X-rays and Compton effect

Announcements:

• Next week’s homework has been posted on the class calendar website. Due next Wednesday.

• Exam solutions have been posted on Desire to Learn.

• Expect to have the exam graded by this weekend and I will post the results when they are complete and handback next week.
X-rays

X-rays have an energy between UV and gamma rays with wavelengths around 0.1 nm (1 Å).

When X-rays were first observed, did not know if they were EM waves like radio waves or visible light or particles like cathode rays (electrons).

The X-ray sources are not nearly as bright as visible light sources so need a diffraction grating to detect interference.
Found by Wilhelm Roentgen (1895) that very penetrating radiation was produced when energetic electrons hit a solid metal target. Radiation observed at side. Bremsstrahlung! Didn’t know whether or not x-rays were really waves.

Good way to see if something is a wave is to use a diffraction grating – problem needed spacing on order 0.1nm.

http://www.colorado.edu/physics/phys2170/
Diffraction grating

The wave fronts are separated by a distance $d \sin \theta$.

When this separation is equal to an integer multiple of the wavelength of the light there is constructive interference.

Constructive interference when $d \sin \theta = m \lambda$

This is a transmission grating; can also have a reflective grating such as on CD’s, mother-of-pearl, or peacock feathers.
Which of the following is a true statement about diffraction gratings?

A. Destructive interference occurs when the path length difference is \((n+\frac{1}{2})\lambda\).

B. For a given \(m\neq 0\), blue light gets scattered at a larger angle than red light.

C. A green laser pointer with \(\lambda = 532\, \text{nm}\) will show a rainbow of colors when sent through an appropriate diffraction grating.

D. More than one of the above

E. None of the above
Clicker question 1

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Diffraction grating

Constructive interference when \( dsin\theta = m\lambda \)

This implies that different wavelength of light \( \lambda \) will get scattered at different angles \( \theta \).

This is what causes white light going into a diffraction grating to split into its component wavelengths (rainbow effect).

http://www.colorado.edu/physics/phys2170/
Bragg diffraction

Diffraction grating spacing must be $\approx$ wavelength of the wave

Very difficult to make 0.1 nm spacing for X-rays

However, atoms in crystalline structures are generally separated by about 0.1 nm so can use these to make diffraction grating.

The extra distance that the deeper wave travels is $2dsin\theta$.

When this equals a multiple of the wavelength you get constructive interference

Constructive interference when $2dsin\theta = n\lambda$
X-ray diffraction

X-ray diffraction off of crystals was observed by Max Laue (Nobel Prize in 1914) and by the father-son Bragg team in 1913 (Nobel prize in 1915).

X-ray crystallography has been widely used to understand the atomic structure of many materials. Most famous is the structure of DNA which was figured out from the X-ray diffraction pattern here.

Graphite and diamond differ only in structure which can be seen in X-ray crystallography.

In 2150 you can do electron diffraction which is very similar
Proved X-rays were waves.

Once X-ray wavelength was known then one could use the radiation of a probe of material structures.

Electrons impinge on a metal and gave characteristic X-ray energies depending on metal. \[ h\nu_{\text{max}} = eV_0 \] - called Duane-Hunt Law

http://www.colorado.edu/physics/phys2170/
Compton effect

We know that X-rays are just a part of the EM wave spectrum.

In 1923 Compton published results showing that X-rays also behave like particles *and* that these photons have momentum.

In classical theory, an EM wave striking a free electron should cause the electron to oscillate at the EM wave frequency and eventually emit light (in all directions) at the *same* frequency.

Starting in 1912, reports were coming in that, for X-rays, some of the emitted light was at a *lower* frequency than the absorbed light.

The photon model is needed to explain this.
Compton effect

Start with an incoming X-ray photon with energy $E_0$ and momentum $p_0$.

The photon hits an electron at rest.

Photon has final energy $E$ and momentum $p$.

Electron has final energy $E_e$ and momentum $p_e$.

Conservation of energy and momentum give us:

$$E_0 + m_e c^2 = E + E_e$$

$$p_0 = p \cos \theta + p_e \cos \phi$$

$$0 = p \sin \theta - p_e \sin \phi$$

Solve for $p_e^2$, namely

$$p_e^2 = p_0^2 + p^2 - 2p_0 p \cos \theta$$

$$[(E_0-E) + m_e c^2]^2 = E_e^2$$
The energy of a photon is $E = \frac{hc}{\lambda}$. If we believe Einstein, photons have energy $E = pc$. Setting these two equal we get $p = \frac{h}{\lambda}$.

Subtracting $E$ squared and momentum squared equations from the last slide reduces to $m_e(p_0 - p) = p_0 p (1 - \cos \theta)$

or

\[
\frac{1}{p} - \frac{1}{p_0} = \frac{1}{m_e c} (1 - \cos \theta)
\]

The energy of a photon is $E = \frac{hc}{\lambda}$. If we believe Einstein, photons have energy $E = pc$. Setting these two equal we get $p = \frac{h}{\lambda}$.

Substituting in gives us

\[
\Delta \lambda = \lambda - \lambda_0 = \frac{h}{m_e c} (1 - \cos \theta)
\]

\[
\lambda_C = \frac{h}{m_e c} = 0.02426 \times 10^{-8} \text{ cm}
\]

Compton Wavelength

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\[ \Delta \lambda = \lambda - \lambda_0 = \frac{h}{m_e c} (1 - \cos \theta) \]

Note that \((1 - \cos \theta) \geq 0\) so the wavelength can only get longer (energy gets lower).

The lost energy goes into electron kinetic energy.

Which of the following scattered photons will have the least energy?

A. A photon which continues forward (+x direction)
B. A photon which emerges at a right angle (+y direction)
C. A photon which comes straight back (-x direction)
D. All of the scenarios above have the same energy photons
Note that \((1 - \cos \theta)\) is \(\geq 0\) so the wavelength can only get \textbf{longer} (energy gets lower).

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If the photon goes straight \((\theta = 0)\) no real collision, no energy loss.

If \(\theta = 180\degree\) it is a head on collision with maximum energy loss.

Compton observed this wavelength shift versus angle proving photons have momentum (1927 Nobel prize).

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Summary of Chapter 4

Blackbody radiation and the photoelectric effect can only be explained by a new quantum theory of light.

The quantum of light is called a photon and it has energy of \( E = hf = hc/\lambda \).

Furthermore, the Compton effect shows that photons carry momentum of \( p = h/\lambda \), consistent with the relativistic energy-momentum relation for massless particles which says \( E = |pc| \).

How do we reconcile this new photon picture with all the evidence for light as a wave (interference, diffraction, etc.)?

Light is both a particle and a wave!