Magnetic Moments and Spin

- Still have several Homeworks to hand back
- Finish up comments about hydrogen atom and start on magnetic moment + spin.
- Eleventh Homework Set is due today and the last one has been posted.
- On Wednesday we will do FCQs – will appreciate a volunteer to take the forms back to the Department of Physics.
- Chapter 8.7-8.9, Chapter 9.3-9.4

Summary of hydrogen wave function

The hydrogen wave function is

$$\psi(r,\theta,\phi) = R_{n\ell}(r)\Theta_{\ell m}(\theta)e^{im\phi} \quad \text{or} \quad \psi(r,\theta,\phi) = R_{n\ell}(r)Y_{\ell m}(\theta,\phi)$$

<u>The quantum numbers are:</u>

n = 1, 2, 3, … = principal quantum number

$$E_n = -Z^2 E_R / n^2$$

ℓ = 0, 1, 2, … n-1 = angular momentum quantum number
= s, p, d, f, …

$$L = \sqrt{\ell(\ell+1)}\hbar$$

m = 0, ±1, ±2, … ±ℓ is the z-component of angular momentum quantum number

$$L_z = m\hbar$$

Hydrogen energy levels

	$\ell = 0$ (s)	$\ell = 1$ (p)	$\ell = 2$ (d)	
n = 3	3s	3p	3d	$E_3 = -E_R / 3^2 = -1.5\,\text{eV}$
n = 2	2s	2p		$E_2 = -E_R / 2^2 = -3.4\,\text{eV}$
n = 1	1s			$E_1 = -E_R = -13.6\,\text{eV}$

What do the wave functions look like?

n = 1, 2, 3, …

ℓ (restricted to 0, 1, 2 … *n*-1)

m (restricted to –ℓ to ℓ)

1s

2s

3s

4s (ℓ=0)

Increasing *n*

Increases distance from nucleus,

Increases # of radial nodes

4p (ℓ=1) 4d (ℓ=2) 4f (ℓ=3, *m*=0)

Increasing ℓ

Increases angular nodes

Decreases radial nodes

$m = -3$

Changes angular distribution

$m = 3$

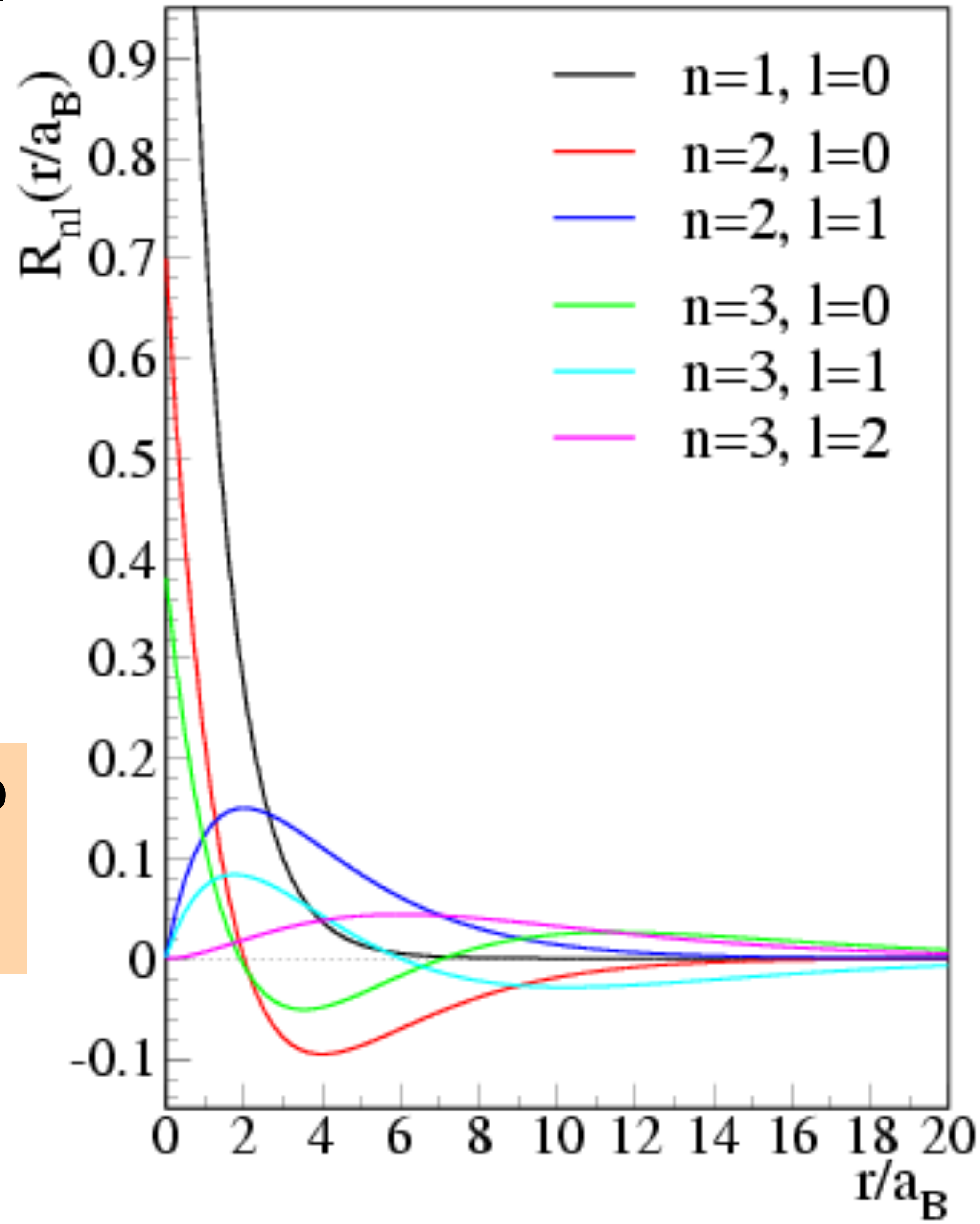
Radial part of hydrogen wave function $R_{nl}(r)$

Radial part of the wave function for n=1, n=2, n=3.

x-axis is in units of the Bohr radius a_B.

Number of radial nodes (R(r) crosses x-axis or |R(r)|^2 goes to 0) is equal to n−ℓ-1

Quantum number *m* has no affect on the radial part of the wave function.



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$|R_{nl}(r)|^2$

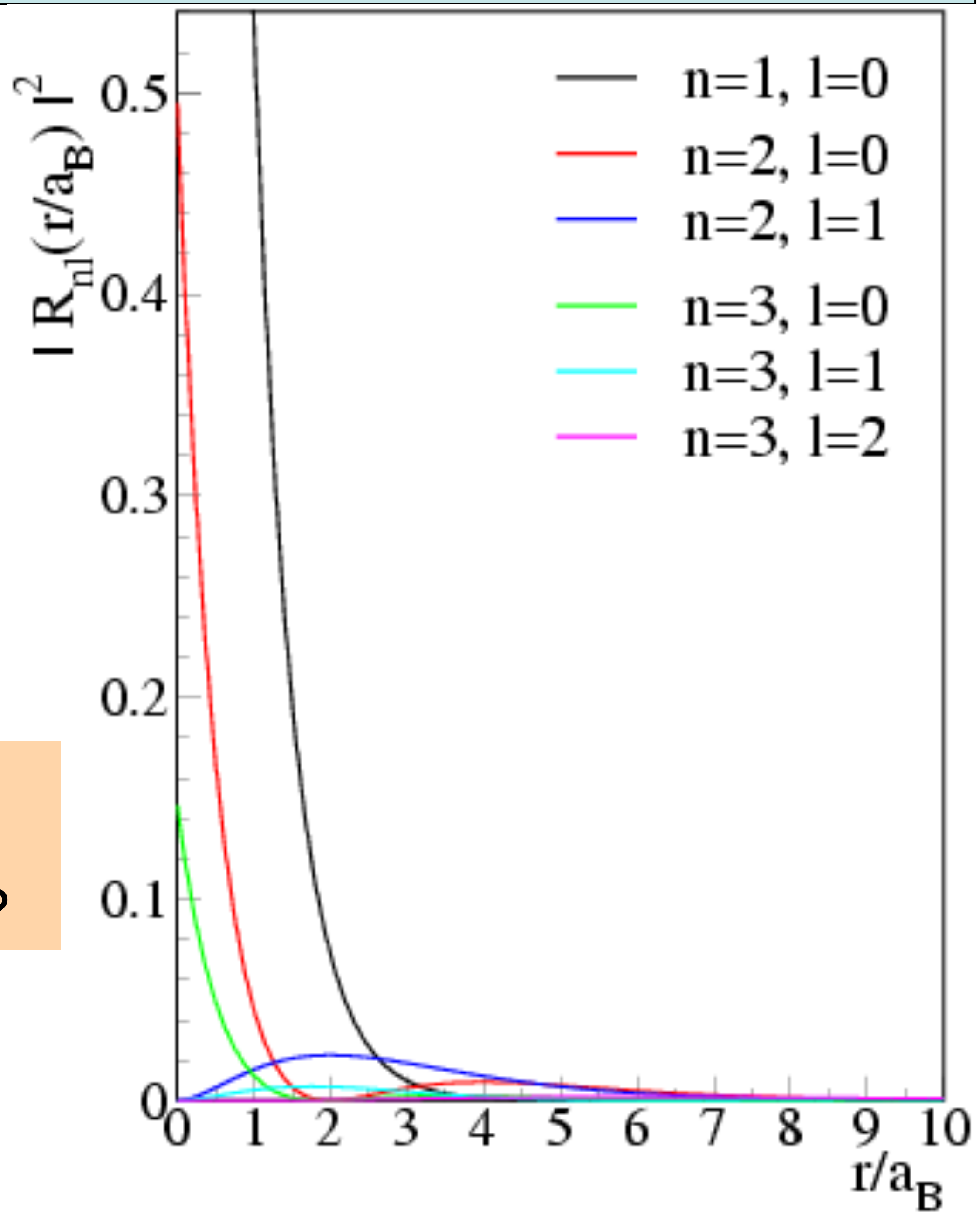
The radial part of the wave function squared

Note that all ℓ=0 states peak at r=0

Since angular momentum is $\vec{r} \times \vec{p}$ the electron *cannot* be at r=0 and have angular momentum.

Does this represent the probability of finding the electron near a given radius?

Not quite.



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Probability versus radius: $P(r) = |R_{nl}(r)|^2 r^2$

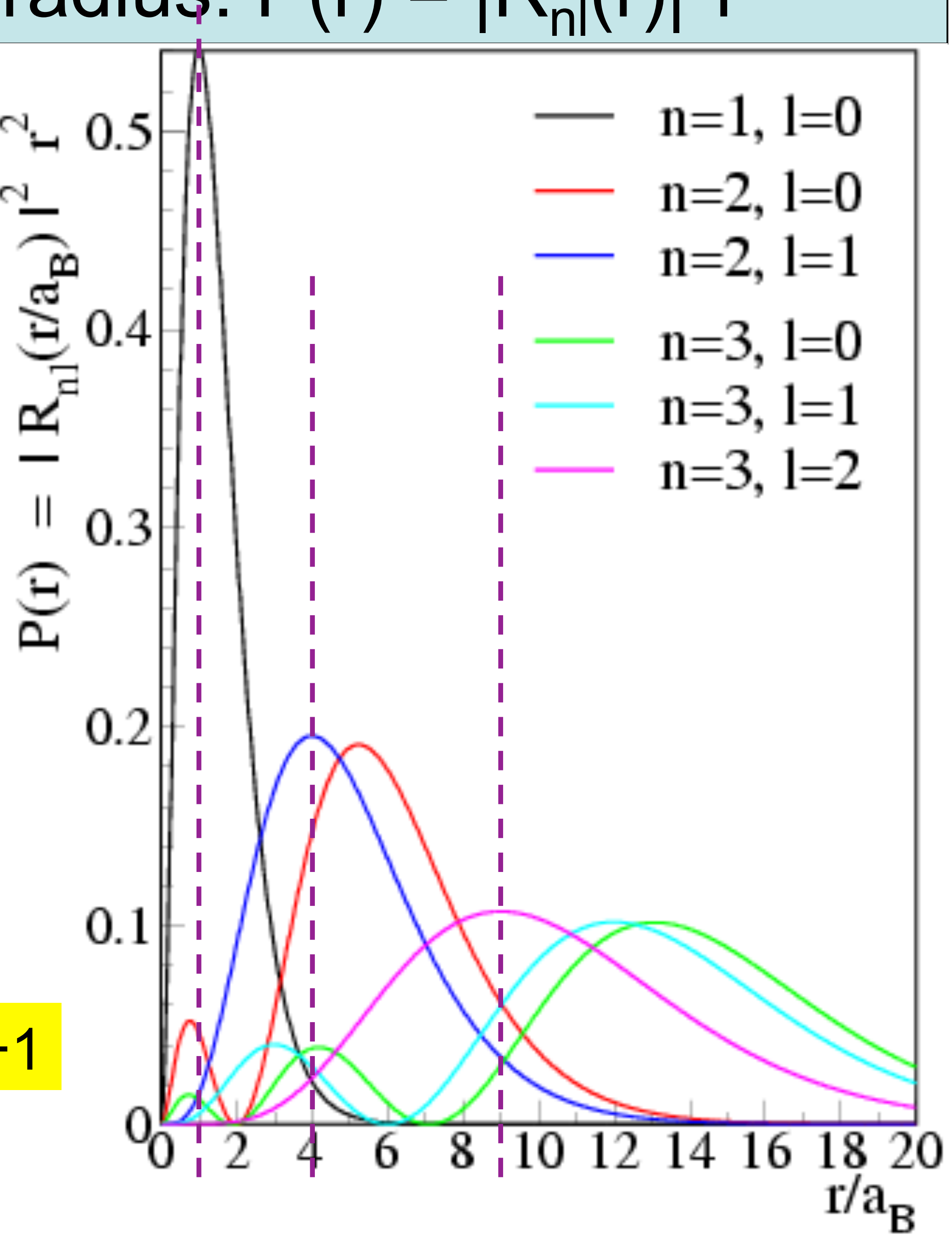
In spherical coordinates, the volume element $\propto r^2$ so probability increases with r^2.

Most probable radius for the n = 1 state is at the Bohr radius a_B.

Most probable radius for all ℓ=n-1 states (those with only one peak) is at the radius predicted by Bohr ($n^2\ a_B$).

Note the average radius increases as *n* increases.

Number of radial nodes = $n-\ell-1$



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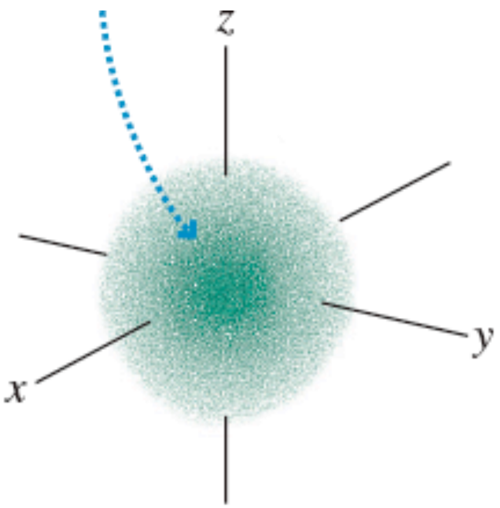
Visualizing the hydrogen atom

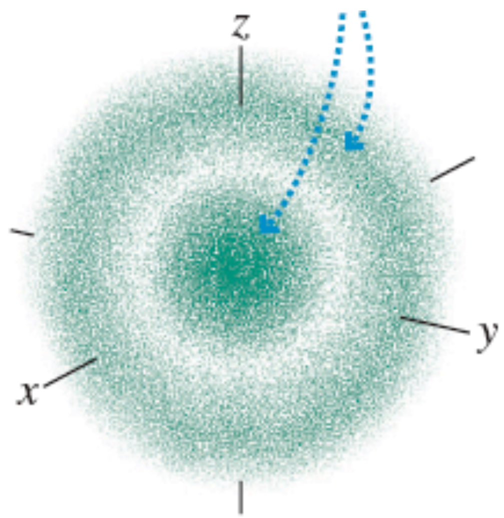
S states: ℓ = 0

s states have no angular momentum and are thus spherically symmetric.

There are n−ℓ−1 radial nodes.
Nodes are where $|\psi|^2 \rightarrow 0$.



1s
$m = 0$

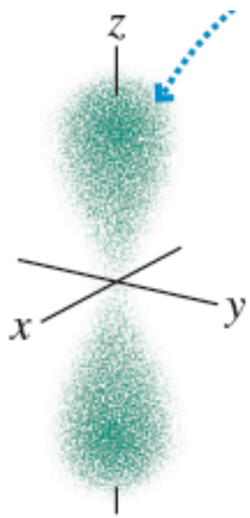


2s
$m = 0$

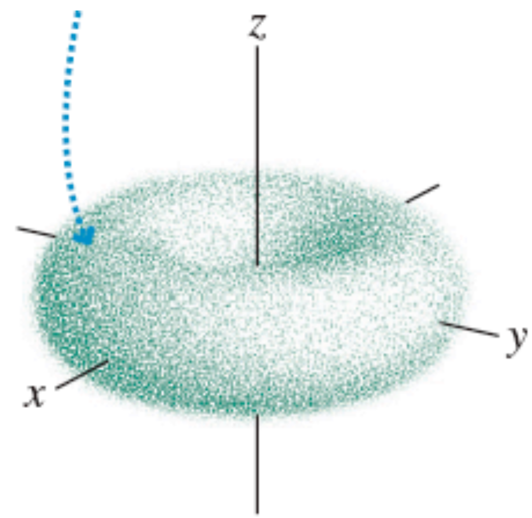
P states: ℓ = 1

For *m*=0, L_z=0 so no rotation about the *z* axis

For *m*=±1, L_z=±ħ so there is rotation about the *z* axis (either clockwise or counter clockwise)



2p
$m = 0$



2p
$m = \pm 1$

Clicker question 1	Set frequency to DA

Schrödinger finds quantization of energy and angular momentum:

n = 1, 2, 3 … ℓ = 0, 1, 2, 3 (restricted to 0, 1, 2 … n-1)

$$E_n = -E_R / n^2 \qquad L = \sqrt{\ell(\ell+1)}\,\hbar$$

How does the Schrödinger result compare to the Bohr result?

I. The energy of the ground state solution is _________

II. The angular momentum of the ground state solution is ________

III. The location of the electron is ________

A. same, same, same

B. same, same, different

C. same, different, different

D. different, same, different

E. different, different, different

Clicker question 1 | Set frequency to DA

Schrödinger finds quantization of energy and angular momentum:

$n = 1, 2, 3 \ldots$ $\ell = 0, 1, 2, 3$ (restricted to $0, 1, 2 \ldots n$-1)

$$E_n = -E_R / n^2$$

$$L = \sqrt{\ell(\ell+1)}\,\hbar$$

How does the Schrödinger result compare to the Bohr result?

I. The energy of the ground state solution is <u>same</u>

II. The angular momentum of the ground state solution is <u>different</u>

III. The location of the electron is <u>different</u>

A. same, same, same

B. same, same, different

C. same, different, different

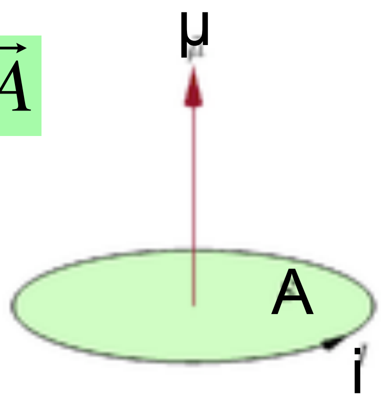
D. different, same, different

E. different, different, different

Bohr got the energy right, but said angular momentum was $L=n\hbar$, and thought the electron was a point particle orbiting around nucleus at a fixed distance.

Spin + Magnetic Moments

$\vec{\mu} = i\vec{A}$

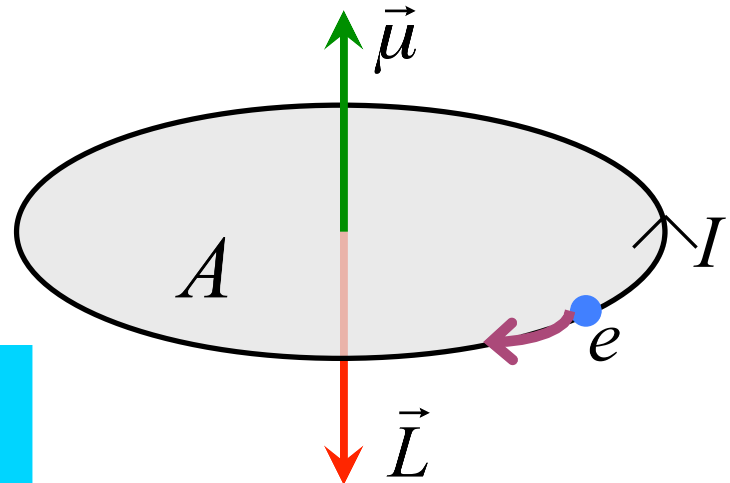


Much is known about electron's spin, but most of it is indirect! It comes from its atoms angular momentum and its magnetic moment. Consider first it's magnetic moment.

If you place a current carrying loop in a magnetic field then it experiences a torque given by $\vec{\tau} = \vec{\mu} \times \vec{B}$
The torque tends to turn the loop so it points in the same direction as the magnetic field B

Magnetic moment

Magnetic field which behaves like a magnetic dipole with a magnetic dipole moment of $\vec{\mu} = IA$. Direction is given by the right hand rule.



An orbiting electron creates a current (in the opposite direction) around an area.

The current depends on electron velocity and the area size depends on the orbit radius.

$$I = \frac{e}{T} = e\frac{v}{2\pi r}$$

Magnetic moment = $\mu = IA = \frac{ev}{2\pi r}\pi r^2 = \frac{1}{2}evr$

Same quantities go into angular momentum: $\vec{L} = m\vec{r} \times \vec{v}$

$\frac{\mu}{L} = \frac{e}{2m_e}$ called gyromagnetic ratio

Magnetic Moments

Turns out we can write the magnetic moment of an atom in terms of the electron's angular momentum:

$$\vec{\mu} = -\frac{e}{2m_e}\vec{L}$$

The potential energy for a current loop in a magnetic field Can be calculated from the work done by a torque as it turns through an angle dθ

$$W = -\int \tau d\theta = -\mu B \int \sin\theta d\theta = \mu B \cos\theta + const.$$

Potential Energy is defined as the negative of this work and we set the constant equal to zero

$$U = -\mu B \cos\theta = -\vec{\mu} \bullet \vec{B}$$

Note PE is a minimum when aligned with B

Magnetic Moments cont.

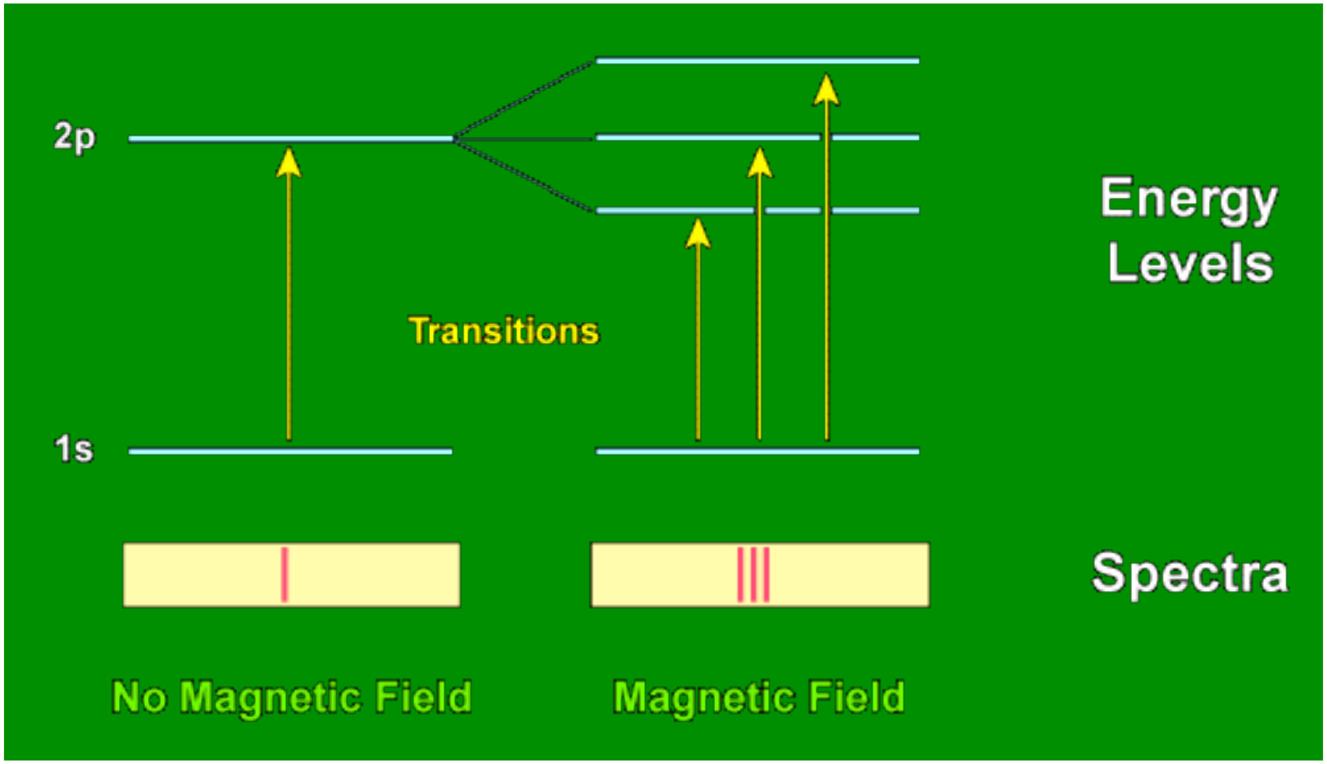
We have derived the magnetic moment using classical arguments, but we get the right answer for the moment if we use the quantum values for L

For a given L, the magnitude is given by $L = \sqrt{\ell(\ell+1)}\hbar$

This means the magnetic moment μ of the orbiting electron has $2\ell + 1$ possible orientations.

Zeeman Effect

Place an atom in a magnetic field and what happens?



Remember PE changed, so $E=E_0+\Delta E$; $\Delta E = -\vec{\mu} \bullet \vec{B}$

Zeeman cont.

$$\Delta E = -\vec{\mu} \bullet \vec{B}$$

$$\Delta E = \left(\frac{e}{2m_e}\right)\vec{L} \bullet \vec{B}$$

Pick magnetic field in the z direction

$$\Delta E = \left(\frac{e}{2m_e}\right)L_z B = \left(\frac{e\hbar}{2m_e}\right)mB$$

$$\mu_B = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24}\ A \bullet m^2$$

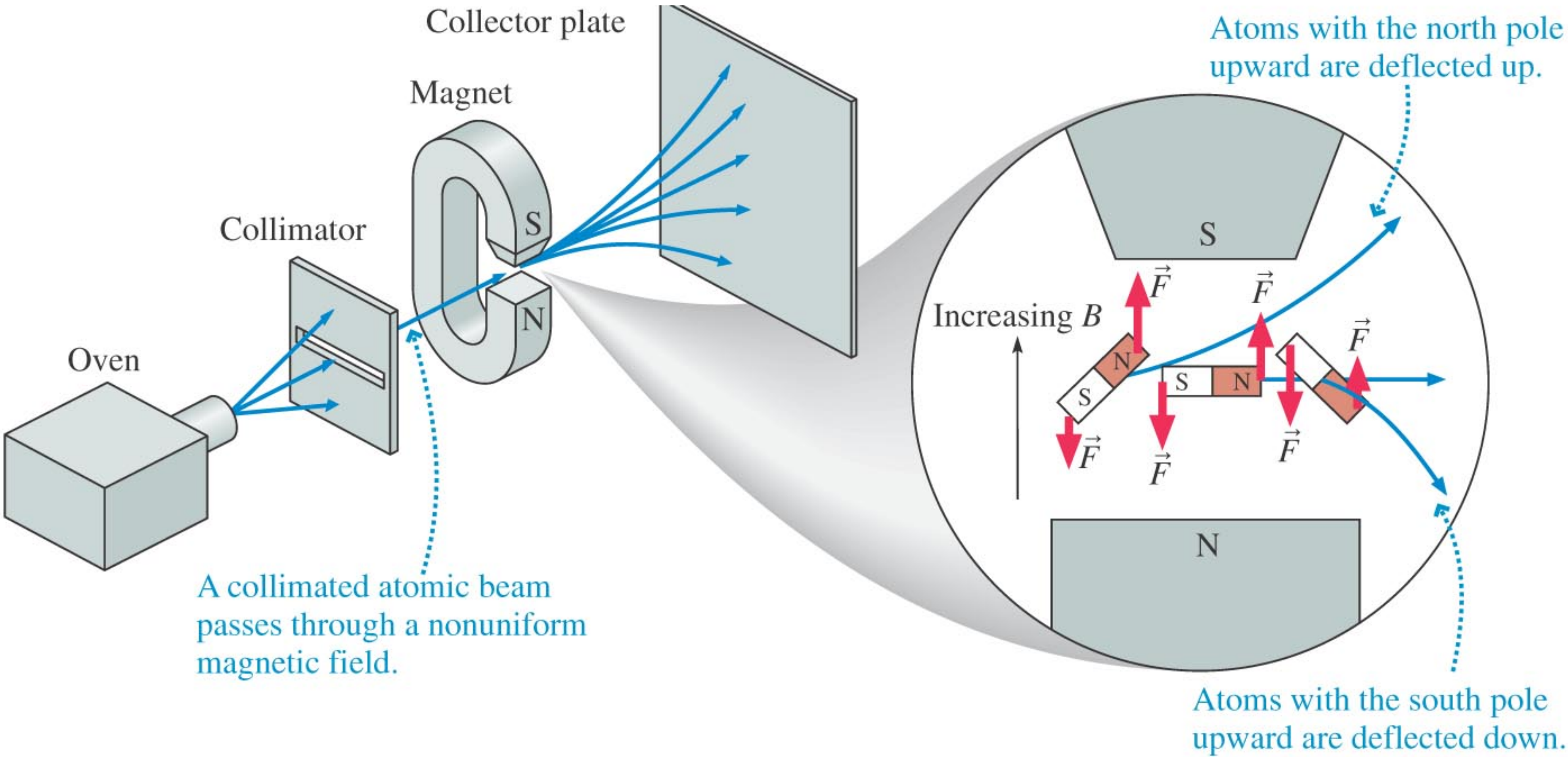
$$\Delta E = m\mu_B B$$

μ_B=5.79 x 10^{-5} eV/T

Stern-Gerlach experiment

Placing a magnetic dipole in an external *uniform* magnetic field $\vec{B}$ causes a torque on the dipole $\vec{\tau} = \vec{\mu} \times \vec{B}$ but **no** net force.

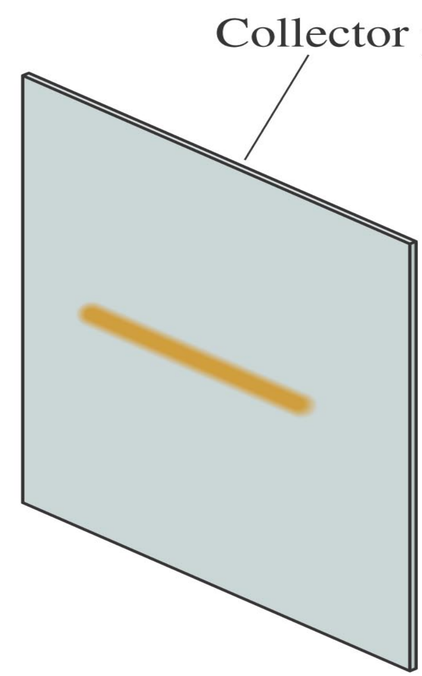
A Stern-Gerlach experiment sends atoms through a *nonuniform* magnetic field which *can* exert a net force on a magnetic dipole.



Clicker question 2 Set frequency to DA

The Stern-Gerlach magnet is oriented so deflections occur in the z direction. Based on what we know so far, if the atoms passing through have no angular momentum $\left(\ell = 0 \text{ so } L = 0\right)$ what will happen?

A. Atoms will be deflected in z direction
B. Atoms will be deflected in x direction
C. Atoms will be deflected in y direction
D. Atoms will not be deflected
E. Need quantum number *m* to tell



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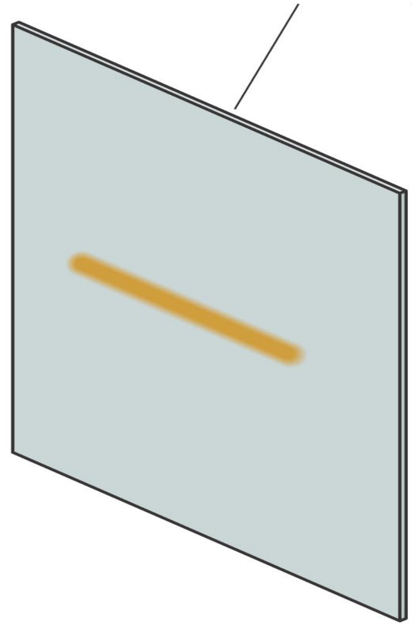


A. Atoms will be deflected in z direction
B. Atoms will be deflected in x direction
C. Atoms will be deflected in y direction
D. Atoms will not be deflected
E. Need quantum number *m* to tell

Atoms with no angular momentum have no magnetic dipole moment

$$\vec{\mu} = -\frac{e}{2m_e}\vec{L}$$

Therefore, they are not affected by the magnet (no torque or force)

Clicker question 3 | Set frequency to DA

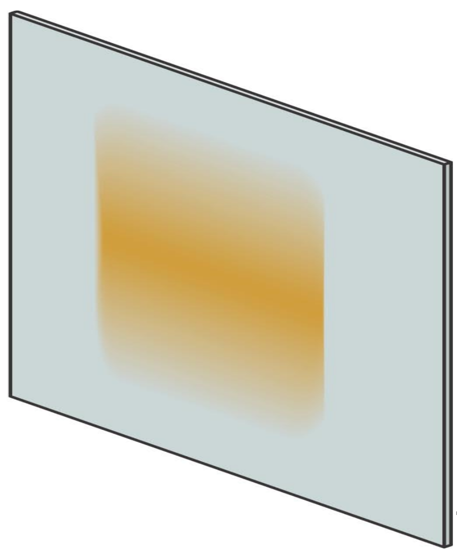
The Stern-Gerlach magnet is oriented so deflections occur in the z direction. If the atoms passing through have angular momentum of $\ell = 1$ so $L = \sqrt{2}\hbar$ but the z-component $L_z = m\hbar$ is unknown, how many possibilities are there for deflection?

A. 0
B. 1
C. 2
D. 3
E. Infinite

$$L_z = -\hbar, 0, \text{or } \hbar$$

Classical result

Classically, an atom with $L = \sqrt{2}\hbar$ can have any value of L_z as long as $|L_z| \le \sqrt{2}\hbar$



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The Stern-Gerlach magnet is oriented so deflections occur in the z direction. If the atoms passing through have angular momentum of $\ell = 1$ so $L = \sqrt{2}\hbar$ but the z-component $L_z = m\hbar$ is unknown, how many possibilities are there for deflection?

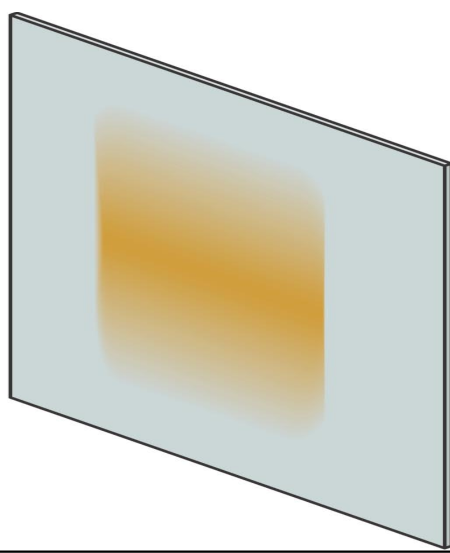
A. 0
B. 1
C. 2
D. 3
E. Infinite

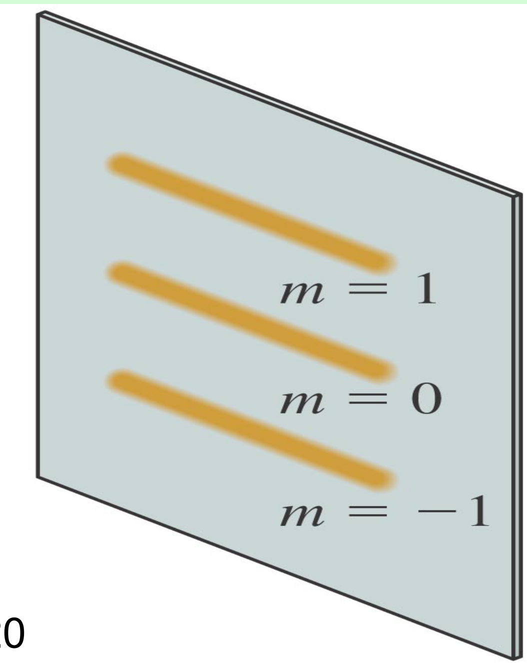
When ℓ=1, the quantum number *m* can only have three possible values (−1, 0, 1) so $L_z = -\hbar, 0, \text{or } \hbar$

The *m* = −1 and *m* = 1 atoms are deflected in opposite directions and the *m* = 0 atoms are not deflected at all.

Classical result

Classically, an atom with $L = \sqrt{2}\hbar$ can have any value of L_z as long as $|L_z| \le \sqrt{2}\hbar$





Clicker Question 4	Set frequency to DA

Q. The spin quantum number for the electron *s* is…

A. 0
B. ½
C. 1
D. Can be more than one of the above
E. None of the above

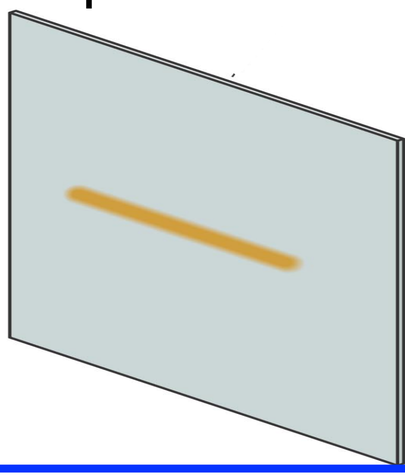
Clicker Question 4 | Set frequency to DA

Q. The spin quantum number for the electron *s* is…

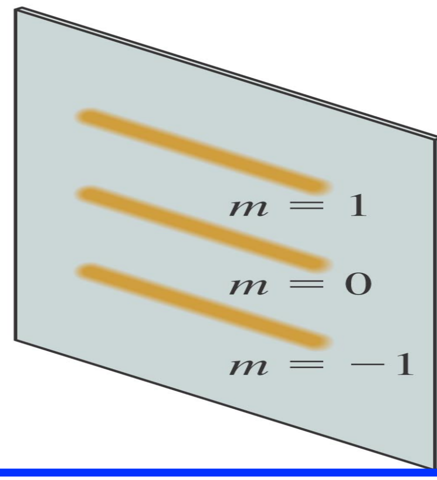
A. 0
B. ½
C. 1
D. Can be more than one of the above
E. None of the above

Result of Stern-Gerlach

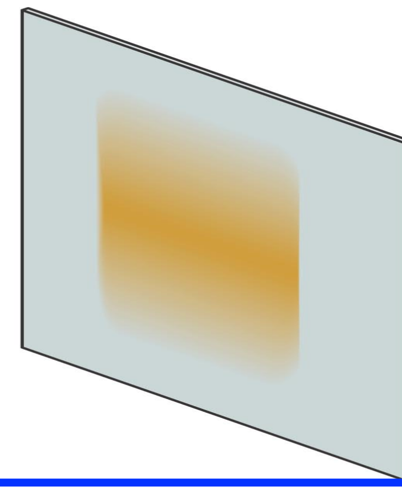
Sending in (ground state) hydrogen atoms which were believed to have ℓ=0, one expects no deflection.



If ℓ≠0, would find 2ℓ+1 bands (odd number)

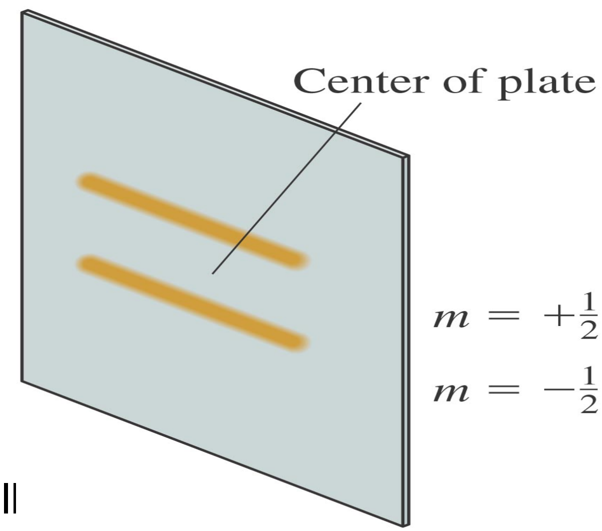


Classically, one would see a broad band



Doing the experiment gave two lines.

Interpretation: ℓ=0 but the electron itself has some <u>*intrinsic*</u> angular momentum which can either be −ħ/2 or ħ/2.



Electron spin

ℓ = 0, 1, 2, … *n*-1 = *orbital* angular momentum quantum number

$$L = \sqrt{\ell(\ell+1)}\hbar$$

m = 0, ±1, ±2, … ±ℓ is the z-component of *orbital* angular momentum

$$L_z = m\hbar$$

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s = *spin* (or *intrinsic*) angular momentum quantum number. The actual spin angular momentum is

$$S = \sqrt{s(s+1)}\hbar$$

Electrons are *s* = ½ (spin one-half) particles. Since this never changes, it is often not specified.

m_s = z-component of *spin* angular momentum and can have values of m_s = −*s*, −*s*+1, … *s*−1, *s*. The actual z-component of spin angular momentum is

$$S_z = m_s\hbar$$

For an electron only two possibilities: m_s = ±*s* = ±½

An electron with m_s = +½ is called *spin-up* or ↑

An electron with m_s = −½ is called *spin-down* or ↓