Finite square well

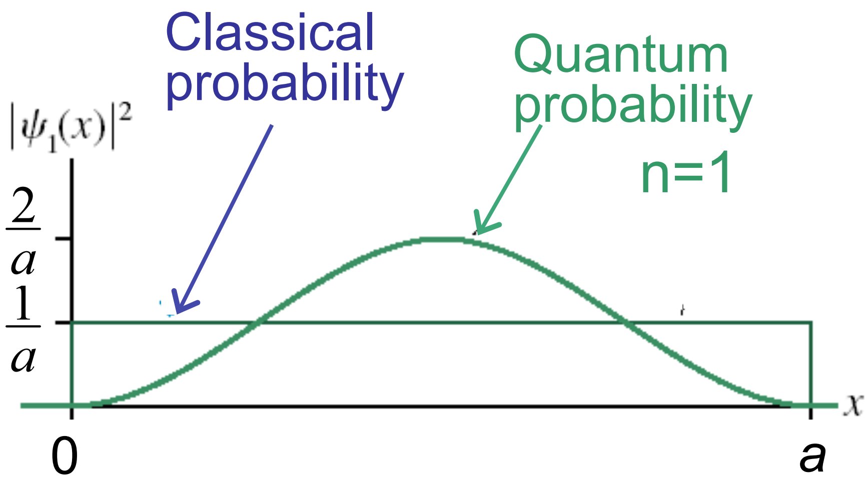
Announcements:

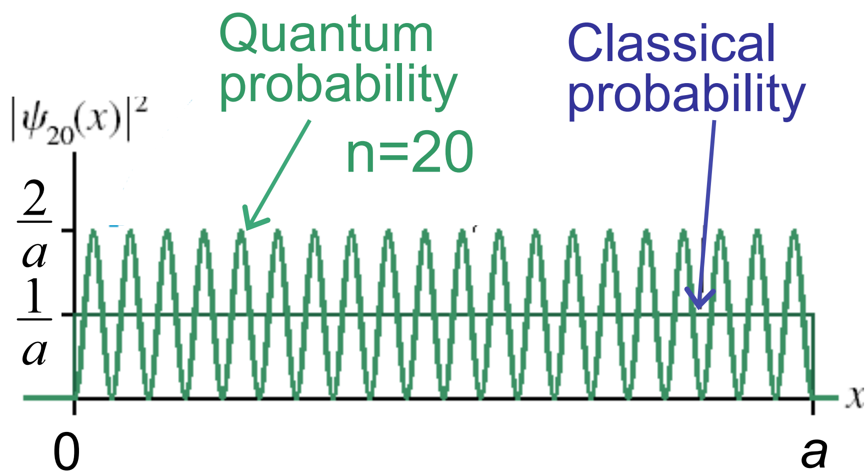
- 2^{nd} exam is next Thursday, Nov. 7 – 7:30pm-9:00pm
- Homework due today.
- Homework #7 to return
- A practice exam will be posted on CULearn sometime on Friday.

Today I will try to answer some questions raised last time, finish up the finite square well.

Correspondence principle

Proposed by Bohr: Quantum physics results should match classical physics results in the appropriate regions (large quantum number *n*).





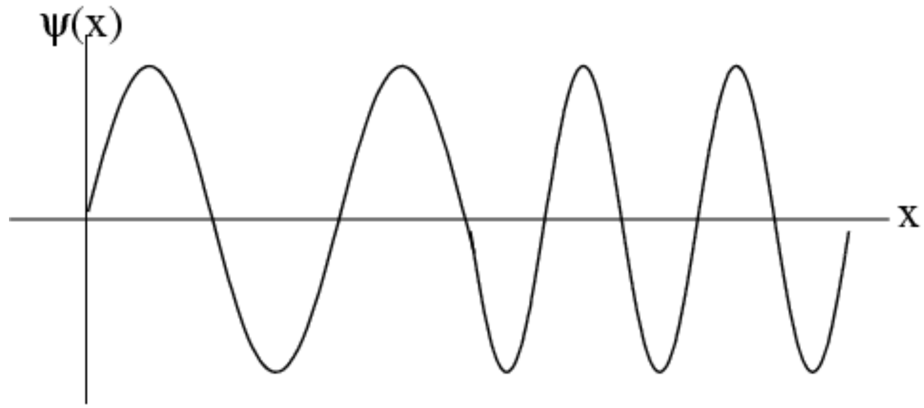
As *n* increases, the quantum probability averages out to flat across the well. This is exactly what is predicted by classical physics.

In HW 4d you will find millions of levels between ground state and average thermal energy for a normal piece of wire. This basically means the energy levels form a continuum as in classical physics.

Only really tiny wires have quantum effects at thermal energies

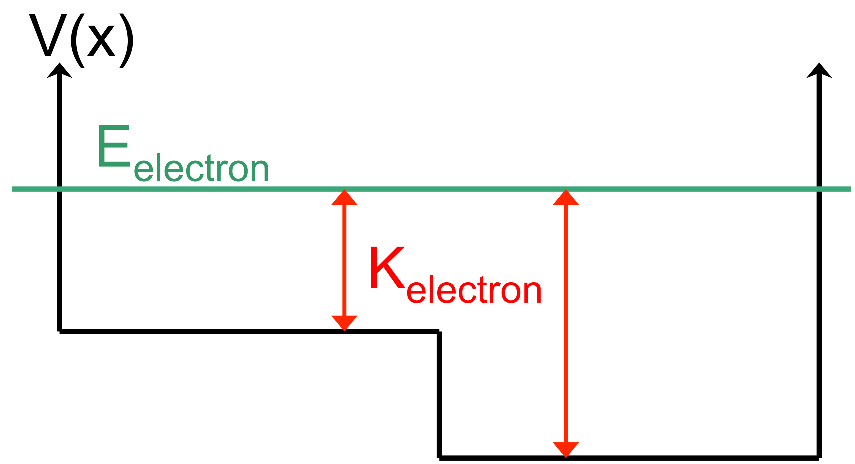
Modifications of square well potential

Wave function from question 6 of this week's homework set .



In question 1 you determine that closer spaced waves in *x* means higher wave number *k*.

From deBroglie, $p=\hbar k$. Kinetic energy = $K = p^2/2m = \hbar^2k^2/2m$, so higher *k* means higher *K*.

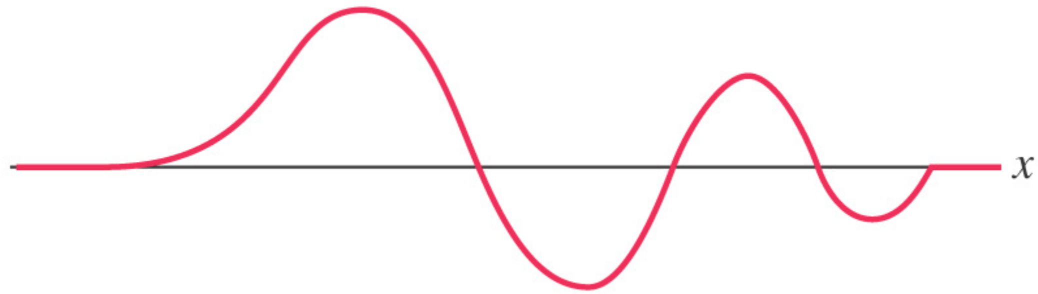


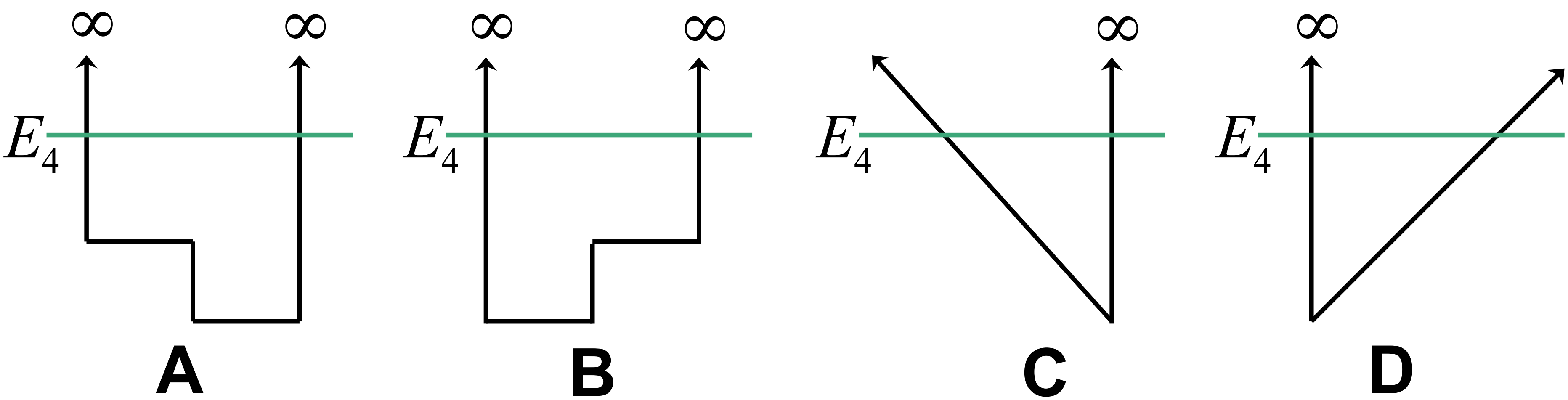
Since there are no outside forces, total energy is conserved.

If kinetic energy goes up, potential energy must go down.

Clicker question 1 | Set frequency to DA

Q. For which of the potentials below is the curve in red a possible n=4 wave function?

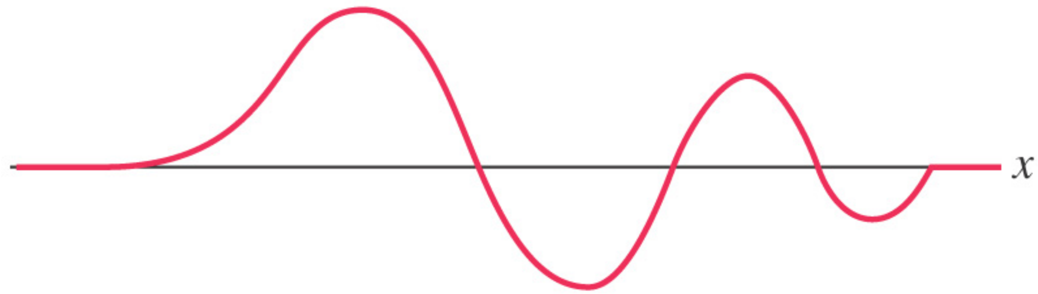


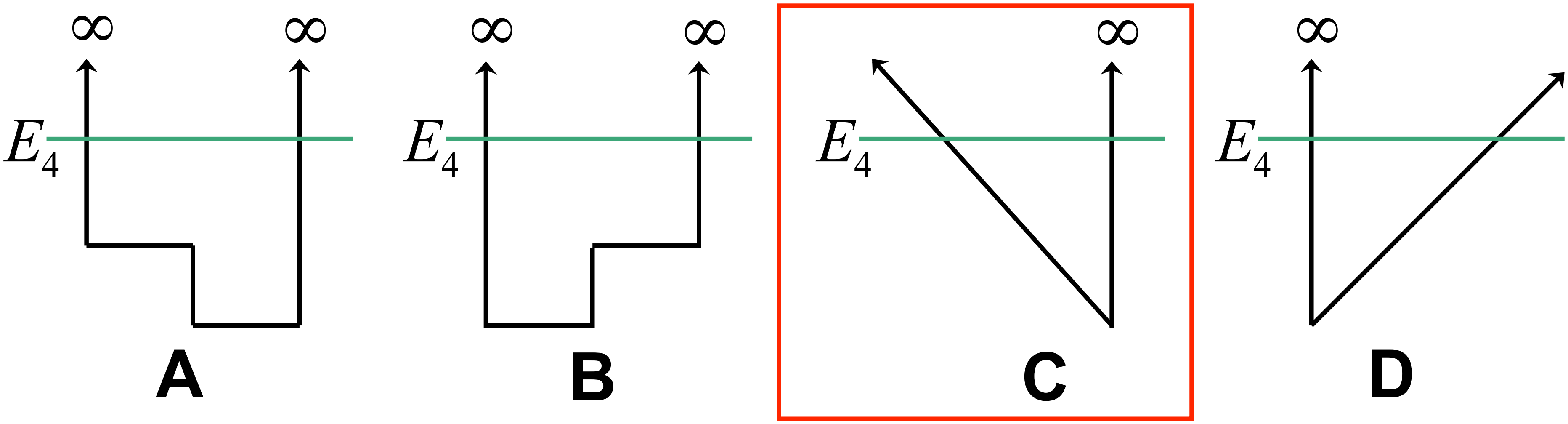


Clicker question 1

Set frequency to DA

Q. For which of the potentials below is the curve in red a possible n=4 wave function?

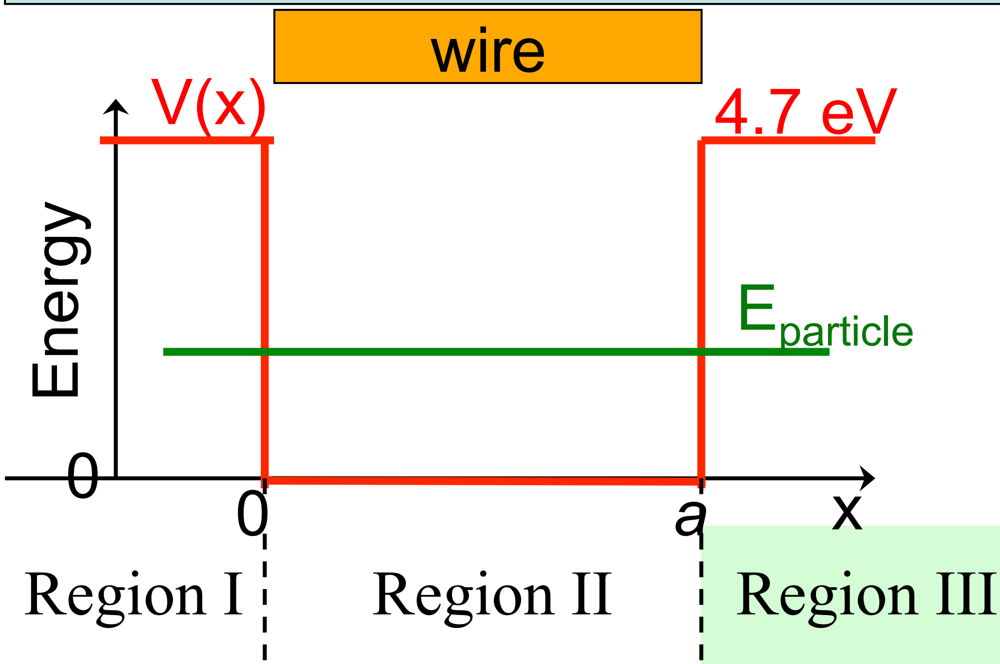




Wave number *k* goes up to the right indicating kinetic energy is going up and potential energy is going down.

Note that left side smoothly goes to 0 indicating a finite well while right side abruptly goes to zero indicating infinite well.

Analyzing the finite square well



Rewritten TISE:

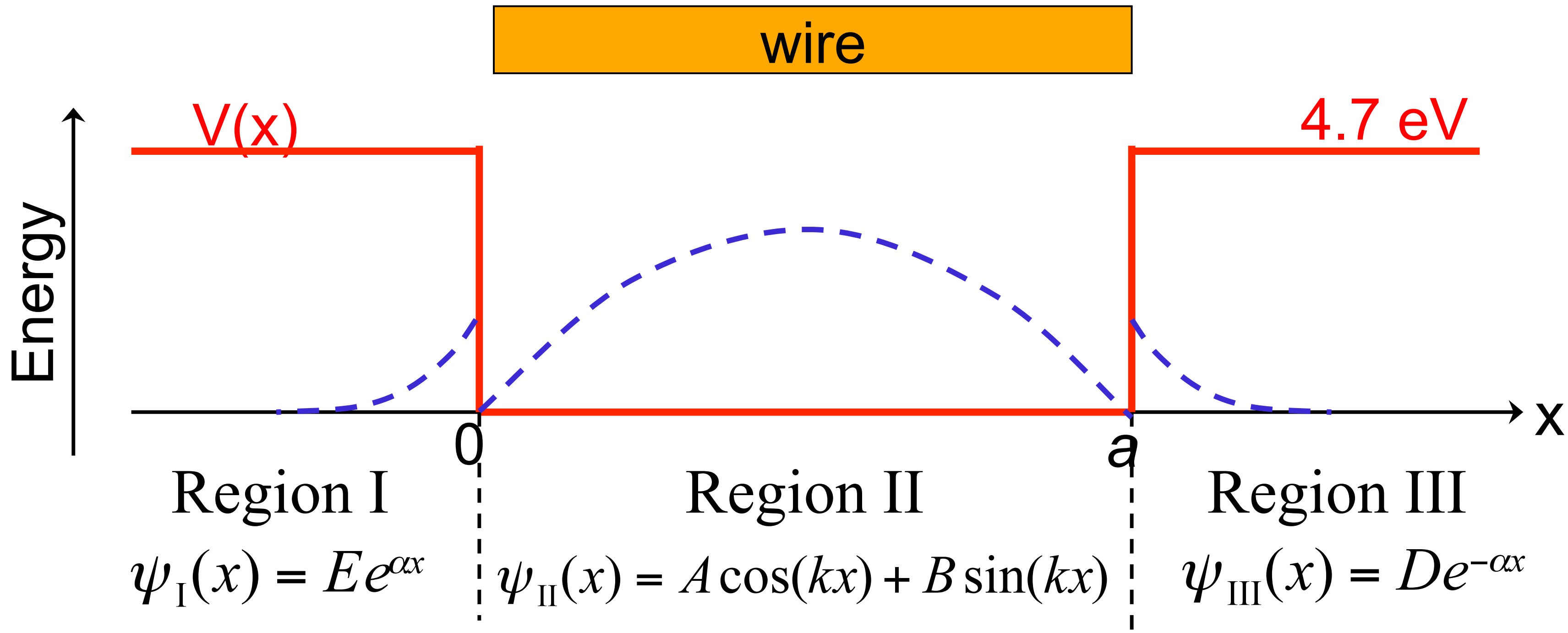
$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}\left[V(x) - E\right]\psi(x)$$

In Region I & III solutions are of the form $e^{\alpha x}$ and $e^{-\alpha x}$.

Assume $\alpha > 0$. Then for Region III, $e^{\alpha x}$ gives exponential growth and $e^{-\alpha x}$ gives exponential decay

Region III: $\psi_{III}(x) = Ce^{\alpha x} + De^{-\alpha x}$

Matching boundary conditions



Region I

$$\psi_{\mathrm{I}}(x) = Ee^{\alpha x}$$

Region II

$$\psi_{\mathrm{II}}(x) = A\cos(kx) + B\sin(kx)$$

Region III

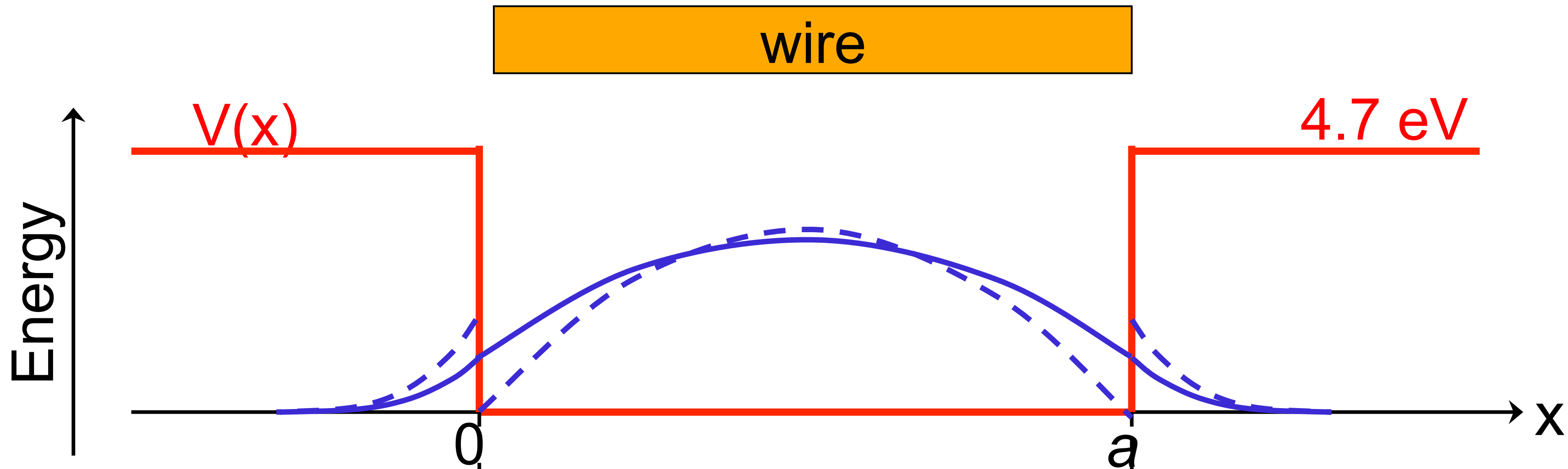
$$\psi_{\mathrm{III}}(x) = De^{-\alpha x}$$

Matching boundary conditions at x=0 and x=a requires:

ψ(x) is continuous so $\psi_{\mathrm{I}}(0) = \psi_{\mathrm{II}}(0)$ and $\psi_{\mathrm{II}}(a) = \psi_{\mathrm{III}}(a)$

$\dfrac{d\psi(x)}{dx}$ is continuous so $\dfrac{d\psi_{\mathrm{I}}(0)}{dx} = \dfrac{d\psi_{\mathrm{II}}(0)}{dx}$ and $\dfrac{d\psi_{\mathrm{II}}(a)}{dx} = \dfrac{d\psi_{\mathrm{III}}(a)}{dx}$

Matching boundary conditions



Region I	Region II	Region III
$\psi_{\mathrm{I}}(x) = Ee^{\alpha x}$	$\psi_{\mathrm{II}}(x) = A\cos(kx) + B\sin(kx)$	$\psi_{\mathrm{III}}(x) = De^{-\alpha x}$

$$\frac{d^2\psi_{\mathrm{I}}(x)}{dx^2} = \alpha^2\psi_{\mathrm{I}}(x) \qquad \frac{d^2\psi_{\mathrm{II}}(x)}{dx^2} = -k^2\psi_{\mathrm{II}}(x) \qquad \frac{d^2\psi_{\mathrm{III}}(x)}{dx^2} = \alpha^2\psi_{\mathrm{III}}(x)$$

$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}(V - E)\psi(x)$$

We didn't actually work out the math; but looked at results.

Evaluating results

$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}(V - E)\psi(x)$$

Outside well: E < V

Inside well: E > V

Outside well: E < V

Potential well is not infinite so particle is not strictly contained

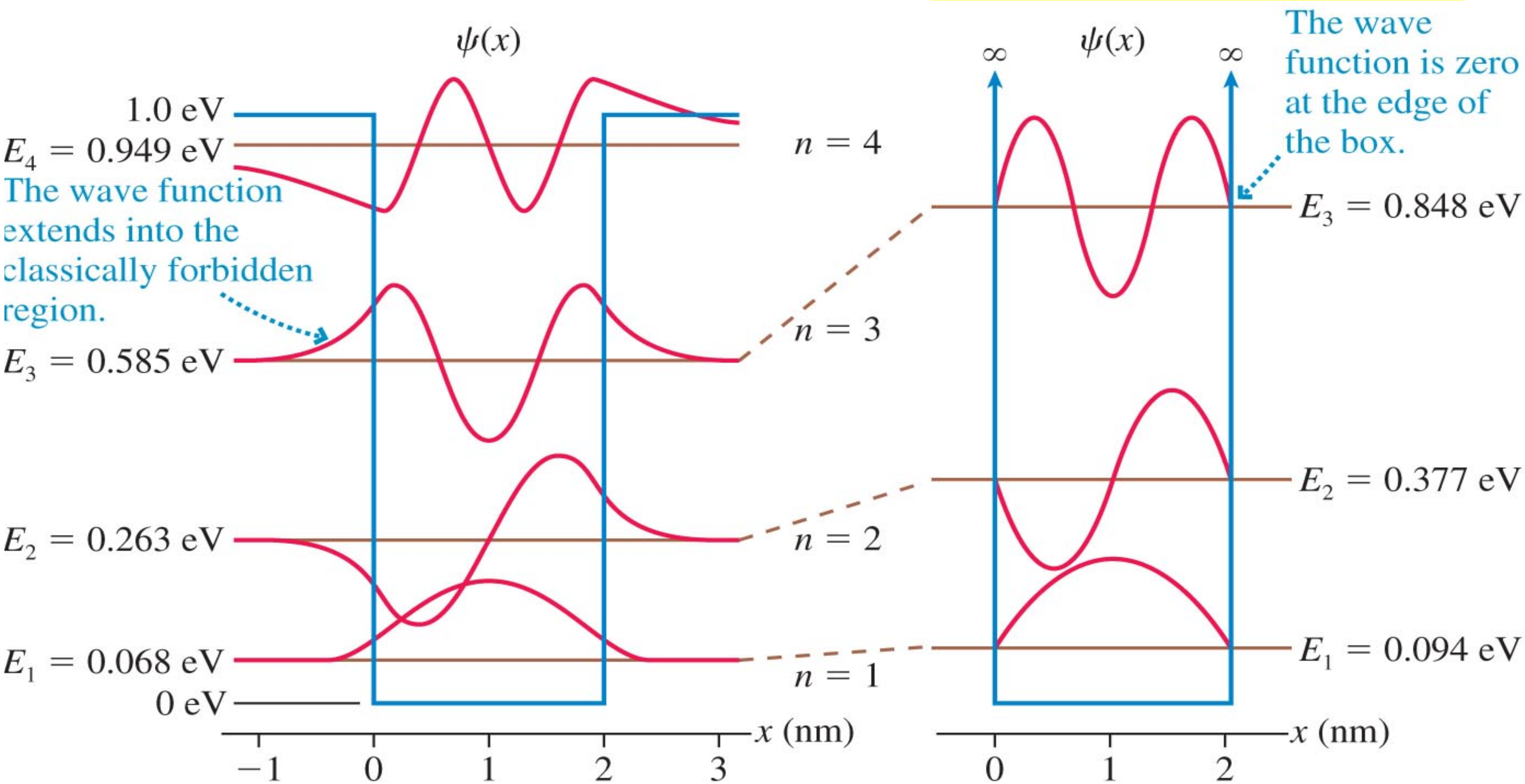
Particle location extends into *classically forbidden region*

In the classically forbidden regions, the particle has total energy **less than** the potential energy!

Comparison of infinite and finite potential wells

Electron in finite square well
(a=2 nm and V=1.0 eV)

Infinite potential well
(a = 2 nm and V = ∞)



Note that energy level *n* has *n* antinodes

Clicker question 2 — Set frequency to AD

How far does the particle extend into forbidden region?

Outside well: E < V

Inside well: E > V

Outside well: E < V

$$\frac{d^2\psi_I(x)}{dx^2} = \alpha^2\psi_I(x)$$

$$\frac{d^2\psi_{III}(x)}{dx^2} = \alpha^2\psi_{III}(x)$$

$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}(V-E)\psi(x) = \alpha^2\psi(x)$$ so $$\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$$

What are the units of α?

A. J

B. J^{-1}

C. m^2

D. m^{-1}

E. $J^{-1/2}$

Clicker question 2 — Set frequency to AD

How far does the particle extend into forbidden region?

Outside well: E < V

Inside well: E > V

Outside well: E < V

$$\frac{d^2\psi_I(x)}{dx^2} = \alpha^2\psi_I(x)$$

$$\frac{d^2\psi_{III}(x)}{dx^2} = \alpha^2\psi_{III}(x)$$

$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}(V - E)\psi(x) = \alpha^2\psi(x)$$ so $$\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$$

What are the units of α?

A. J

B. J^{-1}

C. m^2

D. m^{-1}

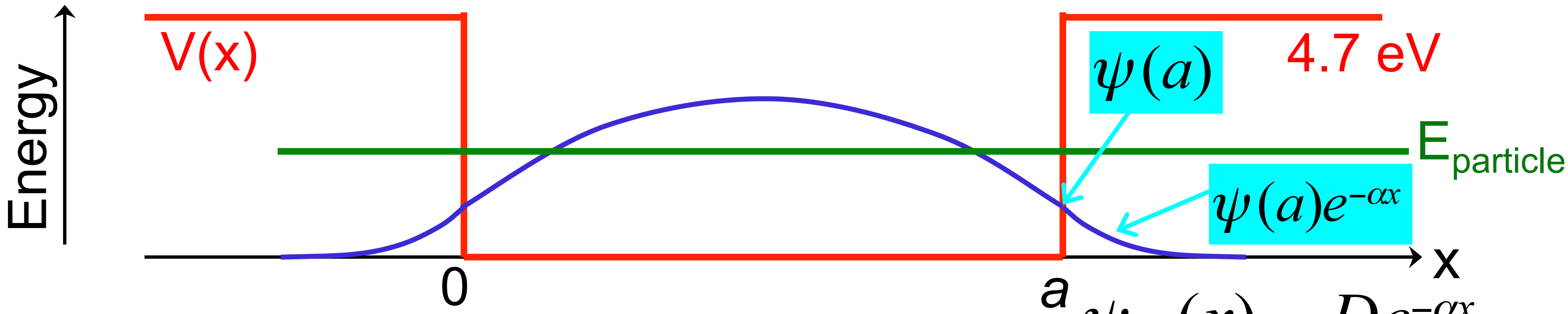
E. $J^{-1/2}$

$$\alpha = \frac{\sqrt{\mathrm{eV}/c^2 \cdot \mathrm{eV}}}{\mathrm{eV}\cdot\mathrm{s}} = \frac{\mathrm{eV}}{\mathrm{eV}\cdot\mathrm{s}\cdot\mathrm{m/s}} = \frac{1}{\mathrm{m}}$$

or note $\psi_{III}(x) = De^{-\alpha x}$ and recall exponent must be dimensionless.

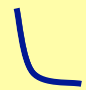
Particles in classically forbidden regions

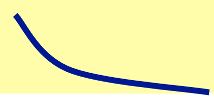
How far does the particle extend into the forbidden region?



$$\psi_{III}(x) = De^{-\alpha x}$$

$$\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$$

Large α means fast drop off 

Small α means slow drop off 

A measure of the *penetration depth* is $1/\alpha = \lambda$

Distance at which $\psi(x)$ is reduced by a factor of 1/*e*.

For an electron with V-E = 4.7 eV this is only 10^{-10} m (size of an atom). Not very far!

What changes would increase the penetration depth?

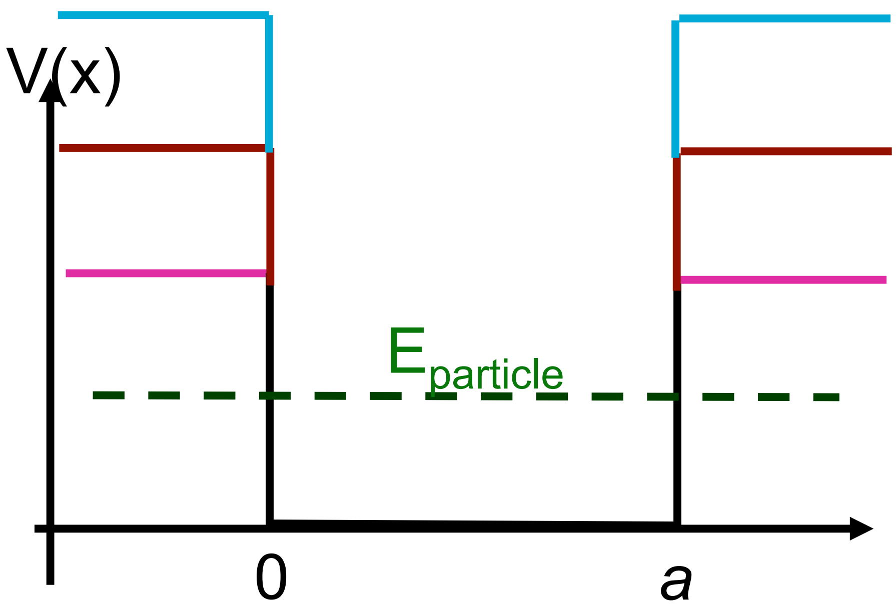
Thinking about α and penetration depth

Consider changing the potential energy curve or the particle energy. What changes increase or decrease the penetration depth: $\lambda = 1/\alpha$

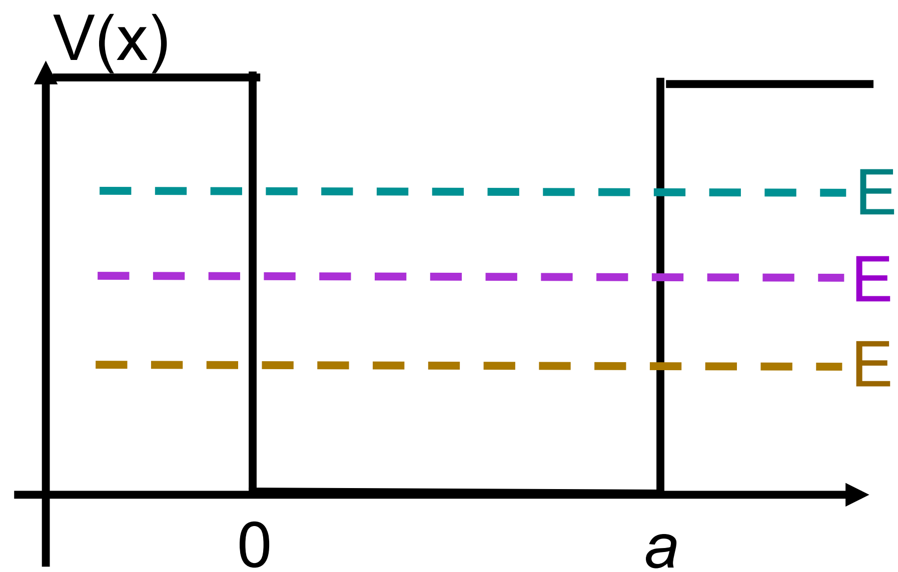
$$\psi_{\text{III}}(x) = De^{-\alpha x}$$

$$\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$$

Changing potential curve.



Changing particle energy

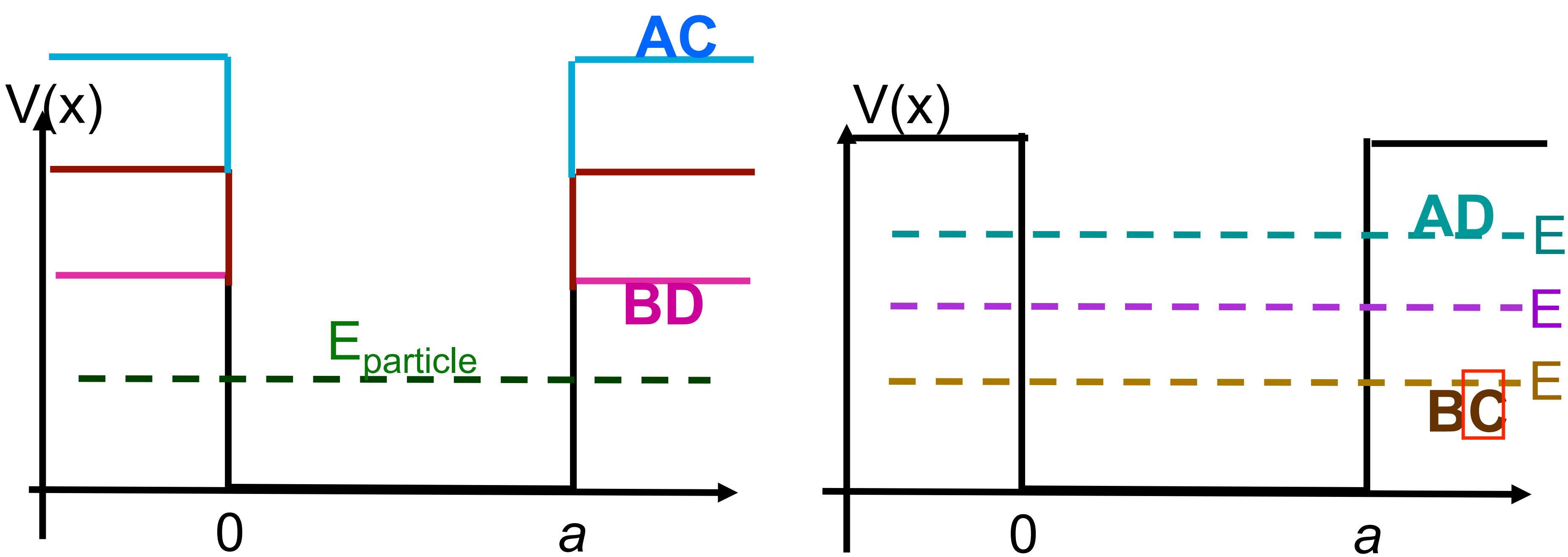


Clicker question 3 | Set frequency to DA

Which of the four possible scenarios (A,B,C,D) would give the *shortest* penetration depth $\lambda = 1/\alpha$

$$\psi_{\text{III}}(x) = De^{-\alpha x}$$

$$\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$$



Clicker question 3 — Set frequency to DA

Which of the four possible scenarios (A,B,C,D) would give the *shortest* penetration depth $\lambda = 1/\alpha$

$$\psi_{III}(x) = De^{-\alpha x}$$

Small λ implies large α.

Large α comes from large V and/or small E

$$\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$$

Answer **C** gives largest value for (V-E).