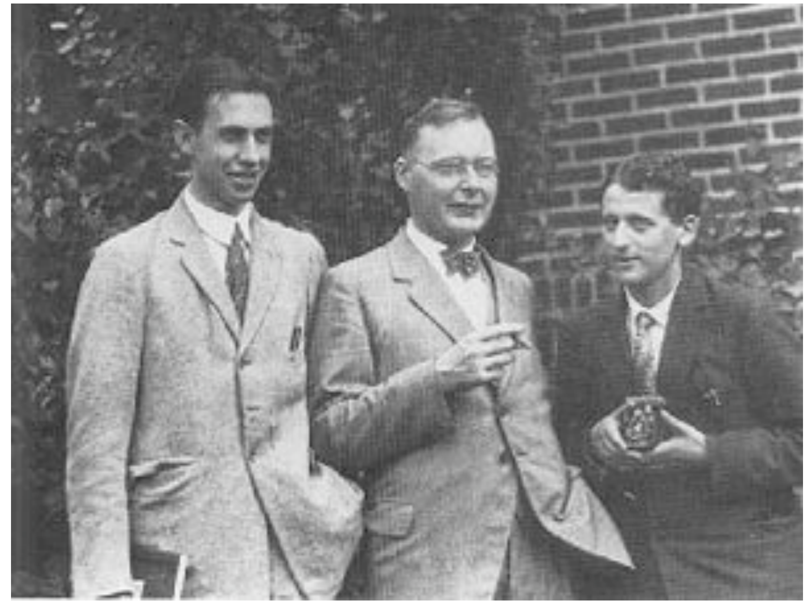
Magnetic Moments and Spin cont.

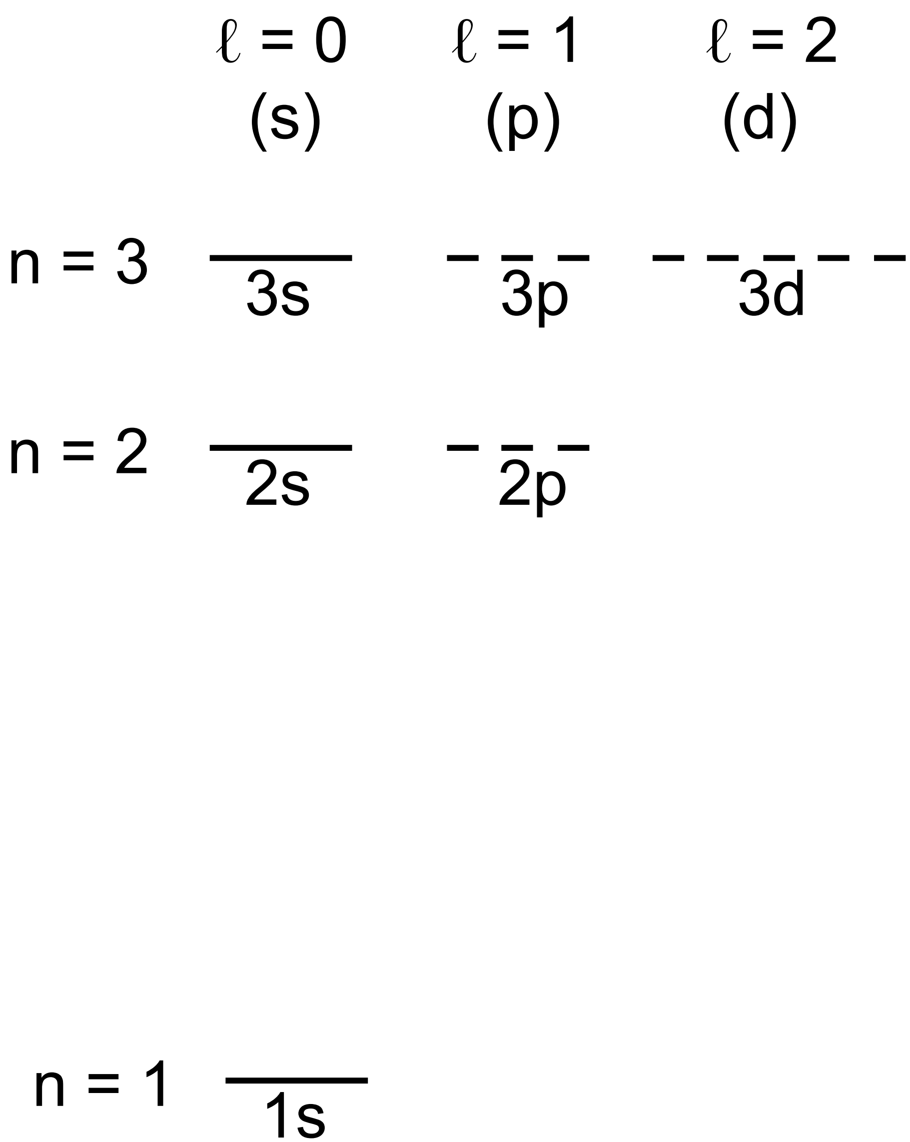
- Still have several homeworks to hand back
- Today we will talk about spin and its effects
- Chapter 8.7-8.9, Chapter 9.1-9.5
- This Wednesday we will fill out the FCQ for the course.
- Colloquium is on NSO



George Uhlenbeck (left)
Samuel Goudsmit (right)

In 1925 came up with idea of intrinsic spin quantum number.

Hydrogen energy levels



$$E_3 = -E_R / 3^2 = -1.5\,\text{eV}$$

$$E_2 = -E_R / 2^2 = -3.4\,\text{eV}$$

Levels are degenerate

$$E_1 = -E_R = -13.6\,\text{eV}$$

The full hydrogen wave function

The *spatial* part of the wave function is $\psi(r,\theta,\phi) = R_{n\ell}(r)Y_{\ell m}(\theta,\phi)$ with quantum numbers *n*, *ℓ*, *m* giving energy $E_n = -Z^2 E_R / n^2$, orbital angular momentum $L = \sqrt{\ell(\ell+1)}\hbar$ and z-component of orbital angular momentum $L_z = m\hbar$

To fully specify the wave function we also need the spin of the electron. This is set by the quantum number m_s which can be either +½ (spin up) or −½ (spin down).

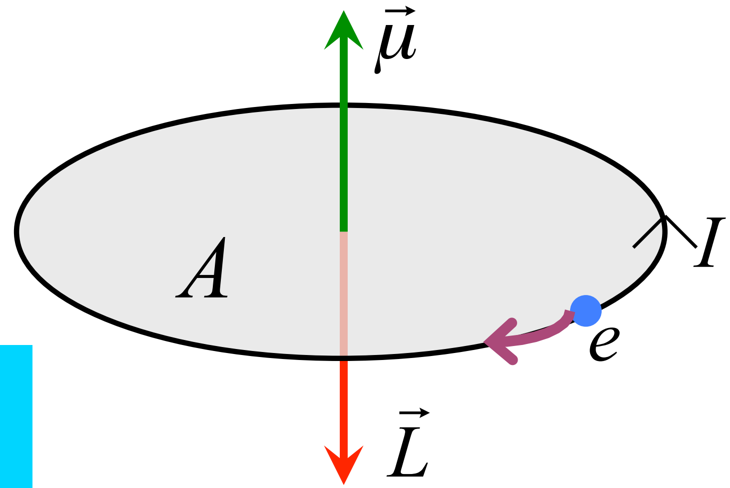
So the full set of quantum numbers that describe the electron in an atom are *n, ℓ, m,* m_s.

Note, the electron is not really spinning. It is a helpful way of thinking about what is technically *intrinsic* angular momentum.

Also, the **total** angular momentum $\vec{J}$ is the sum of orbital and intrinsic angular momentum: $\vec{J} = \vec{L} + \vec{S}$ and $J_z = L_z + S_z$

Magnetic moment

Magnetic field which behaves like a magnetic dipole with a magnetic dipole moment of $\vec{\mu} = I\vec{A}$. Direction is given by the right hand rule.



An orbiting electron creates a current (in the opposite direction) around an area.

The current depends on electron velocity and the area size depends on the orbit radius.

$$I = \frac{e}{T} = e\frac{v}{2\pi r}$$

Magnetic moment =

$$\mu = IA = \frac{ev}{2\pi r}\pi r^2 = \frac{1}{2}evr$$

Same quantities go into angular momentum: $\vec{L} = m\vec{r} \times \vec{v}$

$$\frac{\mu}{L} = \frac{e}{2m_e}$$

called gyromagnetic ratio

Magnetic Moments

Turns out we can write the magnetic moment of an atom in terms of the electron's angular momentum:

$$\vec{\mu} = -\frac{e}{2m_e}\vec{L}$$

The potential energy for a current loop in a magnetic field Can be calculated from the work done by a torque as it turns through an angle dθ

$$W = -\int \tau d\theta = -\mu B \int \sin\theta d\theta = \mu B \cos\theta + const.$$

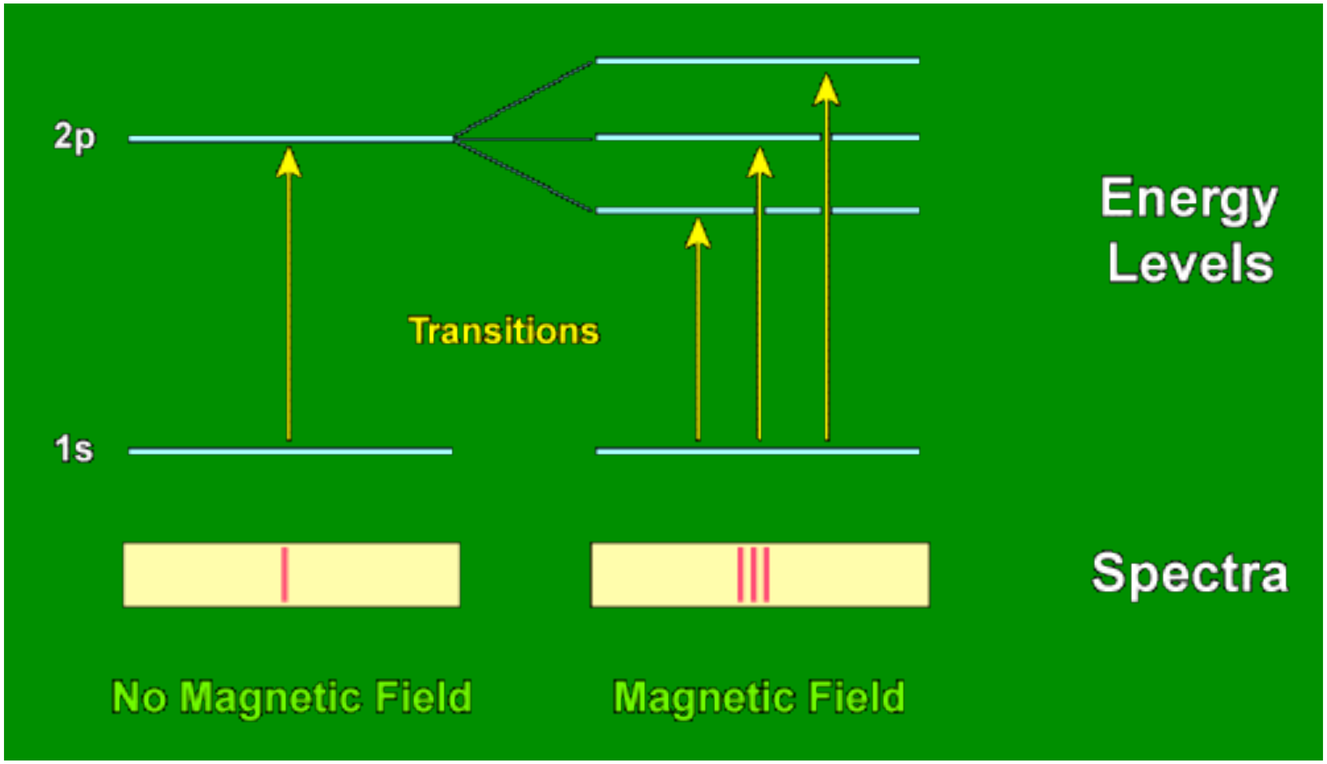
Potential Energy is defined as the negative of this work and we set the constant equal to zero

$$U = -\mu B \cos\theta = -\vec{\mu} \bullet \vec{B}$$

Note PE is a minimum when aligned with B

Zeeman Effect

Place an atom in a magnetic field and what happens?



Remember PE changed, so $E=E_0+\Delta E$; $\Delta E = -\vec{\mu} \bullet \vec{B}$

Zeeman cont.

$$\Delta E = -\vec{\mu} \bullet \vec{B}$$

$$\Delta E = \left(\frac{e}{2m_e} \right) \vec{L} \bullet \vec{B}$$

Pick magnetic field in the z direction

$$\Delta E = \left(\frac{e}{2m_e} \right) L_z B = \left(\frac{e\hbar}{2m_e} \right) mB$$

$$\mu_B = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24} \, A \bullet m^2$$

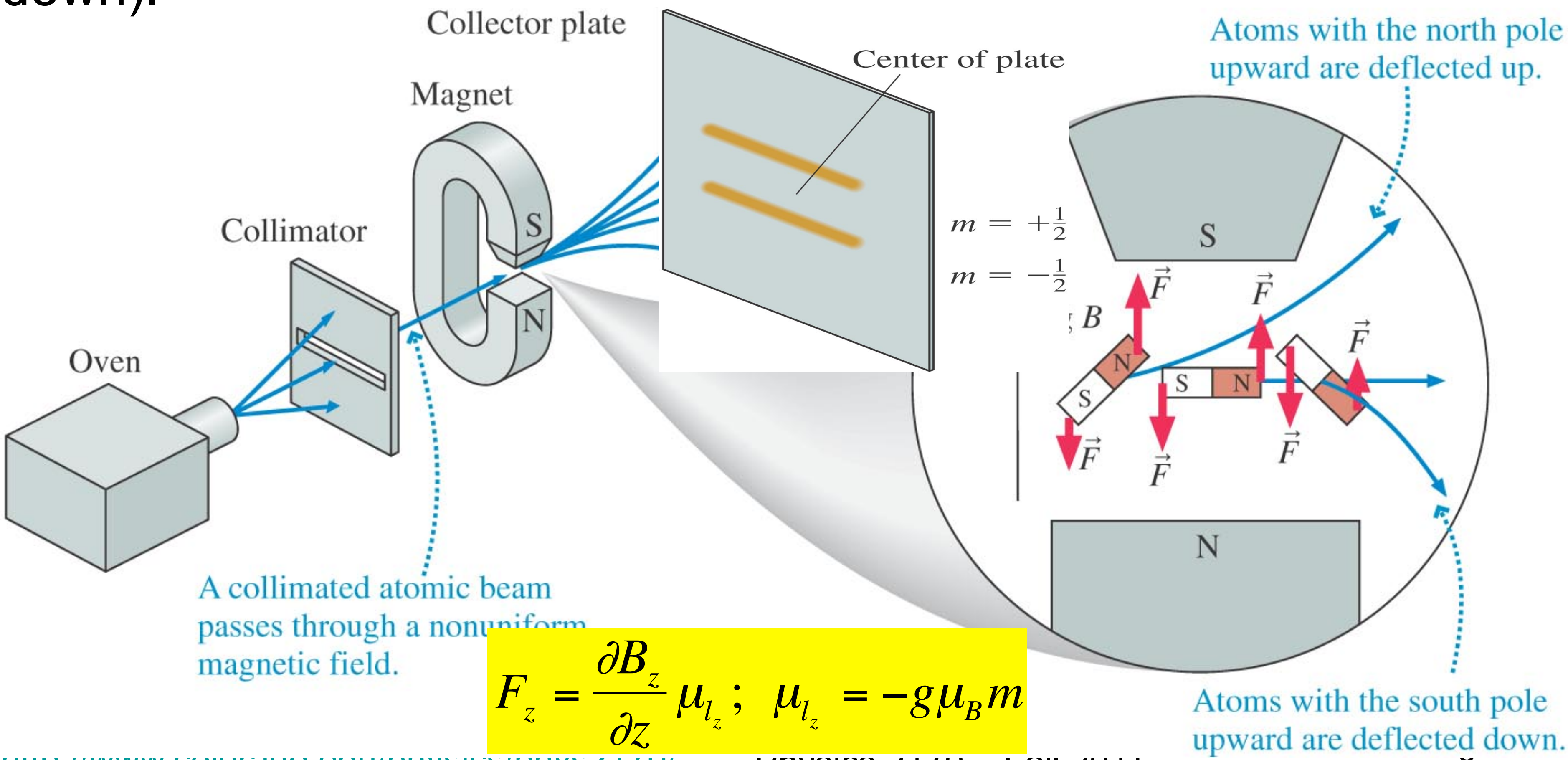
$$\Delta E = m\mu_B B$$

$\mu_B = 5.79 \times 10^{-5}$ eV/T

Stern-Gerlach experiment

A Stern-Gerlach experiment sends atoms through a *nonuniform* magnetic field which exerts a net force on a magnetic dipole.

Sending in hydrogen atoms with total angular momentum just from the electron (½) splits atoms in two samples (spin up and spin down).



$$F_z = \frac{\partial B_z}{\partial z}\mu_{l_z};\ \ \mu_{l_z} = -g\mu_B m$$

Result of Stern-Gerlach

The Stern-Gerlach experiment can be viewed as separating atoms according to their angular momentum direction.

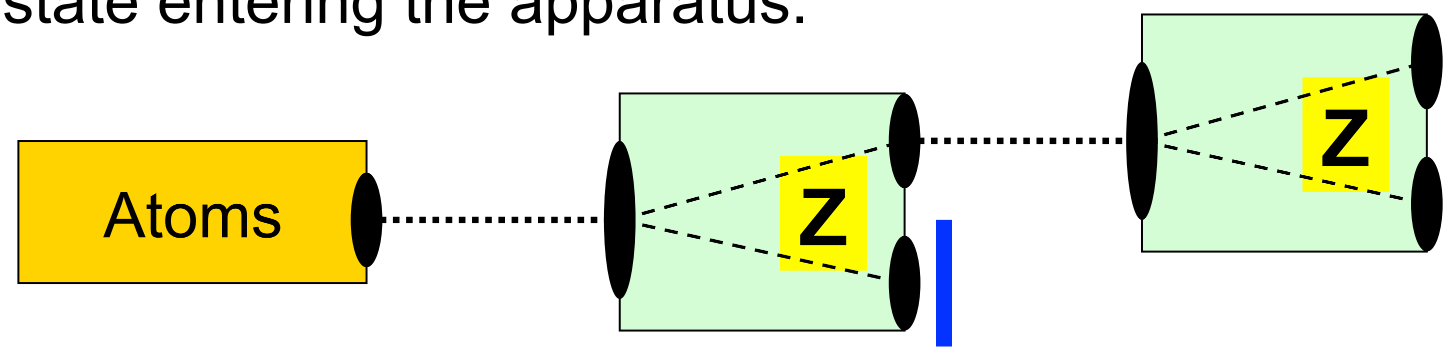
Assume our atoms start out in a random spin state. What fraction of the atoms will emerge from the top/bottom hole?

50% will go through each hole as +z or –z states are selected.

Suppose we block the −z spin atoms and pass the +z spin atoms through another SG system. One important point: in free space, angular momentum is independent of time (like energy).

What fraction of the atoms will emerge from the top/bottom hole?

100% will now come out of the top hole since it is in a +z spin eigenstate entering the apparatus.

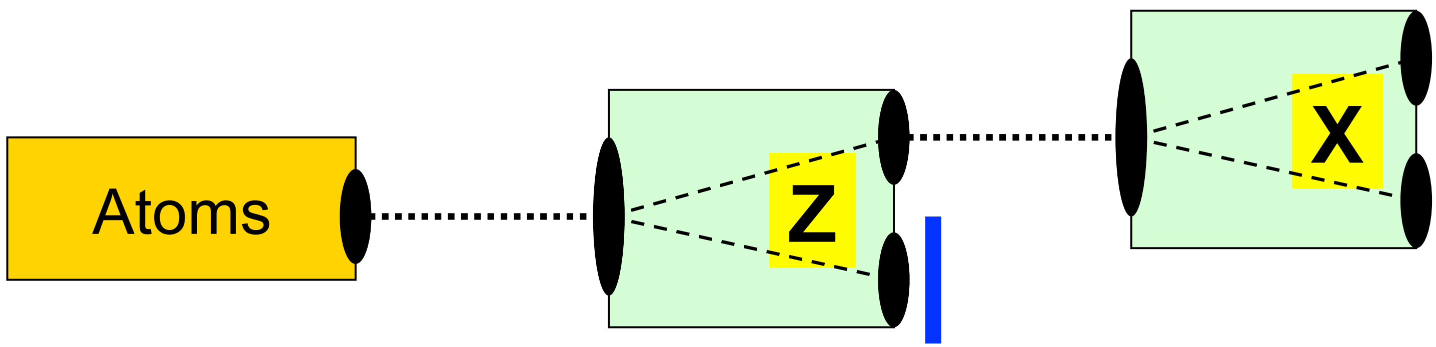


Result of Stern-Gerlach

Suppose we pass the +z spin atoms through a SG system which is oriented in **x** instead of z. One important point: Like position and momentum, we cannot know both the x-spin and z-spin at the same time.

What fraction of the atoms will emerge from the top/bottom hole?

It will be a 50/50 mix. A wave function of +z spin contains no information about the x spin so measuring the spin is just as likely to get +½ as –½.



Result of Stern-Gerlach

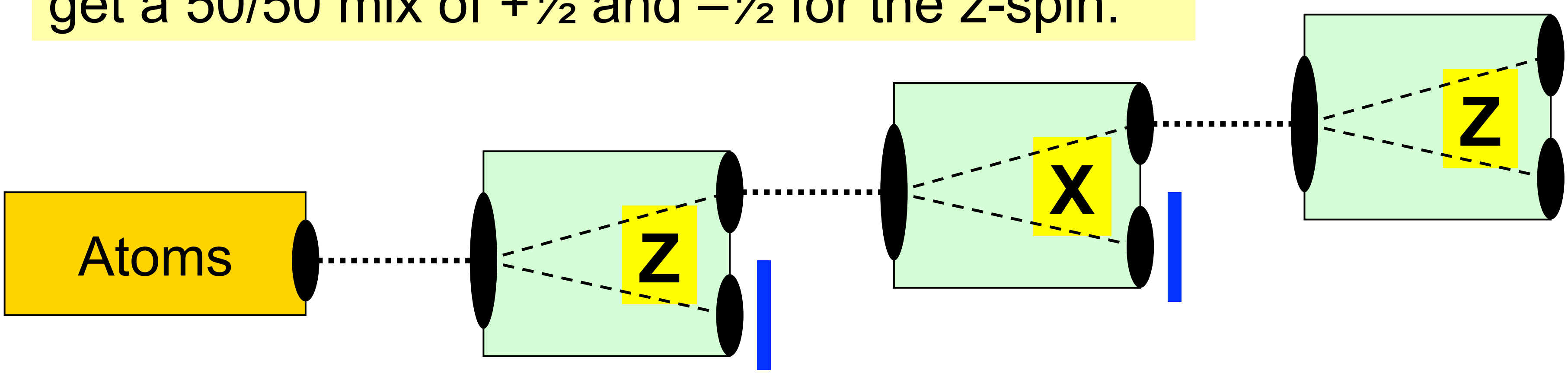
Suppose we now pass the +x spin atoms through a SG system which is oriented again in z.

What fraction of the atoms will emerge from the top/bottom hole?

It will be a 50/50 mix.

After the first magnet we only had +z spin atoms but measuring the x spin caused all knowledge of the z spin to be destroyed.

The wave function for an electron with +x spin contains no information on the z spin so we get a 50/50 mix of +½ and –½ for the z-spin.



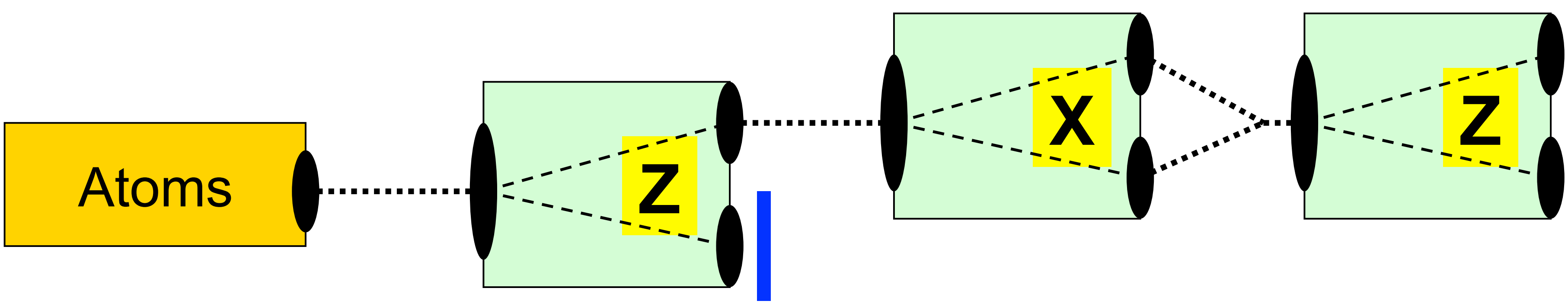
Result of Stern-Gerlach

What if we pass *both* the +x spin and –x spin atoms through the last magnet?

What fraction of the atoms will emerge from the top/bottom hole?

It will be a 100% +z atoms!

If we are careful not to actually *measure* the x spin then the wave function does not collapse to +x or –x. So we preserve the +z spin state from before!



Total Angular Momentum

As the earth orbits the sun, its total angular momentum, **J**, is the sum of two terms, **L** orbital angular momentum and **S**, the spin angular momentum, namely **J= L+S**

The same is true with an atom, with L and S being quantized!

Quantum numbers for an electron in the hydrogen atom are n, ℓ, m, m_s

For a given principal quantum number n, there are $2n^2$ quantum states.

Note protons and neutrons also have spin ½ , photon has spin 1.

Clicker question 1 — Set frequency to DA

Remember degeneracy refers to multiple quantum number combinations with the same energy. For hydrogen, the energy is set by n. For a given n consider all of the combinations of quantum numbers ℓ, m, and m_s. Remember $\ell=0,1\ldots n-1$ and $m=0,\pm1,\pm2\ldots\pm2\ell$ and $m_s=\pm\frac{1}{2}$. How many combinations are there?

A. n
B. $2n$
C. n^2
D. $2n^2$
E. None of the above

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D. $2n^2$
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Before we found out about spin we determined the number of degeneracies for the first three energy levels to be 1, 4, 9. In fact the degeneracy is n^2.

For each of these ℓ and m combinations, there are now two possibilities for m_s and so the degeneracy is doubled to $2n^2$.

Comments about Spin

Have found that the z component of the electron's spin is $\pm\frac{1}{2}\hbar$.

This means analogous to the angular momentum quantum number and we assume Spin quantum number is given by

$$S = \sqrt{s(s+1)}\hbar = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\hbar = \frac{\sqrt{3}}{2}\hbar$$

The electron's spin, S, always has the same value!

Having just discussed angular momentum, how do we think about an electron? Is it a spinning charged sphere?

Spin

Can we think of an electron spinning and having an angular momentum?

Take an electron radius of 10^{-18}m, and mass, m_e

$$L = I\omega;\ I = \frac{2}{5} m_e r^2;\ \omega = v/2\pi r$$

$$\frac{\sqrt{3}}{2}\hbar = \left[\frac{2}{5} m_e r^2\right]\left(\frac{v}{2\pi r}\right) = \frac{m_e r v}{5\pi}$$

$$v = \frac{5\pi\sqrt{3}\hbar}{2m_e r} = \frac{5\pi\sqrt{3}(1.05\times10^{-34}\,J\bullet s)}{2(9.11\times10^{-31}\,kg)(10^{-18}\,m)} \approx 2\times10^{15}\,m/s$$

Way too high – cannot be real!

Electron spin

$\ell = 0, 1, 2, \ldots n\text{-}1$ = *orbital* angular momentum quantum number

$$L = \sqrt{\ell(\ell+1)}\hbar$$

$m = 0, \pm1, \pm2, \ldots \pm\ell$ is the z-component of *orbital* angular momentum

$$L_z = m\hbar$$

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s = *spin* (or *intrinsic*) angular momentum quantum number. The actual spin angular momentum is

$$S = \sqrt{s(s+1)}\hbar$$

Electrons are $s = ½$ (spin one-half) particles.
Since this never changes, it is often not specified.

m_s = z-component of *spin* angular momentum and can have values of $m_s = -s, -s+1, \ldots s-1, s$. The actual z-component of spin angular momentum is

$$S_z = m_s\hbar$$

For an electron only two possibilities: $m_s = \pm s = \pm ½$

An electron with $m_s = +½$ is called *spin-up* or ↑

An electron with $m_s = -½$ is called *spin-down* or ↓

Spin, cont.

$$\vec{\mu}_{orb} = -\frac{e}{2m_e}\vec{L}$$

$$\vec{\mu}_{spin} = -\frac{e}{m_e}\vec{S}$$

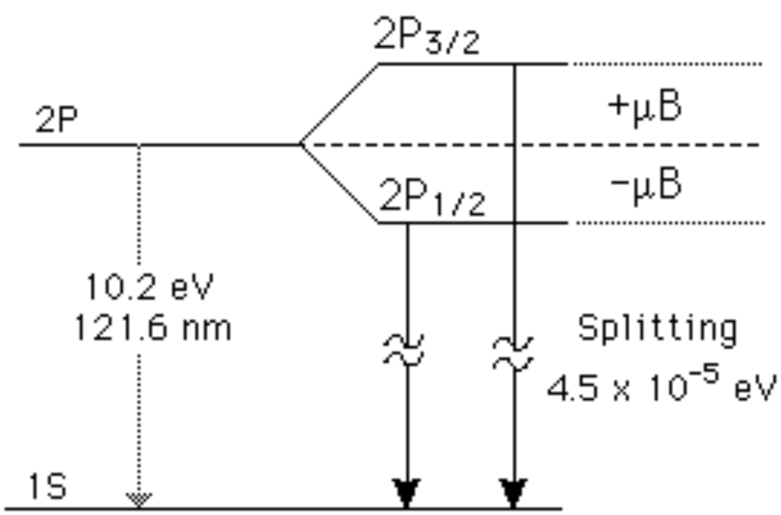
Total magnetic moment for any electron is given by
Sum of orbital and spin magnetic moments

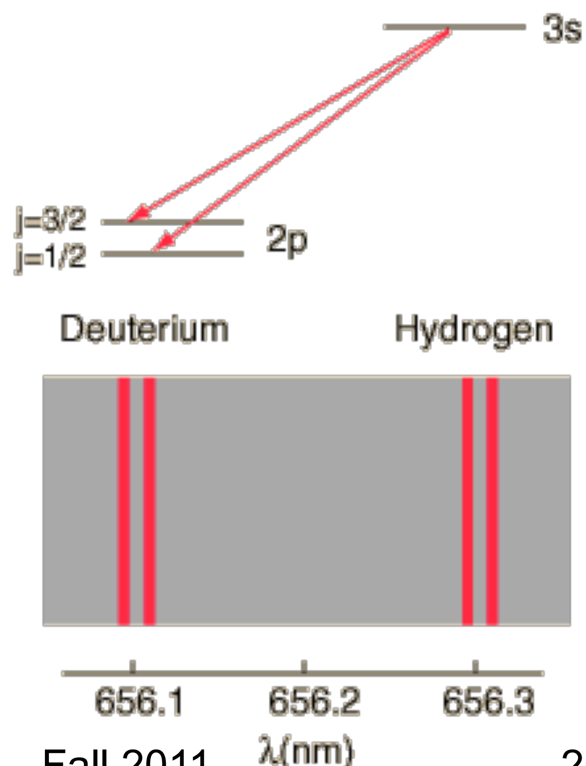
$$\vec{\mu}_{tot} = \vec{\mu}_{orb} + \vec{\mu}_{spin} = -\frac{e}{2m_e}\left(\vec{L} + 2\vec{S}\right)$$

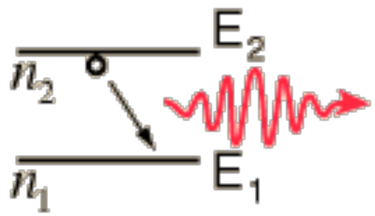
Normal Zeeman effect where spin has no effect;
Anomalous Zeeman effect where spin does contribute

Fine Structure

When the spectral lines of the hydrogen spectrum are examined at very high resolution, they are found to be closely-spaced doublets. This splitting is called fine structure and was one of the first experimental evidences for electron spin. Why?







A downward transition involves emission of a photon of energy:

$$E_{photon} = h\upsilon = E_2 - E_1$$

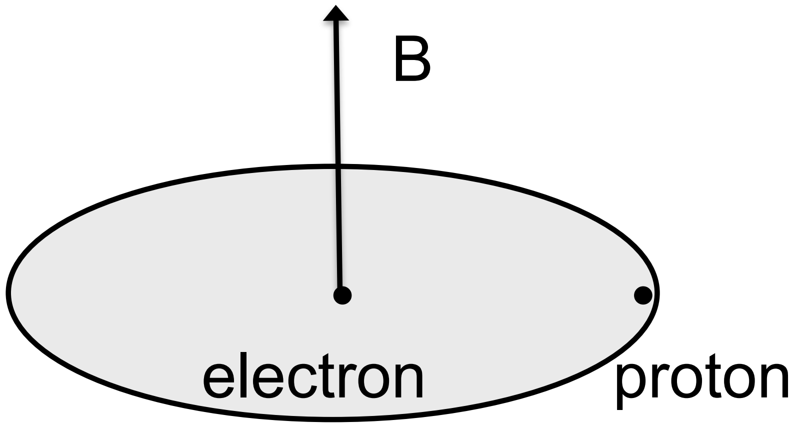
Given the expression for the energies of the hydrogen electron states:

$$h\upsilon = \frac{2\pi^2 m e^4}{h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = -13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]\text{eV}$$

$$\frac{1}{\lambda} = R_H\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$ where $R_H = \frac{2\pi^2 m e^4}{h^2}$ is called the Rydberg constant.

$$R_H = 1.0973731 x 10^7 m^{-1}$$

Spin Orbit Coupling



$$\vec{\mu}_{spin} = -(e/m_e)\vec{S}$$

In electron's rest frame, proton circulates around it and the electron sees a magnetic field; the magnetic field is proportional to **L**, the orbital magnetic field. $B \propto L$

$$\Delta E = -\vec{\mu}_{spin} \bullet B = \frac{e}{m_e}\vec{S} \bullet \vec{B} \propto \vec{S} \bullet \vec{L}$$

Spin and Angular Momentum parallel – energy shifts up; if antiparallel, then shifts down.