Chap. 15: Simple Harmonic Motion

Announcements:

CAPA is due on Tuesday and last set is due Tuesday, May 1.

Will skip sections 15-8, 15-9
On Damped harmonic motion
Forced harmonic motion

Web page: http://www.colorado.edu/physics/phys1110/phys1110_sp12/
Multiple Choice Exam 3

Exam 3 MC Class Statistics

Number of submitted grades: 639 / 681
Minimum: 27.2%
Maximum: 100%
Average: 70.9%
Mode: 80.2%
Median: 72.8%
Standard Deviation: 15.8%

Grade Distribution

Number of Users (%)

Grade Received (%)
Exam 3-Long Answer Class Statistics

Number of submitted grades: 639 / 681
Minimum: 5.3 %
Maximum: 126.3 %
Average: 71.6 %
Mode: 84.2 %
Median: 73.7 %
Standard Deviation: 28.6 %

Grade Distribution
Exam 3 Scores

Exam 3 Total Class Statistics
Number of submitted grades: 639 / 681
Minimum: 27%
Maximum: 105%
Average: 71.1%
Mode: 68%
Median: 72%
Standard Deviation: 15.4%

Grade Distribution

Grade Received (%)
Sum of Exams 1-3

Exams 1-3 Class Statistics

- Number of submitted grades: 677 / 681
- Minimum: 0 %
- Maximum: 98.3 %
- Average: 67.3 %
- Mode: 73.3 %
- Median: 69 %
- Standard Deviation: 16.2 %

My guess of grades from Exams alone

![Grade Distribution Chart]

F  D  C  B  A
We want a solution to the equation \( \frac{d^2x}{dt^2} = -\frac{k}{m} x \)

We now know that for \( x = A \cos(\omega t + \phi) \)

\[
\frac{d^2x}{dt^2} = -A \omega^2 \cos(\omega t + \phi) = -\omega^2 x
\]

So this is a solution as long as \( \omega = \sqrt{\frac{k}{m}} \)

Note that the constants \( A \) and \( \phi \) are not determined by the equations of motion. They will be determined by the initial conditions of the problem.
Mass on a spring

What does the motion of $x = A \cos(\omega t + \phi)$ with $\omega = \sqrt{\frac{k}{m}}$ look like?

Cosine function oscillates between -1 and 1.

The factor $A$ (amplitude) multiplies cosine and sets the maximum displacement that the oscillator reaches from the equilibrium point.
What does the motion of $x = A \cos(\omega t + \phi)$ with $\omega = \sqrt{\frac{k}{m}}$ look like?

$\omega$ is the **angular frequency** (rad/s). It sets how quickly the system oscillates.

The time it takes $\omega t$ to increase by $2\pi$ (a complete cycle) is the **period** $T$.

The frequency $f$ is how many cycles are completed per second. $f = 1/T$.

Frequency SI unit is **hertz** (Hz). Units of cycles/second or s$^{-1}$ are also used.

Note that $\omega = 2\pi f = 2\pi / T$. 
Summary Slide

\[ y = A \sin \omega t = A \sin \sqrt{\frac{k}{m}} t \]

\[ T = \frac{1}{f} \]

\[ f = \frac{\omega}{2\pi} \]

\[ \omega = \sqrt{\frac{k}{m}} \]
Initial conditions

Position versus time: \( x = A \cos(\omega t + \phi) \)

Velocity versus time: \( v = -A \omega \sin(\omega t + \phi) \)

The initial \((t=0)\) position and velocity are:

\[ x_0 = A \cos \phi \quad \quad v_0 = -A \omega \sin \phi \]

Since \( \omega \) is set by \( \omega = \sqrt{k/m} \), we can solve these two equations for \( A \) and \( \phi \) (assuming we have the initial position & velocity).

\[ \frac{v_0}{x_0} = -A \omega \sin \phi \quad \quad \frac{v_0}{A \cos \phi} = -\omega \tan \phi \quad \quad \text{so} \quad \quad \phi = \tan^{-1} \left( -\frac{v_0}{\omega x_0} \right) \]

Adding \( x_0^2 = A^2 \cos^2 \phi \) and \( \frac{v_0^2}{\omega^2} = A^2 \sin^2 \phi \)

gives \( x_0^2 + \frac{v_0^2}{\omega^2} = A^2 \cos^2 \phi + A^2 \sin^2 \phi = A^2 \quad \text{so} \quad A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}} \)

Thus, initial conditions (position & velocity) give us \( A \) & \( \phi \).
Energy considerations

Our analysis of the mass on a spring tells us about the position, velocity, acceleration as a function of time.

Our previous analysis using energy considerations still works too!

Without friction, energy is conserved:  \( E = K + U_{el} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \)

Remember that when \( v = 0, \ x = A \) so the total energy is \( E = \frac{1}{2}kA^2 \)

The maximum velocity occurs at \( x = 0 \) so using conservation of energy we see \( E = \frac{1}{2}mv_{\text{max}}^2 + \frac{1}{2}k0^2 = \frac{1}{2}kA^2 \) so \( v_{\text{max}} = \sqrt{\frac{k}{m}}A = \omega A \)

Of course we could have figured that out from \( v = -A \omega \sin(\omega t + \phi) \)

Generally, use energy to solve problems that do not have time in them. If time is in the problem, you need the motion equations.
Two masses are identical. One is attached to a stiff spring; the other to a floppy spring. Both are positioned at $x=0$ and given the same initial speed. Which spring produces the largest amplitude motion?

A. stiff spring  
B. floppy spring  
C. same for both
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At the beginning, energy is all kinetic energy and is the same for both. Energy is given by:

$$E = \frac{1}{2} m v_{\text{max}}^2 + \frac{1}{2} k 0^2 = \frac{1}{2} m v_{\text{max}}^2$$

But remember that the total energy is also $\frac{1}{2}kA^2$ so

$$E = \frac{1}{2} m v_{\text{max}}^2 = \frac{1}{2} k A^2$$

The spring with the smallest $k$ will have the largest $A$.

A stiff spring has a larger $k$ than a floppy spring.
Clicker question 2

A mass is oscillating back and forth on a spring without friction, as shown. Which answer is most correct. At what point is the total energy of the system a maximum?

A: 0  B: M  C: E  D: All of these, the total energy is constant
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A: 0  B: M  C: E
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Springs conserve energy, it is merely shifting back and forth between kinetic energy and potential energy. At point "O" the KE is maximized, at point E the PE is maximized, but at ALL points the total energy is always the same.
A mass on a spring oscillates with a certain amplitude and a certain period $T$. If the mass is doubled, the spring constant of the spring is doubled, and the amplitude of motion is doubled, the period ..

A: increases
B: decreases
C: stays the same.
D: Not enough information to decide
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A: increases
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**C: stays the same.**
D: Not enough information to decide

\[ T = 2\pi \sqrt{\frac{m}{k}} \]

If you double $m$ AND $k$, $T$ stays exactly the same. The amplitude has *nothing* to do with it!
A particle-on-a-spring oscillates with an amplitude $A$, and a period $T$. If you double the amplitude, the energy of the system becomes

A: twice as big.
B: 4 times as big.
C: 1/2 as big.
D: 1/4 as big.
E: Stays the same
A particle-on-a-spring oscillates with an amplitude $A$, and a period $T$.
If you double the amplitude, the energy of the system becomes

- **A:** twice as big.
- **B:** 4 times as big.
- **C:** 1/2 as big.
- **D:** 1/4 as big.
- **E:** Stays the same

Energy goes like $1/2 k A^2$. If you double $A$, you quadruple the energy.
Vertical mass on a spring

Start with relaxed spring
Define up as positive
Add weight and find new equilibrium where spring force and weight balance so $k\ell - mg = 0$. Call this point $y=0$.

Block is moved to up $y$. What is the net force on block?

$$\sum F = k(\ell - y) - mg = k\ell - mg - ky = 0 - ky = -ky$$

Block is moved to $-y$. Net force on block is… The same! $F = ky$

So $F = -ky$ which is the same force as for a horizontal spring.

Only difference is equilibrium point is not at the relaxed spring position.
Our previous analysis of the pendulum, like the spring, only involved energy considerations:

\[ U_{\text{grav}} = mgL(1 - \cos \theta) \]

Like the spring case, we will now look at the force and use Newton’s 2nd law to figure out the time dependence.

Free body diagram:

- \( T \)
- \( mg \)

### Split weight into \( r, \theta \) components:

\[ \sum F_r = T - mg \cos \theta \]
\[ \sum F_\theta = -mg \sin \theta \]
Small angle approximation

Take a small angle \( \theta \).

At a radius \( r \), the angle covers a distance \( s = r \theta \) (as long as \( \theta \) is measured in radians).

Note that the height of the triangle, \( h = r \sin \theta \), is approximately the same.

So, for small angles \( r \sin \theta \approx r \theta \) which means \( \sin \theta \approx \theta \) (\( \theta \) in radians).

Similar arguments gives us \( \cos \theta \approx 1 \) and thus \( \tan \theta \approx \theta \).
We found the force on a pendulum along the $\theta$ direction is $F_\theta = -mg \sin \theta$. Using the small angle approximation gives us $F_\theta = -mg\theta$.

But note also that $s = L\theta$ so $F_\theta = -\frac{mg}{L}s$.

Now this looks similar to a spring force: $F = -kx$.

Identifying $k$ with $mg/L$ we can simply use all of our results for a mass on a spring.

The angular frequency is $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg}{mL}} = \sqrt{\frac{g}{L}}$.

Everything else is the same.