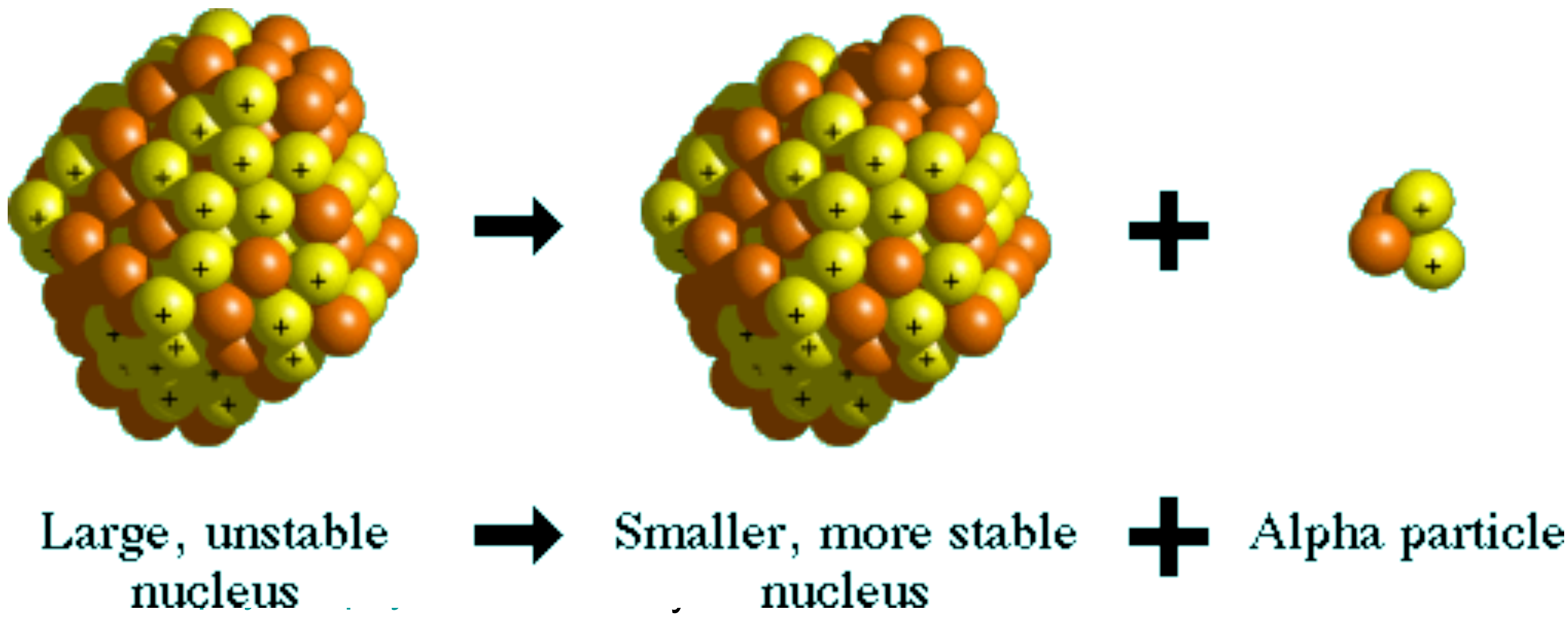
Quantum tunneling: α-decay

Announcements:

Homework #10 is available from Class Calendar.

- Veterans Day –recognizes all who have been in the armed service.
- Exams are graded are ready to be returned, please make sure I added your points correctly.
- Most people did pretty well on the exam.



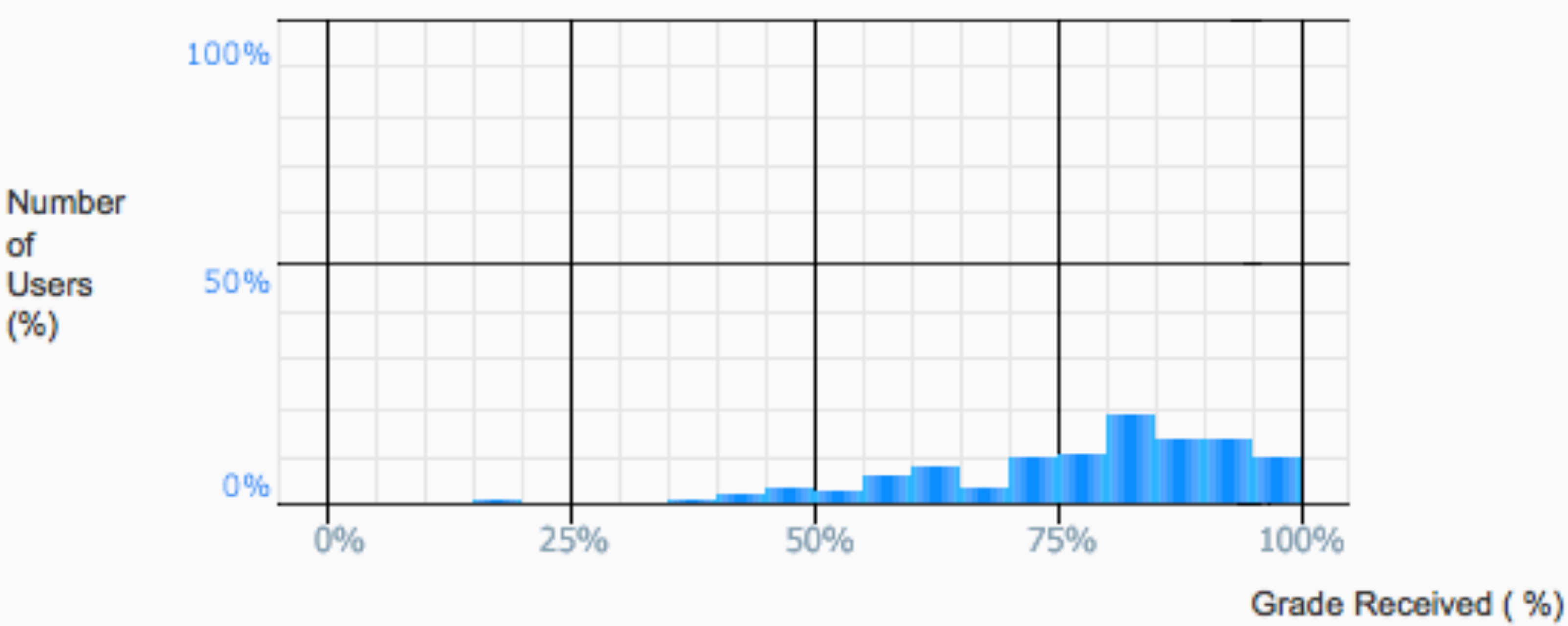
Midterm 2 Statistics

Midterm 2 Class Statistics

Number of submitted grades: 119 / 122
Minimum: 18 %
Maximum: 100 %
Average: 77.04 %

Median: 80 %
Standard Deviation: 15.58 %

Grade Distribution



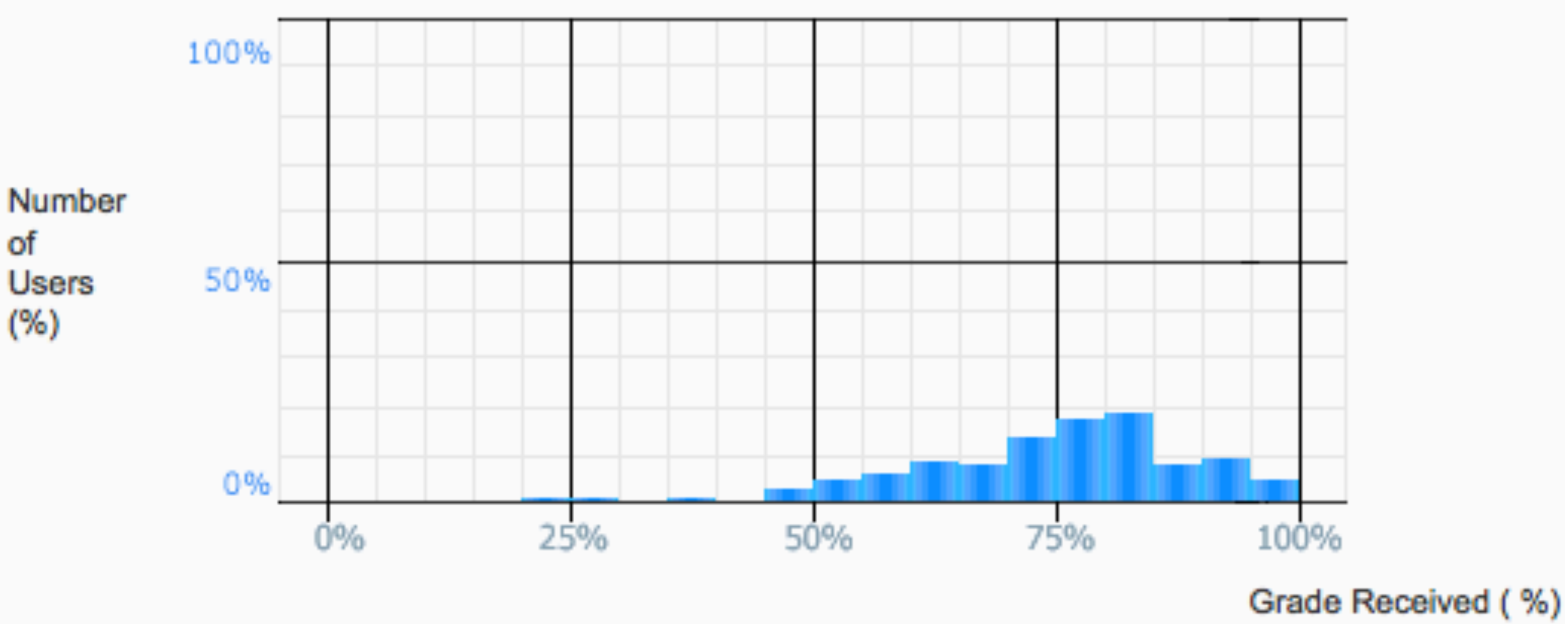
Sum of Both Exams

Exams Class Statistics

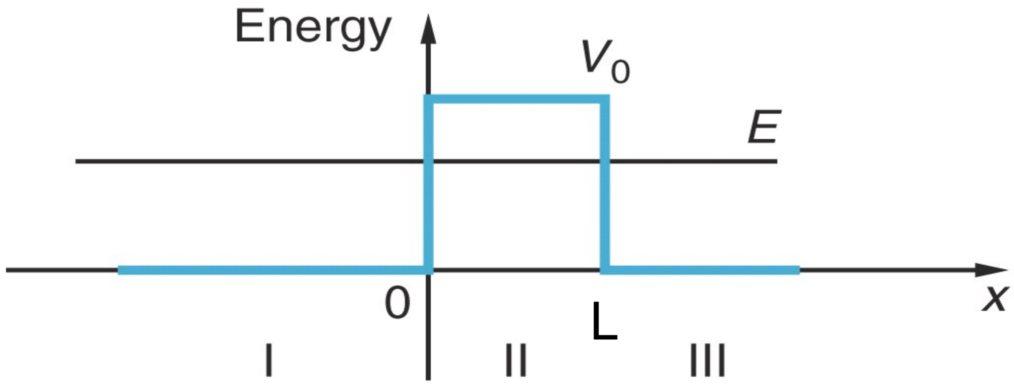
Number of submitted grades: 121 / 122
Minimum: 24 %
Maximum: 100 %
Average: 73.99 %

Median: 76 %
Standard Deviation: 14 %

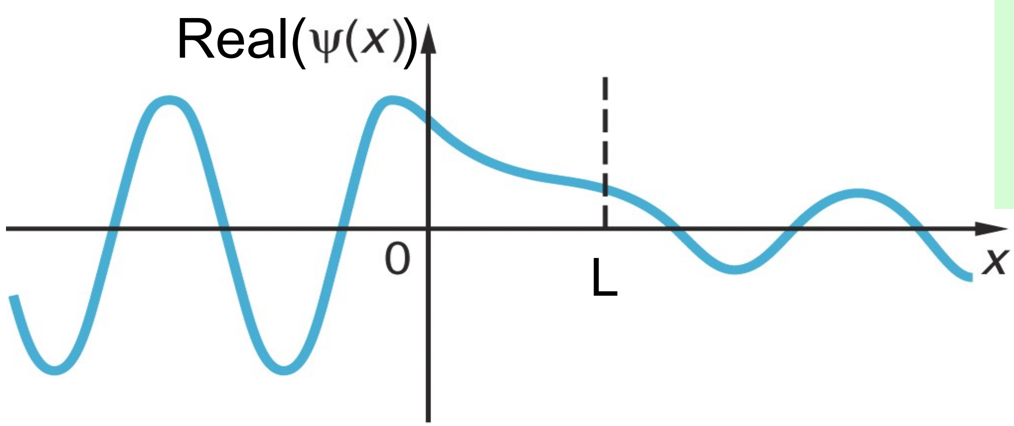
Grade Distribution



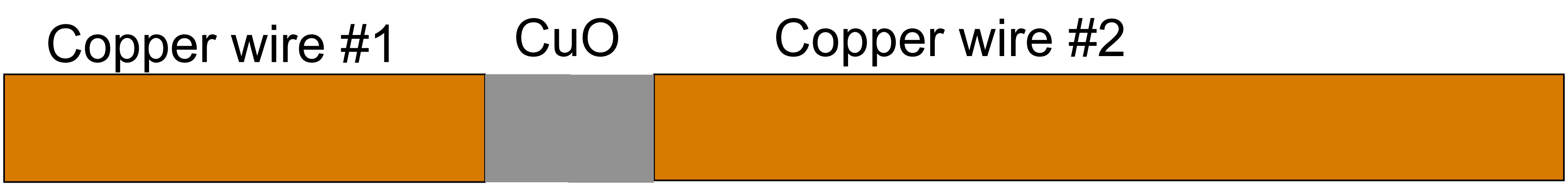
Quantum tunneling



If the potential increase has a finite width, it is a potential *barrier* and the electron can tunnel out of Region I



This is what you were encouraged to investigate in the tutorial



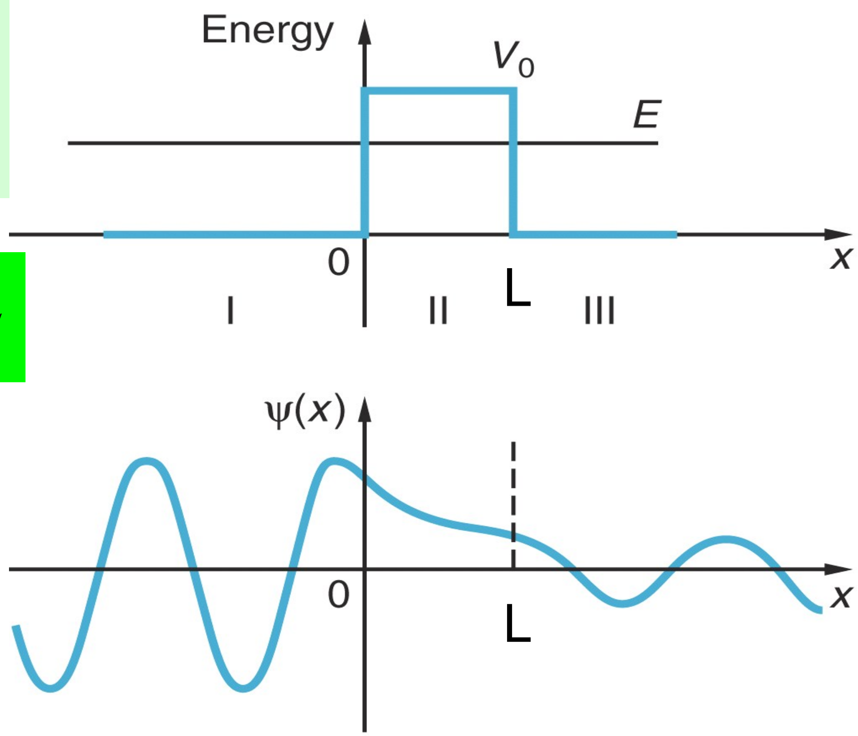
Quantum tunneling probability

<u>The probability of tunneling depends on two parameters:</u>

1. The parameter α measures how quickly the exponential decays and $\lambda=1/\alpha$ is the penetration depth (how far the wave function penetrates).

$$\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$$

2. The width of the barrier *L* measures how far the particles has to travel to get to the other side.



The quantum tunneling probability is $P \approx e^{-2\alpha L}$

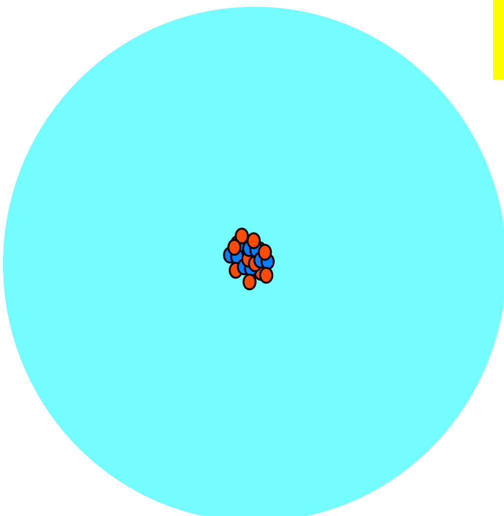
As α increases (penetration depth decreases), probability decreases.

As *L* increases (barrier width increases), probability decreases.

Radioactive decay

Radon-222:
86 protons,
136 neutrons

● Proton (**positive charge**)
● Neutron (**no charge**)



Two competing forces act inside the nucleus:

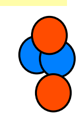
Coulomb force: Protons have the same charge and are very close together so there is a large repulsion from the Coulomb force.

Nuclear force: Protons and neutrons feel the *strong force* which is a very strong **attractive** force but very short range.

Nuclei with many protons and neutrons are generally unstable.

One type of radioactive decay is called alpha decay which releases an α particle)

Alpha particle is 2 neutrons + 2 protons (Helium nucleus)



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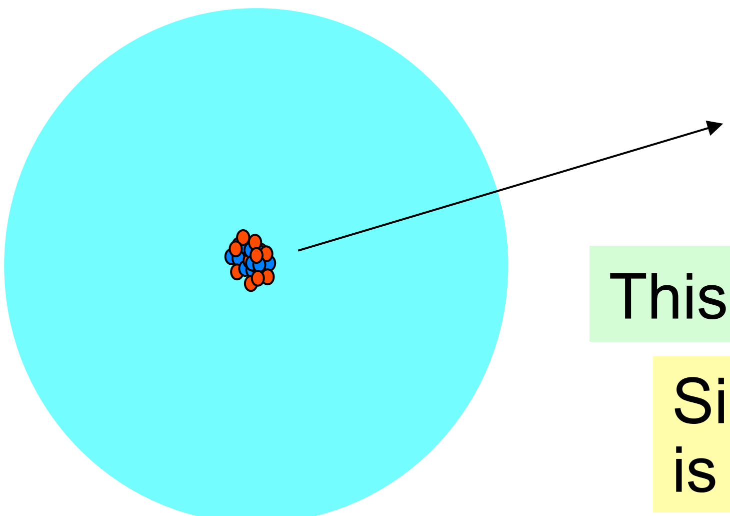


In alpha-decay, an alpha-particle is emitted from the nucleus.

This raises the ratio of neutrons to protons

Since neutrons are neutral, there is no Coulomb repulsion.

Thus, increasing the neutron to proton ratio makes a more stable nucleus.

Analyzing alpha decay

Starting point *always* to look at potential energy curve for particle

Nucleus (Z protons & bunch o' neutrons) = New nucleus (Z-2 protons & bunch o' neutrons) + Alpha particle (2 protons & 2 neutrons)

Look at this system as the distance between the alpha particle and the nucleus changes.

As we bring the α particle closer, what happens to the potential energy? Answering this question will help us figure out the potential energy curve.

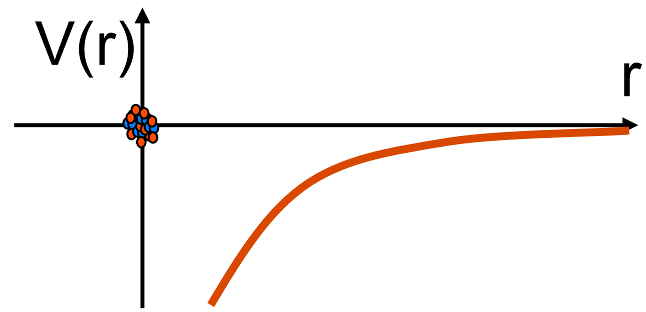
Clicker question 1

Set frequency to DA

As α particle gets closer, what happens to the potential energy? Which is the best representation of the potential energy?

$$V(r) = \frac{kq_1q_2}{r} = \frac{k\overbrace{(Z-2)(e)}^{\text{nucleus}}\overbrace{(2e)}^{\alpha}}{r}$$

A



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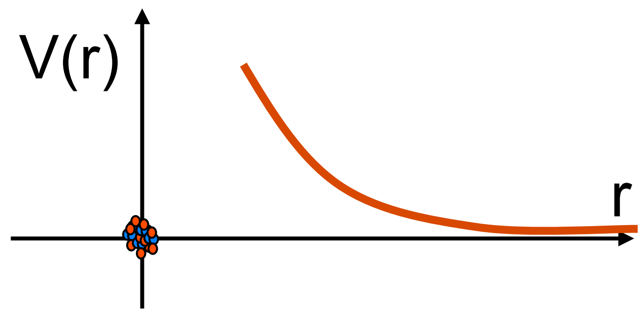

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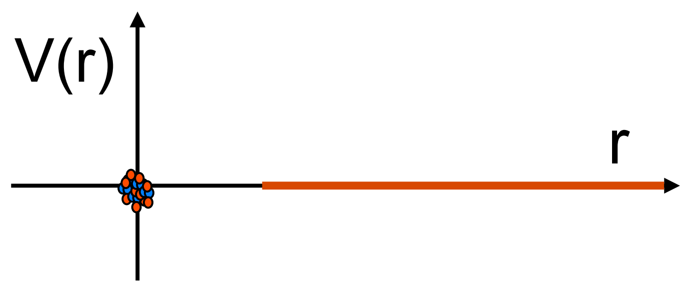
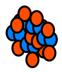


D. Something else

Clicker question 1

Set frequency to DA



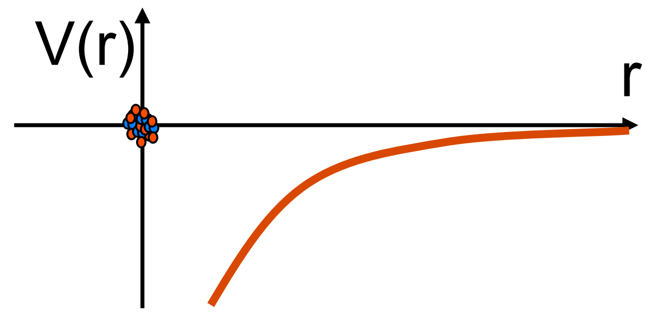
Far away, V=0

As α particle gets closer, what happens to the potential energy? Which is the best representation of the potential energy?

$$V(r) = \frac{kq_1q_2}{r} = \frac{\overbrace{k(Z-2)(e)}^{\text{nucleus}}\overbrace{(2e)}^{\alpha}}{r}$$

Takes energy to push positively charged α towards positively charged nucleus, so potential energy must increase.

A



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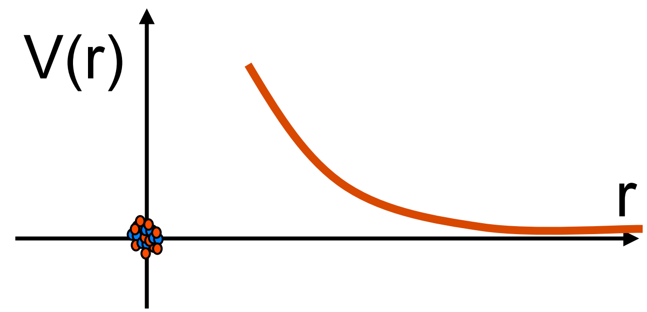

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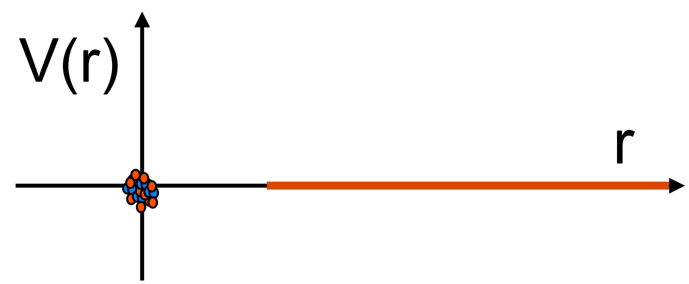
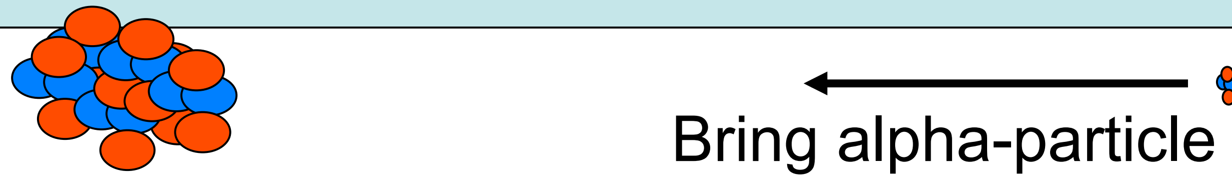
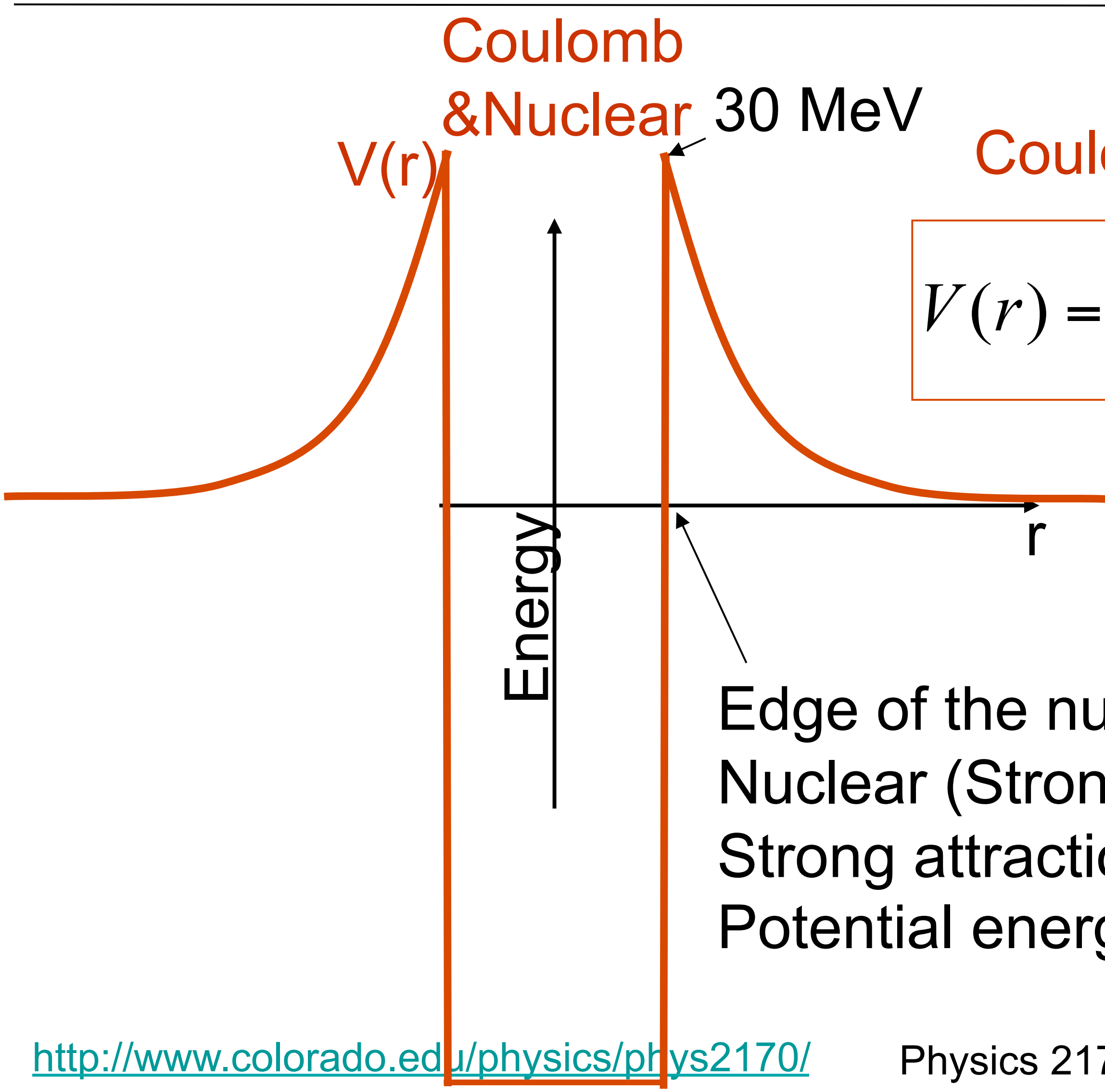


D. Something else



Bring alpha-particle closer

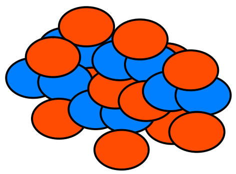


Coulomb force dominates

$$V(r) = \frac{kq_1q_2}{r} = \frac{k(Z-2)(e)(2e)}{r}$$

Edge of the nucleus (~8x10^{-15} m),
Nuclear (Strong) force starts acting
Strong attraction between nucleons
Potential energy drops dramatically

Potential energy curve for alpha decay



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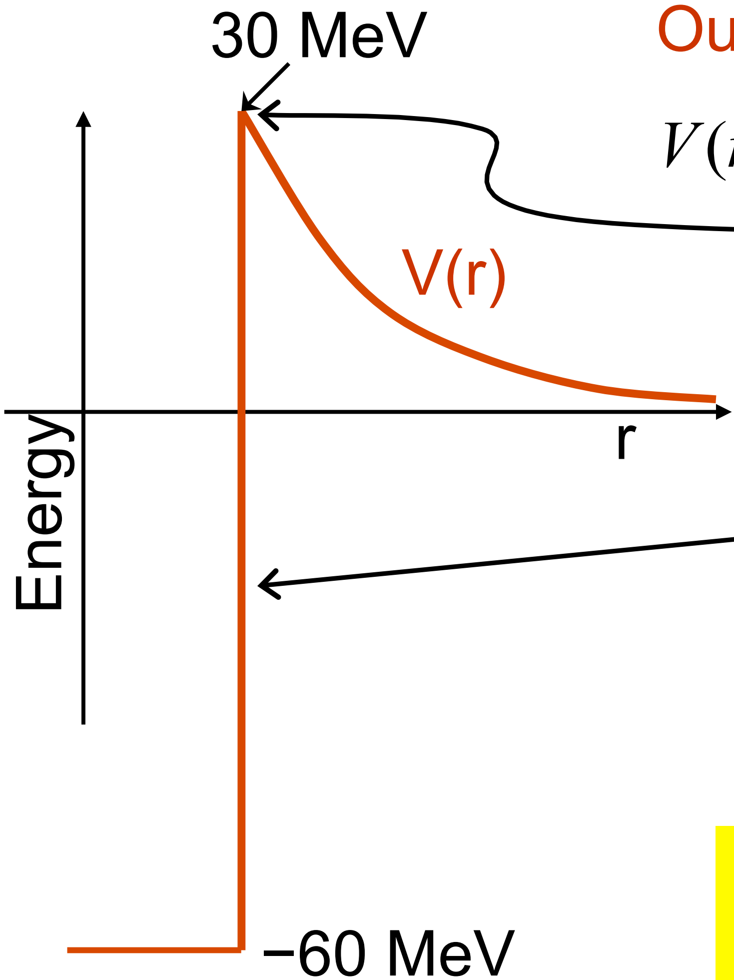
Bring alpha-particle closer



Outside nucleus, Coulomb force dominates

$$V(r) = \frac{kq_1q_2}{r} = \frac{k(Z-2)(e)(2e)}{r} = \frac{ke^2(Z-2)(2)}{r}$$

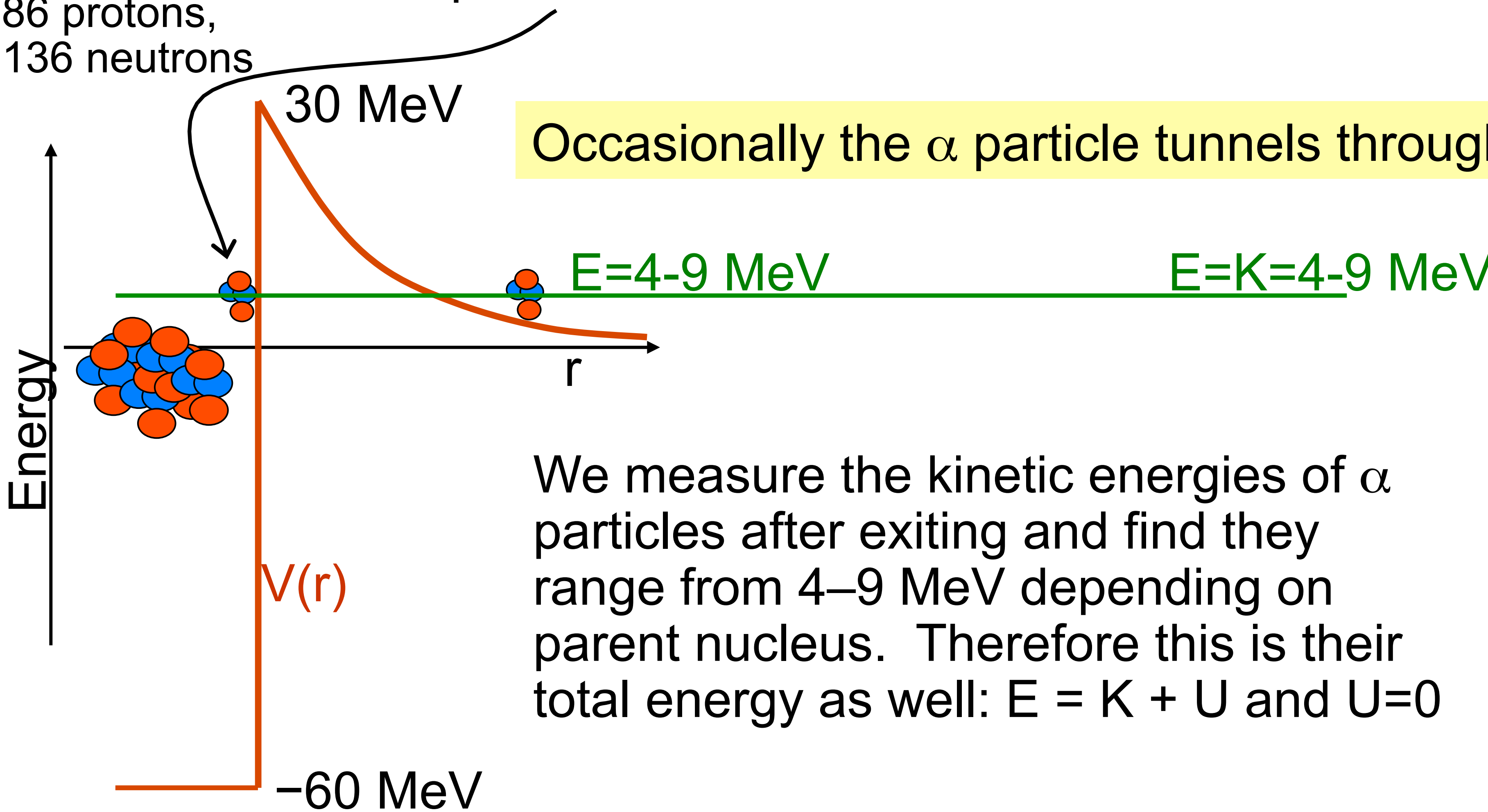
What is the max height of *V(r)*?

Nucleus has radius of 8 fm = $8\text{x}10^{-15}$ m so this is where the strong force takes over.

$$V(8\text{ fm}) = \frac{(1.44\text{ MeV}\cdot\text{fm})(84)(2)}{8\text{ fm}} = 30\text{ MeV}$$

Strong force dominates at *r*<8 fm and behaves like a deep potential well.

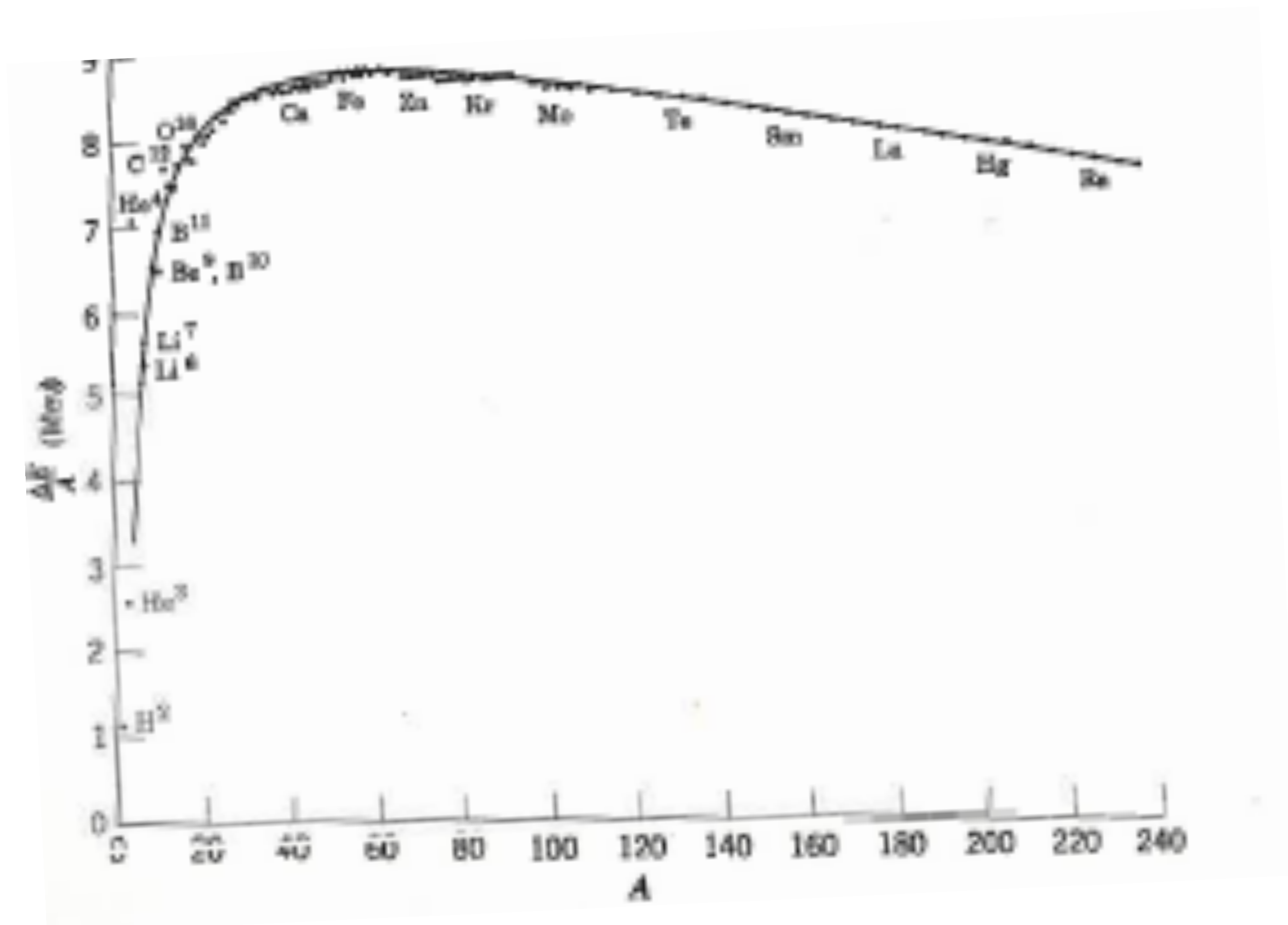
Potential energy curve for alpha decay



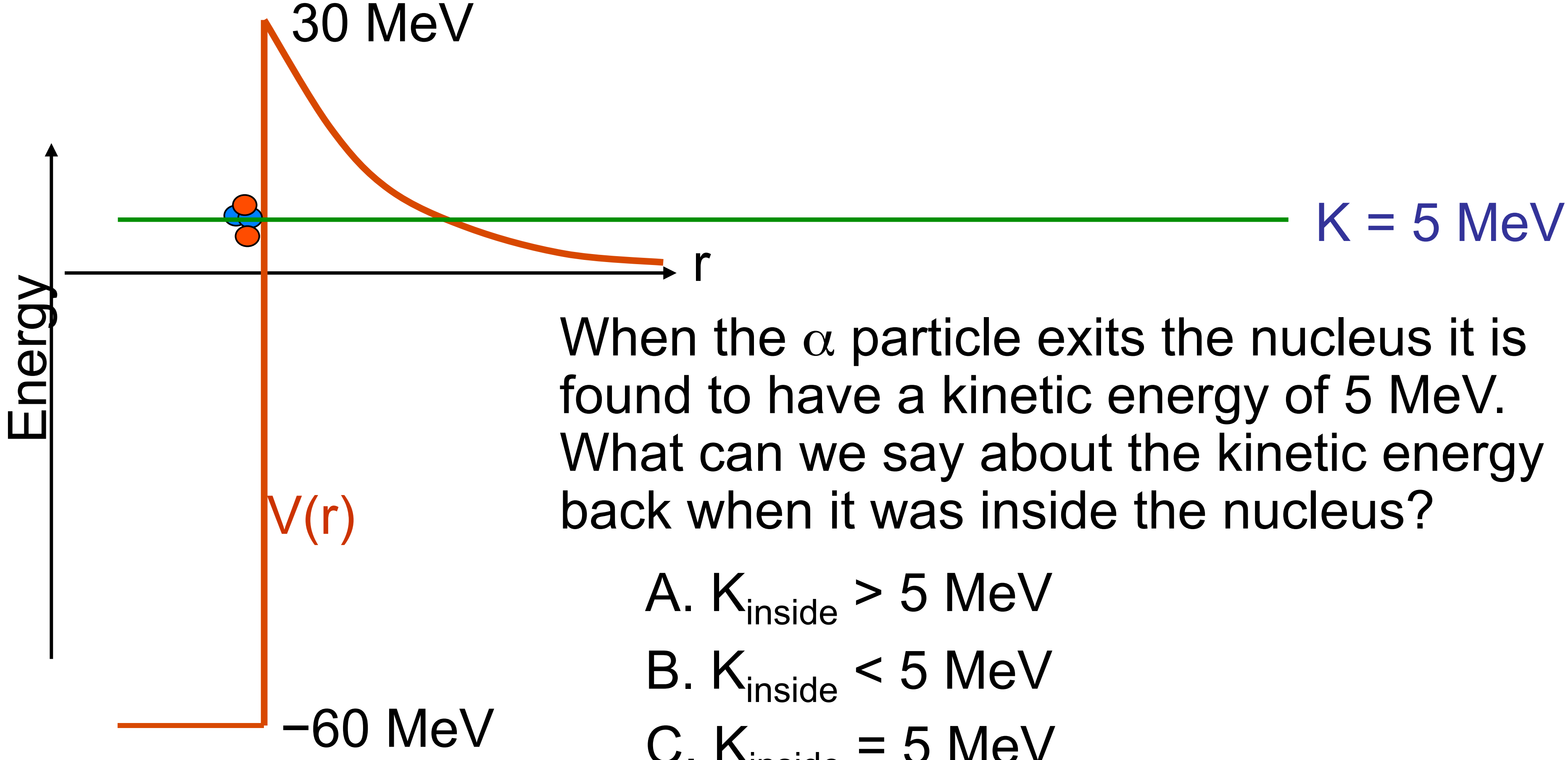
Occasionally the α particle tunnels through

We measure the kinetic energies of α particles after exiting and find they range from 4–9 MeV depending on parent nucleus. Therefore this is their total energy as well: E = K + U and U=0

Binding Energy of Nucleus versus A



Clicker question 2 | Set frequency to DA



When the α particle exits the nucleus it is found to have a kinetic energy of 5 MeV. What can we say about the kinetic energy back when it was inside the nucleus?

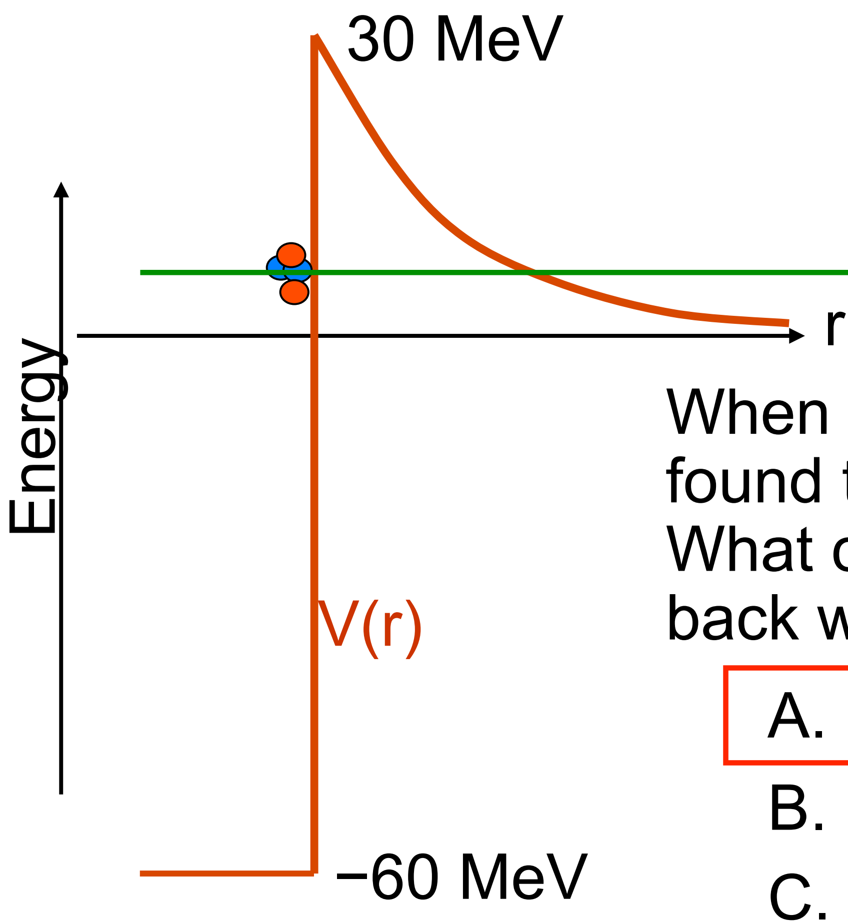
A. K_{inside} > 5 MeV

B. K_{inside} < 5 MeV

C. K_{inside} = 5 MeV

D. Impossible to know

Clicker question 2 | Set frequency to DA



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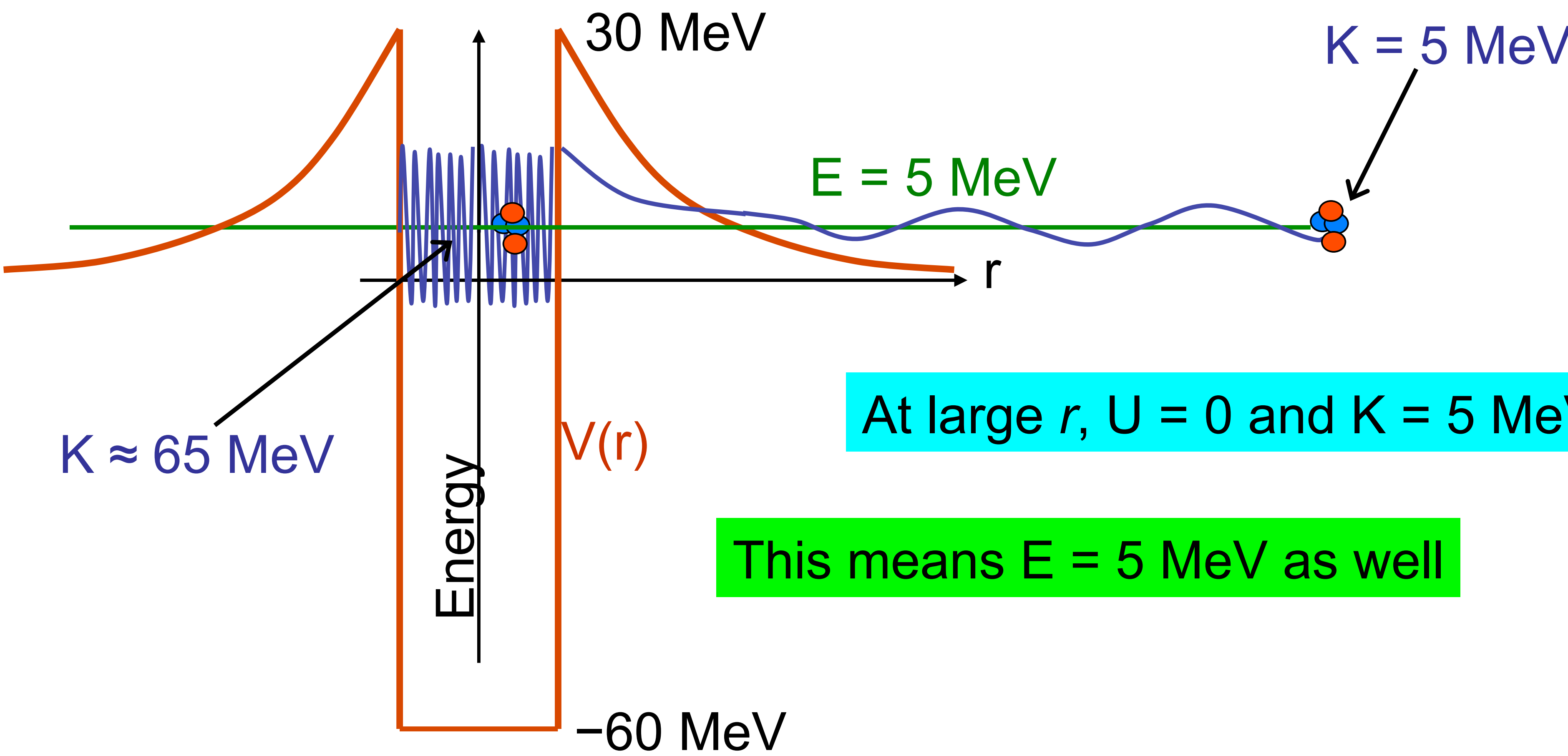
C. K_{inside} = 5 MeV

D. Impossible to know

Energy (E = K + U) is conserved so E = 5 MeV.
Inside it has U=−60 MeV so K = E − U = 65 MeV.

Conservation of energy for the α particle



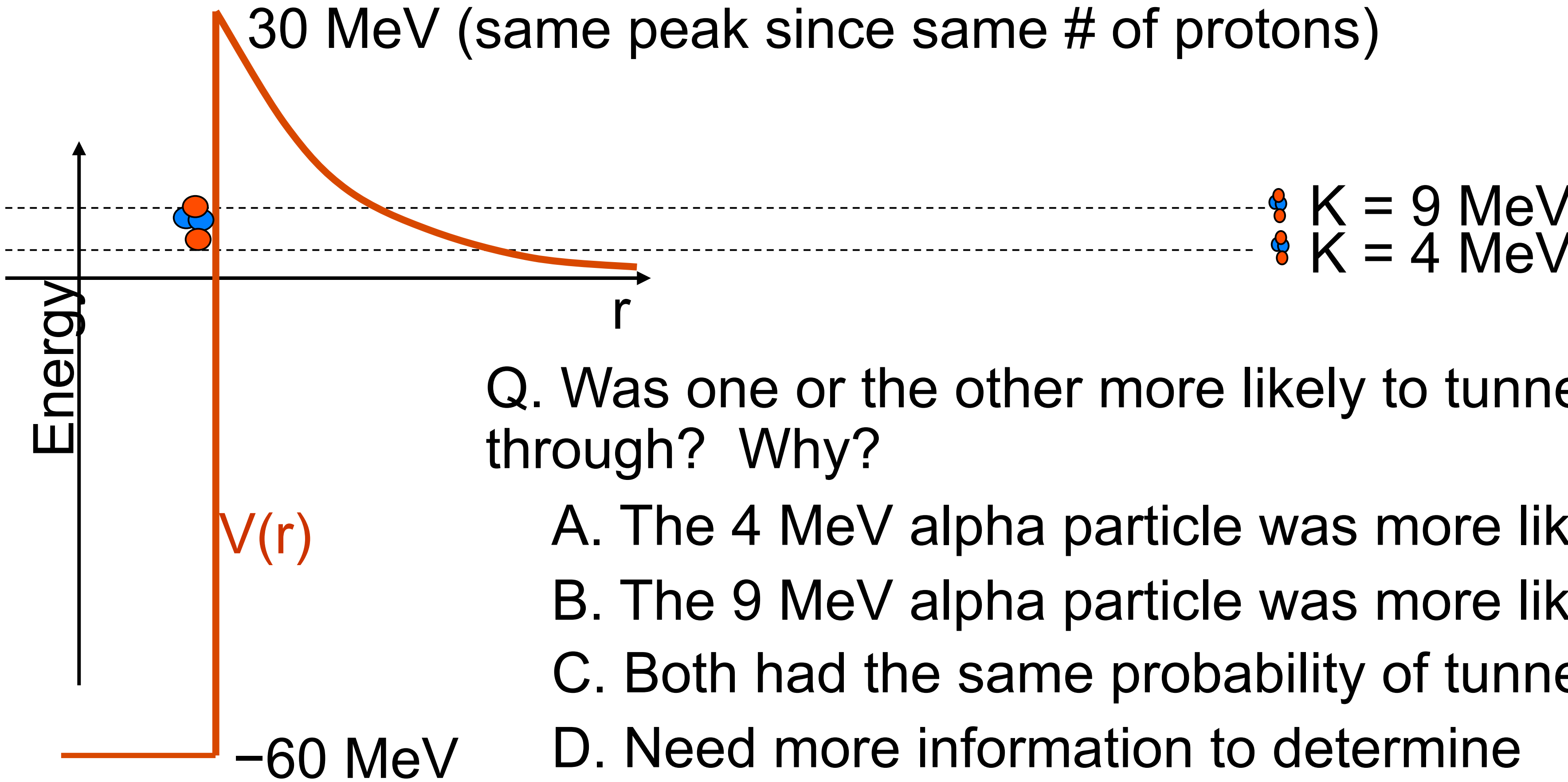
At large *r*, U = 0 and K = 5 MeV

This means E = 5 MeV as well

Inside nucleus, U = −60 MeV and
E = 5 MeV so K = E – U = 65 MeV

Clicker question 3 | Set frequency to DA

Experimentally, we find that different isotopes (same # of protons, different # of neutrons) emit α particles with different energies.



Q. Was one or the other more likely to tunnel through? Why?

A. The 4 MeV alpha particle was more likely
B. The 9 MeV alpha particle was more likely
C. Both had the same probability of tunneling
D. Need more information to determine

Clicker question 3 — Set frequency to DA

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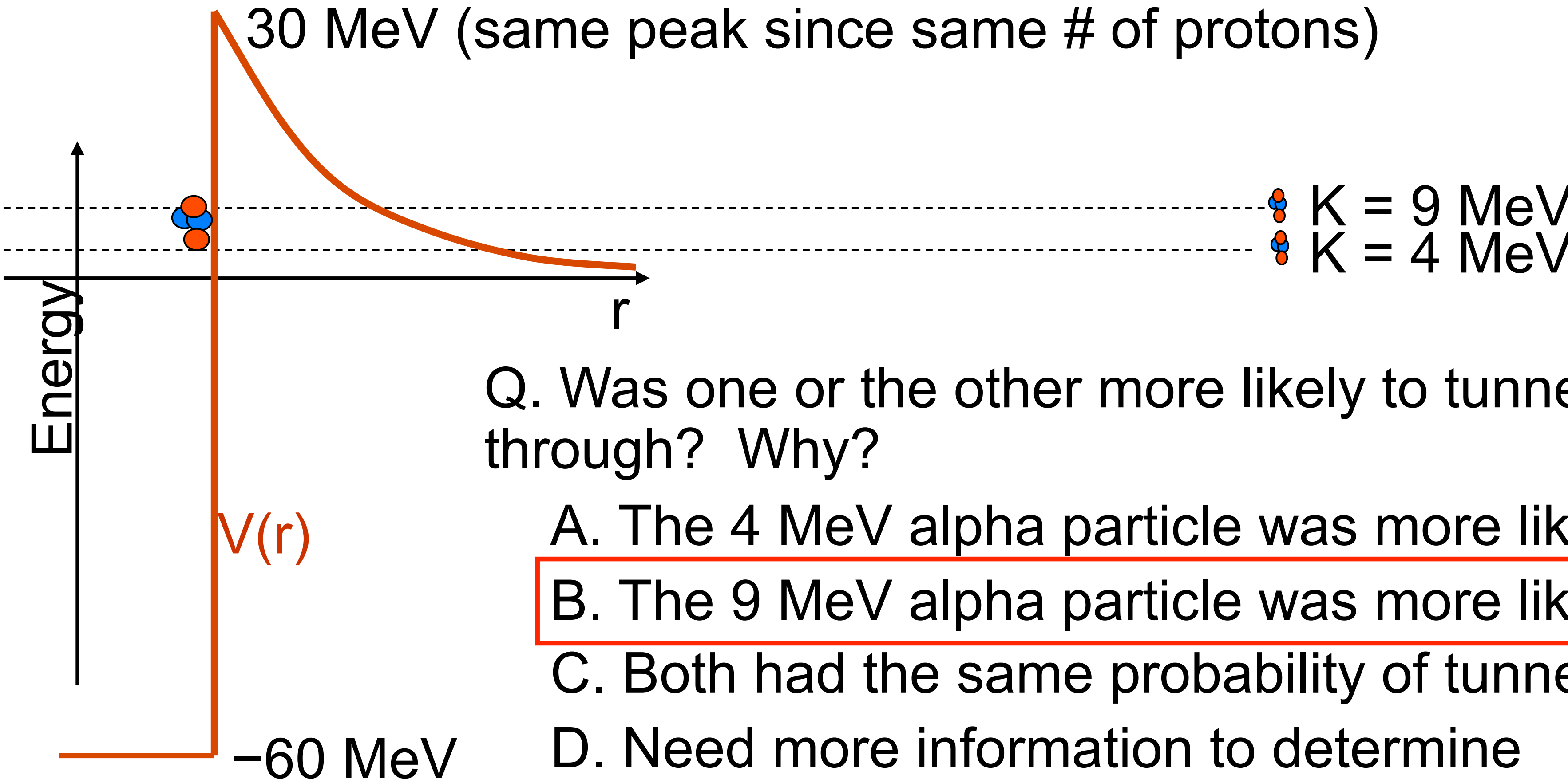

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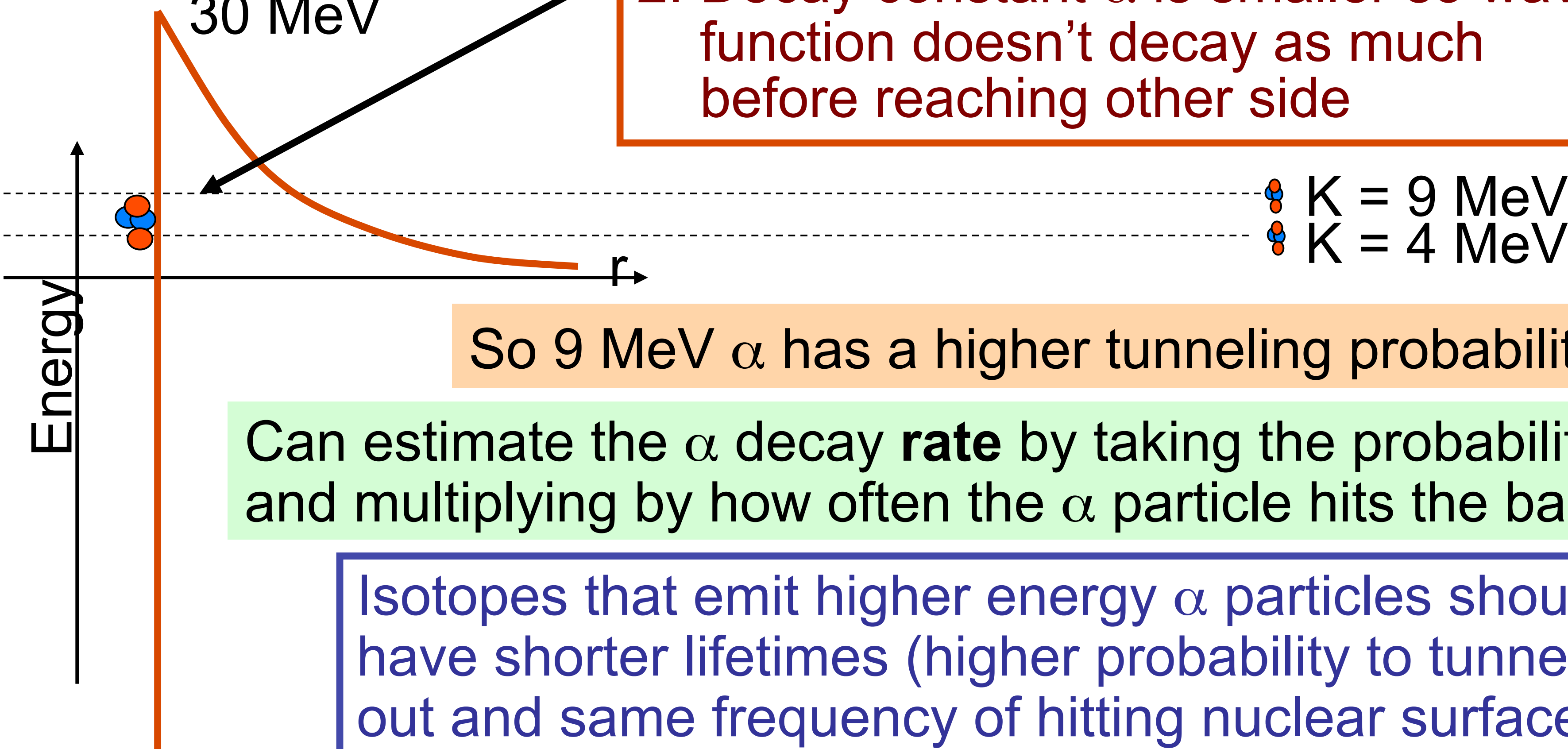
C. Both had the same probability of tunneling

D. Need more information to determine

α decay probability

Probability of tunneling is $P \approx e^{-2\alpha L}$ where $\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$

1. Less distance to tunnel (smaller *L*)
2. Decay constant α is smaller so wave function doesn't decay as much before reaching other side



So 9 MeV α has a higher tunneling probability

Can estimate the α decay **rate** by taking the probability and multiplying by how often the α particle hits the barrier

Isotopes that emit higher energy α particles should have shorter lifetimes (higher probability to tunnel out and same frequency of hitting nuclear surface)

Experimentally confirmed! See text for details.

Lifetime versus Energy