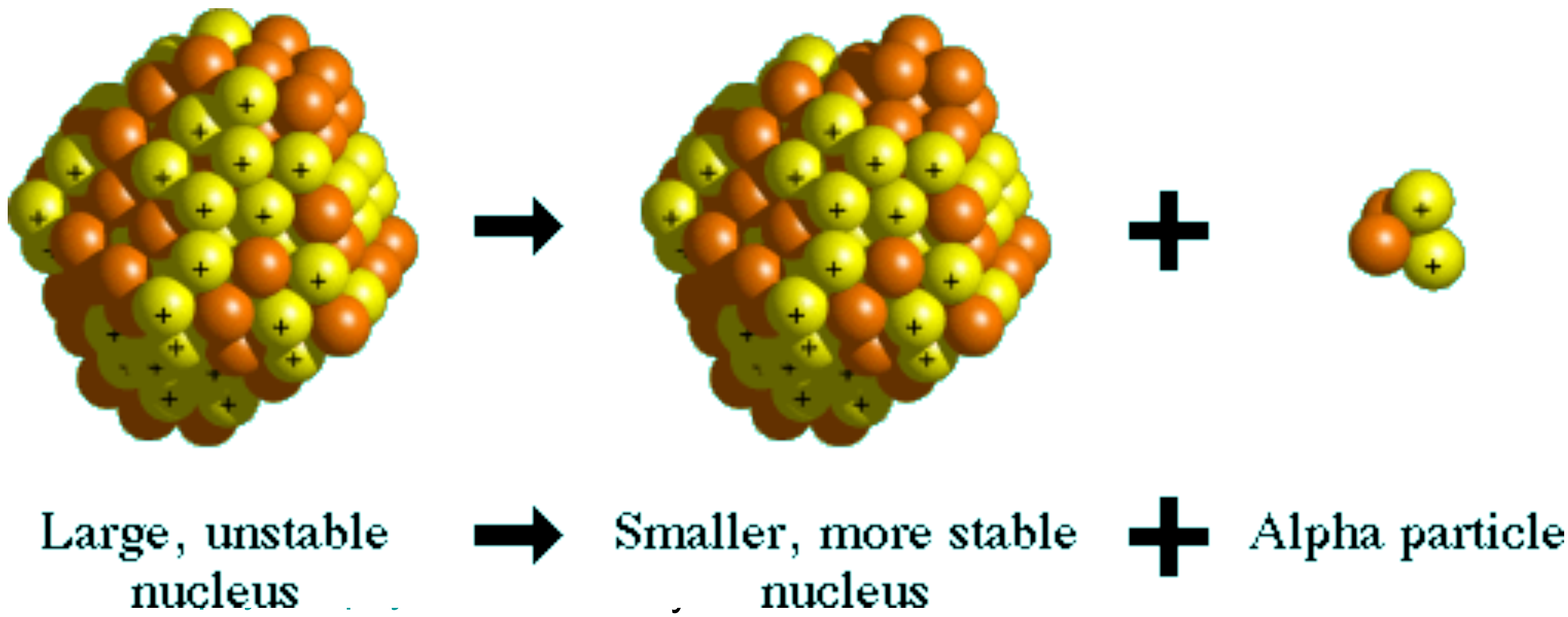
Quantum tunneling: α-decay

Announcements:

- Exam 2 solutions are posted on CULearn
- Next weeks homework is available – will be due a week from today.
- I hope to have the exams graded by class on Monday.
- Cursory glance shows that many people did well on the exam.

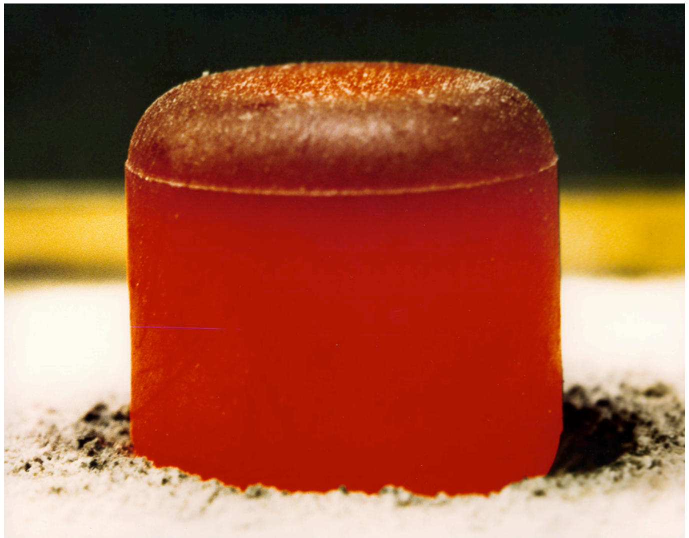


Friday Facts ---WD-40

Maybe you didn't know but WD-40 stands for Water Displacement, 40th attempt. Name was coined by the chemist, Norm Larsen, while he was attempting to concoct a formula to prevent corrosion by displacing water. Norm's persistence paid off when he perfected the formula on his 40th try.

What's an RTG?

- A **radioisotope thermoelectric generator** (**RTG**) is an electrical generator that uses thermocouples to convert heat released by the decay of radioactive material into electricity by the Seebeck effect.



A pellet of $^{238}PuO_2$ to be used in an RTG for either the Cassini or Galileo mission. The initial output is 62 watts.

Clicker Question 1 | Set frequency to AD

Please answer this question on your own.

Q. Which of the following is an example of quantum tunneling?

A. Radioactive decay (α decay)
B. Photoelectric effect
C. Scanning tunneling microscope
D. Time dilation
E. More than one of the above

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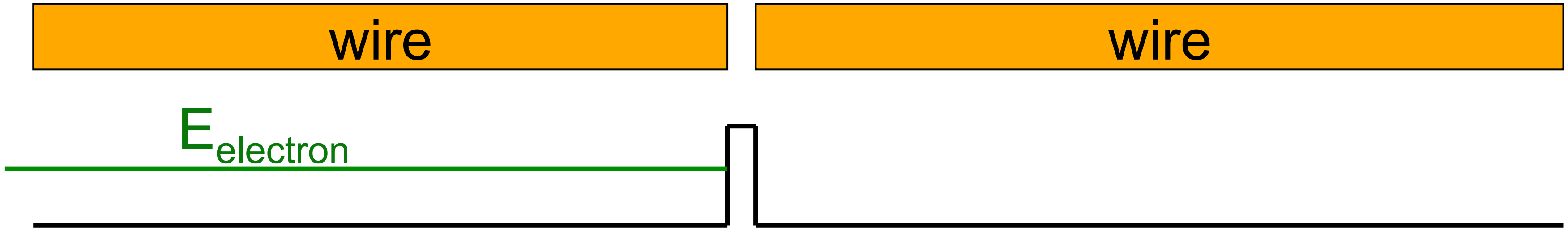
Quantum tunneling through potential barrier

Consider the slightly simpler case of two very long wires separated by a small gap:



This is an example of a <u>*potential barrier.*</u>

Q*uantum tunneling* occurs when a particle which does not have enough energy to go over the potential barrier somehow gets to the other side of the barrier.

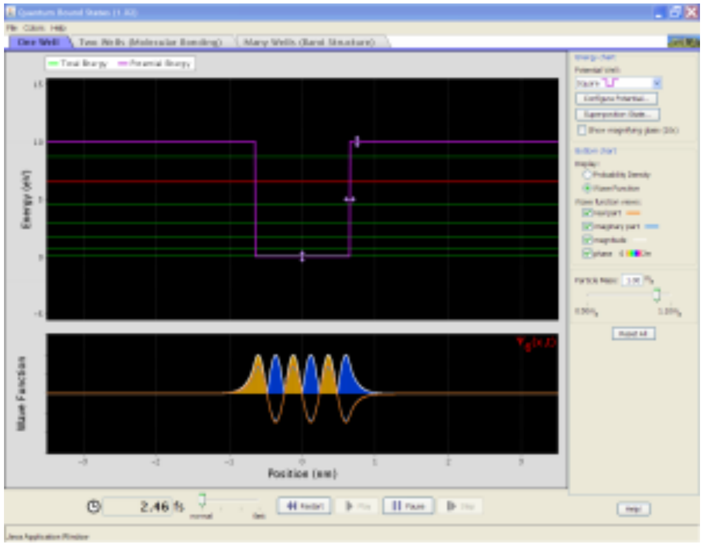
This is due to the particle being able to penetrate into the *classically forbidden region*.

If it can penetrate far enough (the barrier is thin enough) it can come out the other side.

Quantum bound state simulation

http://phet.colorado.edu/simulations/sims.php?sim=Quantum_Bound_States

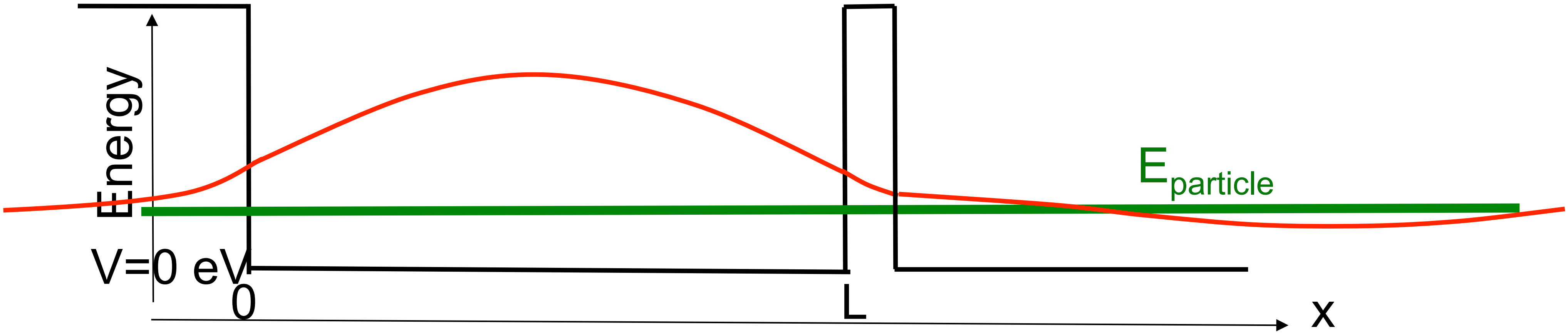
The quantum bound state simulation can be used to figure out and visualize wave functions and probabilities for various potential curves.



Will need to look at this site for next week's homework!

On to quantum tunneling

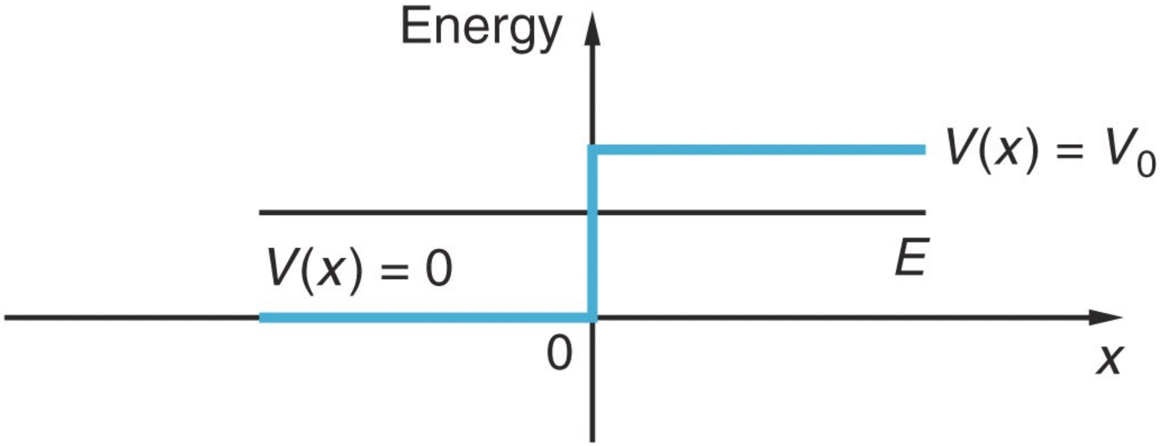
The thinner or shorter the barrier, the easier it is to tunnel …

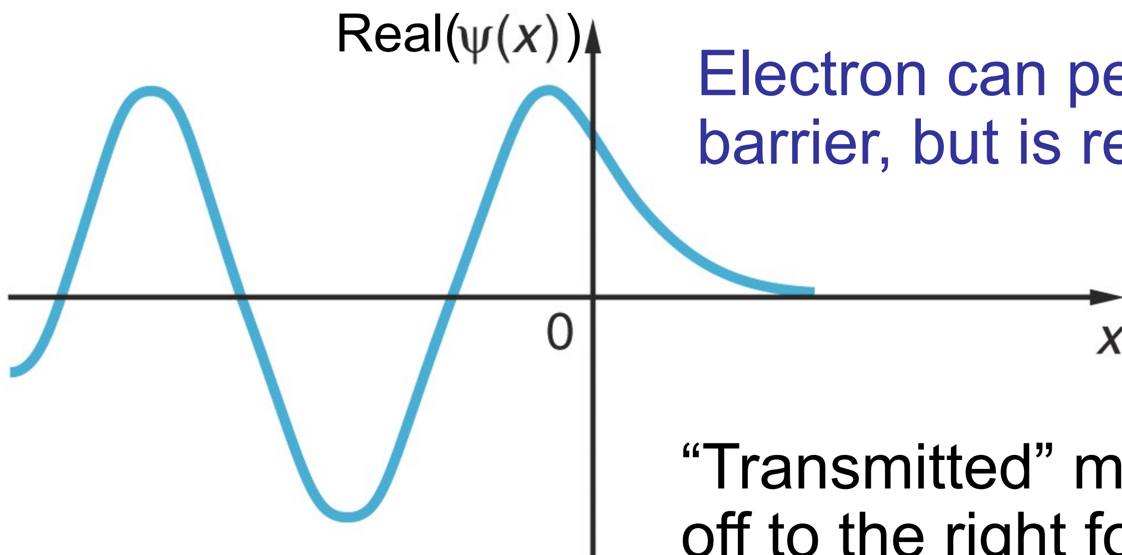


Examples:

- Electron in wire going through air gap (Tutorial)
- Alpha decay: Explained by Gamow and seen in smoke detectors, radon, space probe power, and assassinations
- Scanning tunneling microscope
- Getting shocked just before touching the door knob

Electron encounters potential step

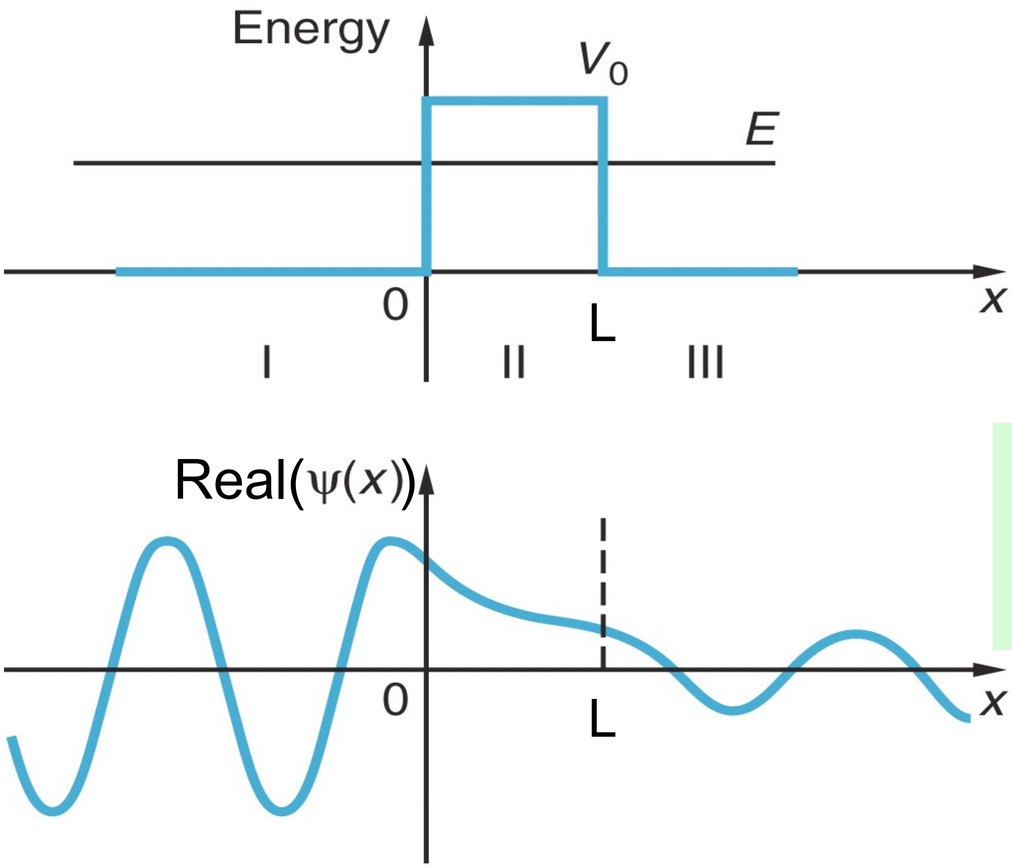




Electron can penetrate into the barrier, but is reflected eventually.

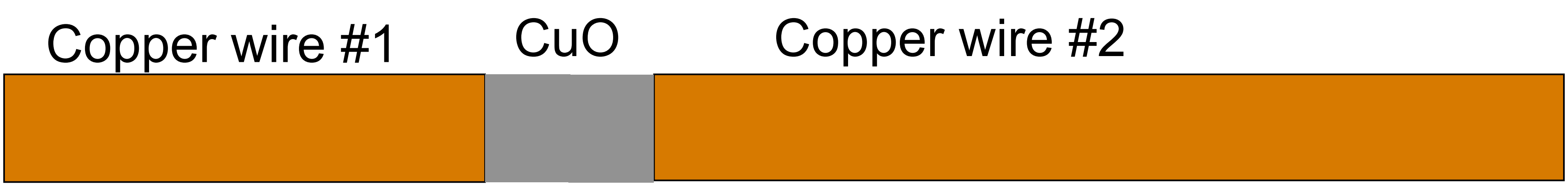
"Transmitted" means continues off to the right forever, i.e. the wave function does not go down to zero.

Quantum tunneling



If the potential increase has a finite width, it is a potential *barrier* and the electron can tunnel out of Region I

This is what you were encouraged to investigate in the tutorial



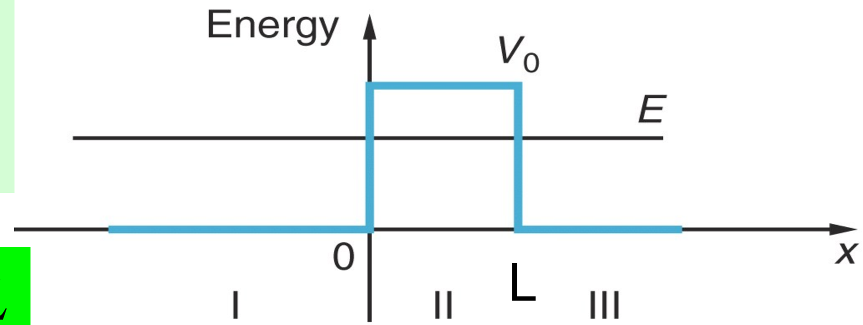
Quantum tunneling probability

<u>The probability of tunneling depends on two parameters:</u>

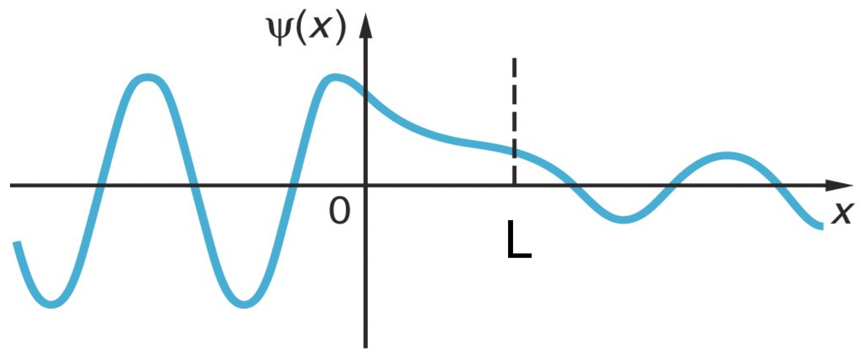
1. The parameter α measures how quickly the exponential decays and $\lambda=1/\alpha$ is the penetration depth (how far the wave function penetrates).

$$\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$$

2. The width of the barrier *L* measures how far the particles has to travel to get to the other side.



The quantum tunneling probability is $P \approx e^{-2\alpha L}$

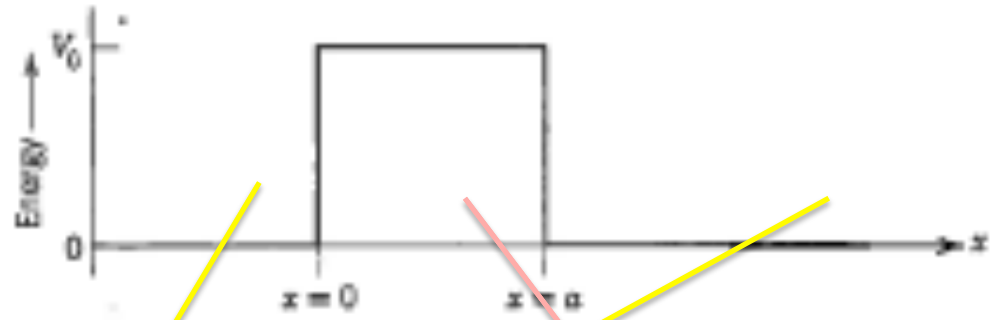


As α increases (penetration depth decreases), probability decreases.

As *L* increases (barrier width increases), probability decreases.

Barrier Potential Boundary Conditions

$$V(x) = \begin{cases} V_0, & 0 < x < a \\ 0, & x < 0,\ x > a \end{cases}$$



$$\psi(x) = Ae^{iK_1x} + Be^{-iK_1x} \qquad x < 0$$

$$\psi(x) = Ce^{iK_1x} + De^{-iK_1x} \qquad x > a$$

$$\text{with } K_1 = \sqrt{2mE}\,/\hbar$$

$$\psi(x) = Fe^{K_2x} + Ge^{-K_2x} \qquad 0 < x < a\ ;\ \ E < V_0$$

$$\psi(x) = Fe^{iK_2x} + Ge^{-iK_2x} \qquad 0 < x < a\ ;\ \ E > V_0$$

$$\text{with } K_2 = \sqrt{2m(E - V_0)}\,/\hbar \qquad E > V_0$$

$$\text{with } K_2 = \sqrt{2m(V_0 - E)}\,/\hbar \qquad E < V_0$$

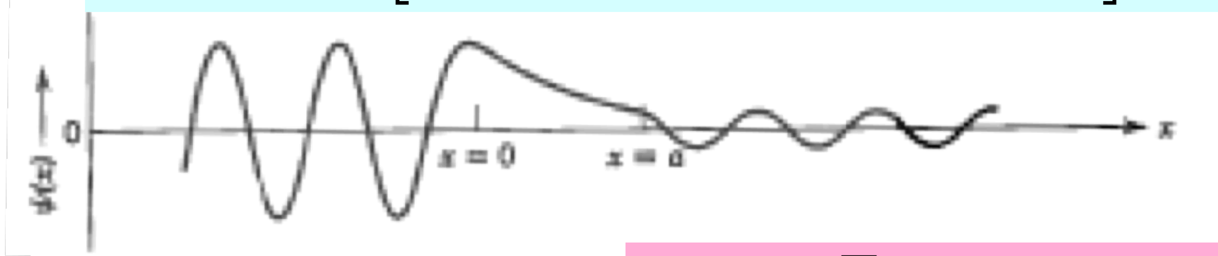
Can set D=0,
But not G!

Barrier Cont. E<V

Matching the wave function and first derivatives at x=0 and x=a, defines A,B,C, F, G

$$T = \frac{v_1 C^* C}{v_1 A^* A} = \left[1 + \frac{\sinh^2 K_2 a}{(4E/V_0)(1-E/V_0)}\right]^{-1} \quad \mathrm{E} < \mathrm{V}_0$$

$$T = \frac{v_1 C^* C}{v_1 A^* A} = \left[1 + \frac{\sinh^2\left(\sqrt{\frac{2mV_0 a^2}{\hbar^2}(1-E/V_0)}\right)}{(4E/V_0)(1-E/V_0)}\right]^{-1} \quad \mathrm{E} < \mathrm{V}_0$$



If sinh x >>1, then

$$T \cong 16\frac{E}{V_0}(1-E/V_0)e^{-2K_2 a}$$

Barrier Cont. E>V

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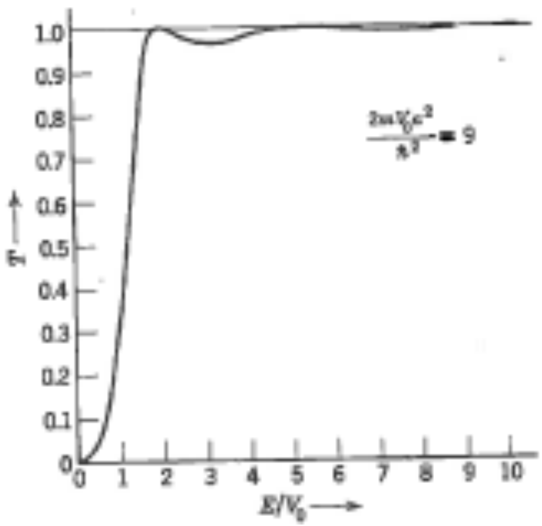
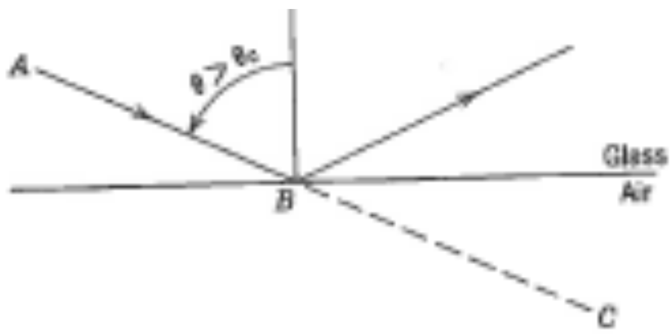
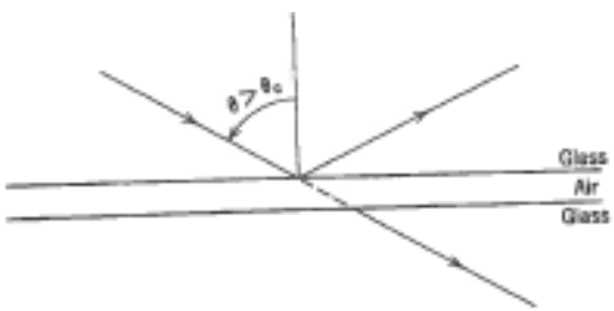


T=1, when

$K_1 a = \pi,\ 2\pi,\ 3\pi, ...$

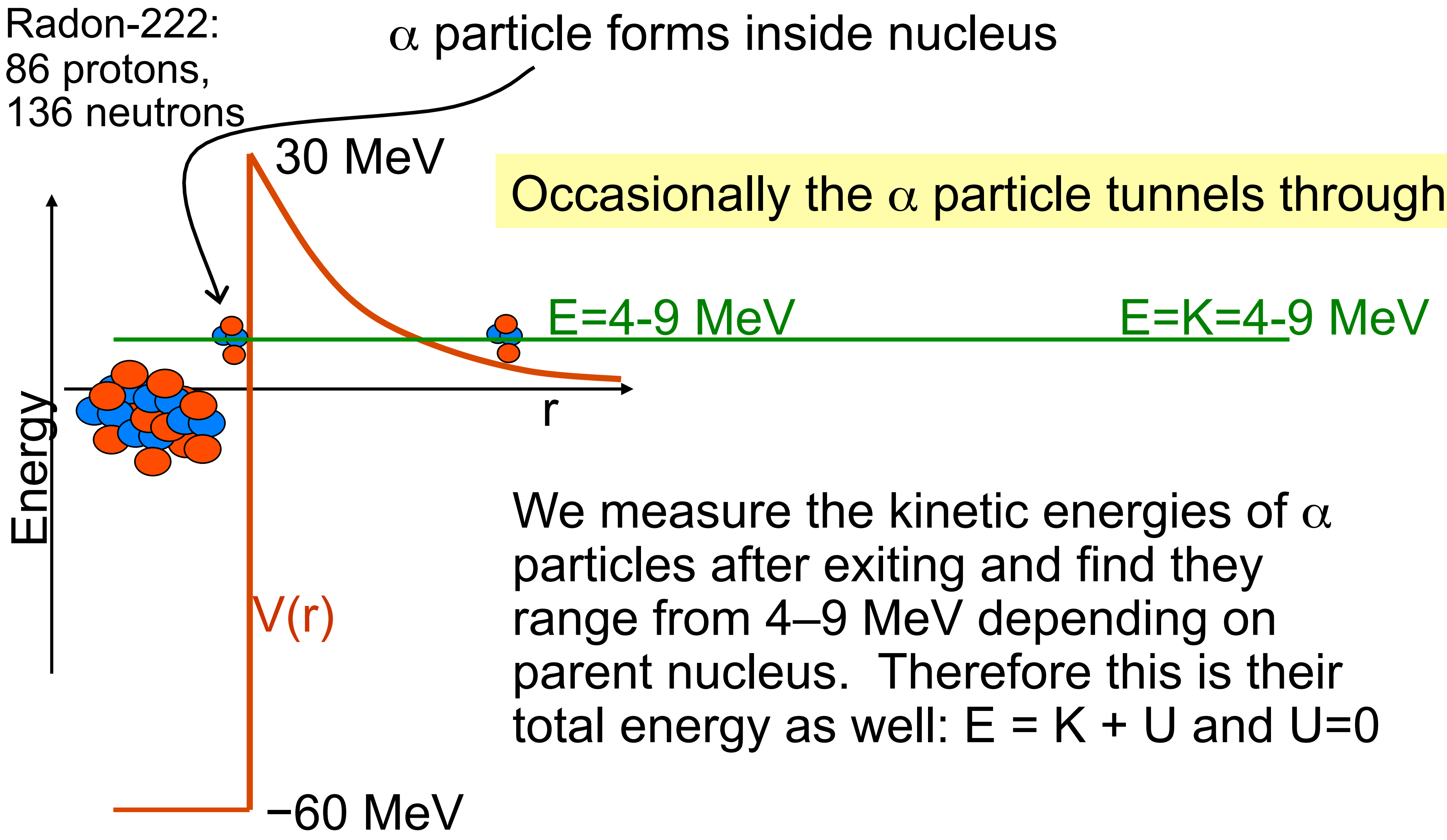
Total Internal Reflection





Frustrated total internal Reflection – found by Newton around 1700

Potential energy curve for alpha decay



Occasionally the α particle tunnels through

We measure the kinetic energies of α particles after exiting and find they range from 4–9 MeV depending on parent nucleus. Therefore this is their total energy as well: E = K + U and U=0