( m \) with velocity \( u \) and \( \gamma_u = 1 / \sqrt{1 - u^2 / c^2} \)

This gives a rest energy of \( E_{\text{rest}} = mc^2 \)

Define kinetic energy: \( KE = E - E_{\text{rest}} = \gamma_u m c^2 - mc^2 = (\gamma_u - 1)mc^2 \)

Energy-momentum relationship: \( E^2 = (pc)^2 + (mc^2)^2 \)

For a massless particle like a photon, this reduces to \( E = |pc| \)

One other equation is \( \beta = pc / E \)

This is it for equations in Chapter 2. Usually use conservation of energy and conservation of momentum to solve problems.
Clicker question 3

A Λ particle with mass 1116 MeV/c² decays at rest to a proton with mass 938 MeV/c² and a pion with mass 140 MeV/c². What can we say about the proton and pion momentum and energy?

A. The proton and pion have the same magnitude momentum and energy.
B. The proton and pion have the same energy but the proton has more momentum.
C. The proton and pion have the same momentum but the proton has more energy.
D. The proton and pion have equal and opposite momenta and the proton has more energy.
E. Need more information
A $\Lambda$ particle with mass 1116 MeV/c$^2$ decays at rest to a proton with mass 938 MeV/c$^2$ and a pion with mass 140 MeV/c$^2$. What can we say about the proton and pion momentum and energy?

A. The proton and pion have the same magnitude momentum and energy.
B. The proton and pion have the same energy but the proton has more momentum.
C. The proton and pion have the same momentum but the proton has more energy.
D. The proton and pion have equal and opposite momenta and the proton has more energy.
E. Need more information

Conservation of momentum requires the proton and pion momentum add to 0 so must be equal and opposite. Since $E^2 = (pc)^2 + (mc^2)^2$, the proton has more energy due to larger mass.
Properties of wave functions

Wave functions are complex valued and are not directly observable.

Probability density is observable:

\[ |\psi(x)|^2 = \psi^*(x)\psi(x) = \psi_{\text{real}}^2(x) + \psi_{\text{imag}}^2(x) \]

The probability of finding the particle at some infinitesimal distance \( \delta x \) around \( x \) is:

\[ |\psi(x)|^2 \delta x \]

Probability of particle being between \( a \) and \( b \) is

\[ \int_a^b |\psi(x)|^2 \, dx \]

The particle must be somewhere with a probability of 100%. This is the normalization condition:

\[ \int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = 1 \]

In 1D, \(|\psi(x)|^2\) has dimension of 1/Length

\( \psi(x) \) is just the spatial part of the wave function.

\( \Psi(x,t) \) is the full wave function.

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The wave equation for electromagnetic waves is:
\[
\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}
\]
Only works for massless particles with \(v = c\).

For massive particles, need the time dependent Schrödinger equation (TDSE):

\[
- \frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x, t)}{\partial x^2} + V(x, t)\Psi(x, t) = i\hbar \frac{\partial \Psi(x, t)}{\partial t}
\]

Kinetic energy + Potential energy = Total energy

For time independent potentials, \(V(x)\): \(\Psi(x, t) = \psi(x)\phi(t)\)

In this case, the time part of the wave function is: \(\phi(t) = e^{-iEt/\hbar}\)

The spatial part \(\psi(x)\) can be found from the time independent Schrödinger equation (TISE):

\[
- \frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)
\]
Free particle

When \( E > V \) (everywhere) you have a free particle. We deal with the case of \( V = 0 \) but it can be applied to other cases as well.

Free particles have oscillating solutions

\[
\psi(x) = A \cos(kx) + B \sin(kx) \quad \text{or} \quad \psi(x) = Ce^{ikx} + De^{-ikx}
\]

To get the full wave function we multiply by the time dependence:

\[
\Psi(x, t) = \psi(x)e^{-i\omega t} = Ce^{i(kx - \omega t)} + De^{-i(kx + \omega t)}
\]

This is the sum of two waves with momentum \( \hbar k \) and energy \( \hbar \omega \).

The \( C \) wave is moving right and the \( D \) wave is moving left.
Infinite square well

Like the free particle, E>V but only in region 0<x<a.

Functional form of solution is also oscillating:
\[ \psi(x) = A \cos(kx) + B \sin(kx) \]

But this time have boundary conditions

Putting in \( x = 0 \) gives \( \psi(0) = A \) so \( A = 0 \)

Putting in \( x = a \) gives \( \psi(a) = B \sin(ka) \)

To get \( \sin(ka) = 0 \) requires \( ka = n\pi \)

Get the condition \( k = \frac{n\pi}{a} \) or \( \lambda = \frac{2a}{n} \)

Find that energies are quantized: \[ E_n = \frac{n^2\pi^2\hbar^2}{2ma^2} \]

After applying normalization condition we get \[ \psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \]
Penetration depth $\lambda = 1/\alpha$ measures how far particle can be found in the classically forbidden region.

Energy levels:

- $E_1 = 0.068$ eV
- $E_2 = 0.263$ eV
- $E_3 = 0.585$ eV
- $E_4 = 0.949$ eV

Note that energy level $n$ has $n$ antinodes.

The wave function is zero at the edge of the box.

The diagram shows the wave function $\psi(x)$ for different energy levels with corresponding antinodes and penetration depths.
Consider two very long wires separated by a small gap:

This is an example of a potential barrier.

Quantum tunneling occurs when a particle which does not have enough energy to go over the potential barrier somehow gets to the other side of the barrier. This is due to the particle being able to penetrate into the classically forbidden region. If it can penetrate far enough (the barrier is thin enough) it can come out the other side.
As in the quantum tunneling tutorial, an electron is moving from left to right when it encounters a potential barrier. Given what is drawn, rank the kinetic energy in the three regions.

A. A>B>C
B. A>C>B
C. A=C>B
D. A=B>C
E. None of the above
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Energy is conserved and since \( V=0 \) at A and C, any electrons that tunnel will have the same kinetic energy at C as at A.
\( \alpha \) decay probability

Probability of tunneling is \( P \approx e^{-2\alpha L} \) where \( \alpha = \frac{\sqrt{2m(V - E)}}{\hbar} \)

1. Less distance to tunnel (smaller \( L \))
2. Decay constant \( \alpha \) is smaller so wave function doesn’t decay as much before reaching other side

So 9 MeV \( \alpha \) has a higher tunneling probability

Can estimate the \( \alpha \) decay rate by taking the probability and multiplying by how often the \( \alpha \) particle hits the barrier

Isotopes that emit higher energy \( \alpha \) particles should have shorter lifetimes (higher probability to tunnel out and same frequency of hitting nuclear surface)

Experimentally confirmed! See text for details.
Clicker question 5

Scanning tunneling microscope (STM)

What does the potential at the tip look like?
A. higher than the sample
B. same as the sample
C. lower than the sample
D. tilts downward from left to right
E. tilts upward from left to right

Sample metal

Tip

+V

-I

V

x

energy

sample

tip

E_{\text{electron}}
Scanning tunneling microscope (STM)

What does the potential at the tip look like?
A. higher than the sample
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C. lower than the sample
D. tilts downward from left to right
E. tilts upward from left to right

Tip

Sample metal

I

+V

−V

x

energy

eV (from applied voltage)

sample tip

E_{electron}

http://www.colorado.edu/physics/phys2170/
Atomic wavefunctions and quantum numbers

Each atomic electron can be identified by four quantum numbers:

- \( n = 1, 2, 3, \ldots \) = principal quantum number
- \( \ell \) gives total orbital angular momentum: \( L = \sqrt{\ell(\ell + 1)} \hbar \)
- \( m \) gives z-component of orbital angular momentum: \( L_z = m \hbar \)
- \( m_s = \pm \frac{1}{2} \) gives the z-component of spin: \( S_z = m_s \hbar \)

The atom itself has angular momentum which is the vector sum of orbital and intrinsic angular momenta of the electrons. \( \vec{J} = \vec{L} + \vec{S} \)

Thus, the Stern-Gerlach experiment actually measures the z-component of the total angular momentum: \( J_z = L_z + S_z \)
Clicker question 6

Set frequency to DA

\[ E_n = -\frac{Z^2 E_R}{n^2} \]

\( n = 1, 2, 3, \ldots = \) Principal Quantum Number

\( \ell = 0, 1, 2, \ldots n-1 = \) orbital angular momentum quantum number

\( m = 0, \pm 1, \pm 2, \ldots \pm \ell \) is the z-component of orbital angular momentum

\( m_s = \pm \frac{1}{2} \) gives the z-component of spin:

\[ S_z = m_s \hbar \]

A hydrogen atom electron is excited to an energy of \(-13.6/4\) eV. How many different quantum states could the electron be in?

A. 2
B. 3
C. 4
D. 8
E. more than 8
Clicker question 6

Set frequency to DA

\[ E_n = -\frac{Z^2 E_R}{n^2} \]

\( n = 1, 2, 3, \ldots \) = Principal Quantum Number

\( \ell = 0, 1, 2, \ldots n-1 \) = orbital angular momentum quantum number

\( m = 0, \pm 1, \pm 2, \ldots \pm \ell \) is the z-component of orbital angular momentum

\[ L = \sqrt{\ell(\ell + 1)\hbar} \]

\[ L_z = m\hbar \]

\( m_s = \pm \frac{1}{2} \) gives the z-component of spin: \( S_z = m_s\hbar \)

A hydrogen atom electron is excited to an energy of \(-13.6/4\) eV. How many different quantum states could the electron be in?

A. 2
B. 3
C. 4
D. 8
E. more than 8

E = \(-13.6/4\) eV means \( n^2 = 4 \) so \( n = 2 \)

For \( n = 2 \), \( \ell = 0 \) or \( \ell = 1 \).

For \( \ell = 0 \), \( m = 0 \).

For \( \ell = 1 \), \( m = -1, 0, \) or \( 1 \).

For any of these four states \( m_s \) can be \( \pm \frac{1}{2} \), doubling the number of states.
Multielectron atoms

Using an approximation for $Z$ which we call $Z_{\text{eff}}$, we can use our hydrogen results to get

$$E_n = -Z_{\text{eff}}^2 E_R / n^2$$

$$r_{mp} \approx n^2 a_B / Z_{\text{eff}}$$

$Z_{\text{eff}} \approx 1$ for an outermost electron alone in an energy shell (fully screened by inner electrons) and $Z_{\text{eff}} \approx Z$ for inner electrons (no screening).

Using this approximation gives the energy levels which tells us how the energy levels fill up with electrons.

Pauli Exclusion Principle does not allow two electrons to have the same quantum numbers.

Each orbital is defined by quantum numbers $nlm$ and can have two electrons since $m_s = \pm \frac{1}{2}$ (spin up or spin down).

This explains why electrons are in the energy levels they are and explains the periodic table of the elements.
A quantum system can be in a superposition of states such as a superposition of energy states \( \psi = c_1\psi_1 + c_2\psi_2 + \cdots + c_n\psi_n \)

The states are orthogonal so \( \left| \psi \right|^2 = c_1^2\psi_1^2 + c_2^2\psi_2^2 + \cdots + c_n^2\psi_n^2 \)

When a quantity (like energy) is measured the wave function collapses into a definite state \( \psi_i \) is simply \( c_i^2 \)

An energy eigenstate is special because it does not change in time (why it is also called a stationary state)

Other eigenstates like a positions eigenstates do change in time; a particle initially in a position eigenstate does not remain in that state
Which of these is a true statement about quantum mechanics?

A. Energies are always quantized
B. Nothing can be known precisely
C. A quantum system may not have a precisely defined energy
D. More than one of the above
E. None of the above
Which of these is a true statement about quantum mechanics?

A. Energies are always quantized
B. Nothing can be known precisely
C. A quantum system may not have a precisely defined energy
D. More than one of the above
E. None of the above

A. The energy of a particle is only quantized when the particle is confined (infinite square well, finite square well, atom, simple harmonic oscillator). It comes from boundary conditions.

B. If you know a particle is in an energy eigenstate you know the energy precisely.

C. A particle can be in a superposition of energy eigenstates so it has a mixture of energies
Other eigenstates

There are actually eigenstates of any measurable quantity; not just energy.

If you measure a quantity other than energy (like position, momentum, angular momentum, …) then the particle will be in an eigenstate of that quantity (and not necessarily of energy).

If we measure the position in an infinite square well the wave function collapses to a state of definite position (instead of definite energy).
Other eigenstates

Remember that the probability density for energy eigenstates does not depend on time (stationary state).

Measuring the energy puts the particle in an energy eigenstate and it stays there until disturbed (for example by a position measurement)

This is not generally true for other eigenstates.

The position eigenstate is not a stationary state.

So a short time after measuring the position, the particle is no longer in a position eigenstate.
Spin, cont.

Total magnetic moment for any electron is given by the sum of orbital and spin magnetic moments:

\[ \vec{\mu}_{tot} = \vec{\mu}_{orb} + \vec{\mu}_{spin} = -\frac{e}{2m_e} \left( \vec{L} + 2\vec{S} \right) \]

Normal Zeeman effect where spin has no effect; Anomalous Zeeman effect where spin does contribute.

\[ U = -\mu B \cos \theta = -\vec{\mu} \cdot \vec{B} \]