Announcements:

- Exam Scores for the Multiple Choice are posted on D2L.
- Look at the answer sheet and see if your score seems correct – there might be an incorrect version number that you selected.
- We should have the Long Answer graded and posted by Wednesday and exams will be returned in recitation on Thursday.

http://www.colorado.edu/physics/phys1110/phys1110_sp12/
The exam was hard. Apparently too hard, but remember everything will be curved and this is only 70% of the exam.

As soon as we have Long Answers posted, then we will add up the scores from Exam 1 and Exam 2 to make a histogram. Hopefully, I’ll have the results to show on Wednesday.

I do expect that you can solve all of the problems on the exam. I encourage you to go through the problems yourself and understand how to solve them.
Second Exam Multiple Choice Results
Consider an object of mass $m$ at the end of a massless rod of length $R$ spinning around an axis with angular velocity $\omega$. What is the kinetic energy of the mass?

$$K = \frac{1}{2} mv^2 = \frac{1}{2} mR^2 \omega^2 = \frac{1}{2} I \omega^2$$

For a particle which is a distance $R$ from the axis (or pivot) the **moment of inertia** is $I = mR^2$

As the particle moves further out ($R$ increases), $\omega$ doesn’t change but velocity increases so kinetic energy increases. This comes from the moment of inertia increasing.
Rings and cylindrical shells (hollow cylinders) behave like a bunch of little particles all located at a distance of $R$ from the axis.

$$I = MR^2$$

This is only true for the axis shown.

$$I = \frac{2}{5} MR^2$$ (solid sphere)

$$I = \frac{2}{3} MR^2$$ (hollow sphere)
Parallel axis theorem

The moments of inertia for a ring, hoop, solid sphere, and hollow sphere were for an axis through the center of mass.

What if the axis is not through the center of mass?

If the axis is parallel to the center of mass axis then can use the parallel axis theorem:

\[ I_P = I_{CM} + Md^2. \]

- \( I_{CM} \) = moment of inertia for an axis through the center of mass.
- \( d \) = distance between the two parallel axes.
A small wheel and a large wheel are connected by a belt. The small wheel is turned at a constant angular velocity $\omega_s$. How does the magnitude of the angular velocity of the large wheel $\omega_L$ compare to that of the small wheel $\omega_s$?

A. $\omega_L > \omega_s$

B. $\omega_L < \omega_s$

C. $\omega_L = \omega_s$
A small wheel and a large wheel are connected by a belt. The small wheel is turned at a constant angular velocity $\omega_s$. How does the magnitude of the angular velocity of the large wheel $\omega_L$ compare to that of the small wheel?

A. $\omega_L > \omega_s$
B. $\omega_L < \omega_s$
C. $\omega_L = \omega_s$

The speed of the belt is constant: $v$

Small wheel: $v = R_s \omega_s$

Large wheel: $v = R_L \omega_L$

Since $R_S < R_L$ it must be that $\omega_L < \omega_s$
Cross Product or Vector Product

\[ \hat{i} \times \hat{i} = 0 \quad \hat{j} \times \hat{j} = 0 \quad \hat{k} \times \hat{k} = 0 \]

\[ \hat{i} \times \hat{j} = \hat{k} \quad \hat{j} \times \hat{k} = \hat{i} \quad \hat{k} \times \hat{i} = \hat{j} \]

\[ \hat{j} \times \hat{i} = -\hat{k} \quad \hat{k} \times \hat{j} = -\hat{i} \quad \hat{i} \times \hat{k} = -\hat{j} \]

\[ a \times b = (a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}) \times (b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}) \]

\[ = a_1 b_1 \hat{i} \times \hat{i} + a_1 b_2 \hat{i} \times \hat{j} + a_1 b_3 \hat{i} \times \hat{k} + \]

\[ a_2 b_1 \hat{j} \times \hat{i} + a_2 b_2 \hat{j} \times \hat{j} + a_2 b_3 \hat{j} \times \hat{k} + \]

\[ a_3 b_1 \hat{k} \times \hat{i} + a_3 b_2 \hat{k} \times \hat{j} + a_3 b_3 \hat{k} \times \hat{k} \]

\[ = a_1 b_1 \hat{0} + a_1 b_2 \hat{k} + a_1 b_3 (-\hat{j}) + a_2 b_1 (-\hat{k}) + a_2 b_2 \hat{0} + a_2 b_3 \hat{i} + a_3 b_1 \hat{j} + a_3 b_2 (-\hat{i}) + a_3 b_3 \hat{0} \]

\[ = (a_2 b_3 - a_3 b_2) \hat{i} + (a_3 b_1 - a_1 b_3) \hat{j} + (a_1 b_2 - a_2 b_1) \hat{k}. \]
Cross Product or Vector Product

The definition of the cross product can also be represented by the determinant of a formal matrix:

\[
\mathbf{a} \times \mathbf{b} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
a_1 & a_2 & a_3 \\
b_1 & b_2 & b_3
\end{vmatrix}.
\]

\[
\mathbf{a} \times \mathbf{b} = \mathbf{i}a_2b_3 + \mathbf{j}a_3b_1 + \mathbf{k}a_1b_2 - \mathbf{i}a_3b_2 - \mathbf{j}a_1b_3 - \mathbf{k}a_2b_1.
\]

\[
\mathbf{a} \times \mathbf{b} = (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}.
\]

\[
|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| |\mathbf{b}| \sin \theta
\]
Force causes linear acceleration while torque causes angular acceleration

- Force applies large torque
- Force applies no torque
- Force applies small torque

Use Greek letter tau for torque: \( \tau \)

\[ \tau = rF \sin \phi \] where \( r \) is the distance between the force application and the axis, \( F \) is the force magnitude, and \( \phi \) is the angle between \( r \) and \( F \).
Calculating torque 3 ways

\[ \tau = rF \sin \phi \]

distance \( r \) times force times sine of angle between them.

\[ \tau = r \left( F \sin \phi \right) = rF \tan \phi \]

distance \( r \) times the perpendicular (or tangent) force

\[ \tau = F(r \sin \phi) = Fl \]

Force times the perpendicular distance. Also called lever arm or moment arm.
Three forces labeled A, B, C are applied to a rod which pivots on an axis thru its center. Which force causes the largest magnitude torque?  \( \sin 45^\circ = \cos 45^\circ = 1/\sqrt{2} = 1/1.414 \)

A. A  
B. B  
C. C  
D. Two are tied for largest  
E. All three are the same
Three forces labeled A, B, C are applied to a rod which pivots on an axis thru its center. Which force causes the largest magnitude torque? \(\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2} = 1/1.414\)

A. A  
B. B  
C. C  
D. Two are tied for largest  
E. All three are the same

\[
\begin{align*}
A: & \quad LF \sin 45^\circ = \frac{LF}{1.4} \\
B: & \quad \frac{L}{2} F \sin 90^\circ = \frac{LF}{2} \\
C: & \quad \frac{L}{4} 2F \sin 90^\circ = \frac{LF}{2}
\end{align*}
\]

A generates the largest magnitude torque
Torque is a vector

Torque is the **cross product** (or vector product) of the vector $\vec{r}$ which is the distance **from** the axis to the force application and the force vector $\vec{F}$: 

$$\vec{\tau} = \vec{r} \times \vec{F}$$

The cross product produces a vector from two other vectors which is perpendicular to the plane formed by the two vectors.

The direction of the cross product vector comes from another application of the right hand rule. Start with fingers pointing along first vector and wrap fingers in direction which points along the second vector.

Can review 3.8 of H+R for more information and has additional pictures which show the resultant vector’s location.
Newton’s 2\textsuperscript{nd} law for angular motion

Consider a force pushing on mass $m$ perpendicular to the rod $R$. At the moment that is into the screen but the force is always tangent to the motion.

Newton’s 2\textsuperscript{nd} law for this is $F_{\text{tan}} = ma_{\text{tan}}$

But we know that $a_{\text{tan}} = r\alpha$ so $F_{\text{tan}} = mr\alpha$

Multiplying both sides by $r$: $rF_{\text{tan}} = mr^2\alpha$

But $rF_{\text{tan}}$ is the torque applied and $mr^2$ is the moment of inertia for this situation so $\tau = I\alpha$

This is the equivalent of Newton’s 2\textsuperscript{nd} law for angular motion
A mass is hanging from the end of a horizontal bar which pivots about an axis through its center and is initially stationary. The bar is released and begins rotating. As the bar rotates from horizontal to vertical, the magnitude of the torque on the bar…

A. increases
B. decreases
C. remains constant
A mass is hanging from the end of a horizontal bar which pivots about an axis through its center and is initially stationary. The bar is released and begins rotating. As the bar rotates from horizontal to vertical, the magnitude of the torque on the bar...

A. increases  
B. decreases  
C. remains constant

\[ \tau = rF \sin \phi \]

\( r \) and \( F \) stay the same but \( \phi \) decreases from 90° when horizontal to 0° when vertical
Summary of torque so far

Torque is the **cross product** (or vector product) of the vector $\vec{r}$ which is the distance from the axis to the force application and the force vector $\vec{F}$: $\vec{\tau} = \vec{r} \times \vec{F}$

In many cases we only care about the magnitude $\tau = rF \sin \phi$ where $\phi$ is the angle between the $\vec{r}$ and $\vec{F}$ vectors.

SI units of torque are newton·meter (N·m). This has the same dimensions as energy but only use Joules for energy.

Newton’s 2nd law $\vec{F} = m\vec{a}$ can be rewritten as $\vec{\tau} = I\vec{\alpha}$ for cases of rotational motion.
A 2 kg weight is attached to the end of a rope coiled around a pulley with mass of 6 kg and radius of 0.1 m.

What is the acceleration of the weight?

Weight free body diagram: \( T - mg = -ma \) \((a > 0)\)

For the pulley, rope (and therefore the tension force) comes off at 90° relative to \( R \) so \( \tau = RT \).

Disk moment of inertia is \( \frac{1}{2} MR^2 \) so \( \tau = I\alpha \) becomes \( RT = \frac{1}{2} MR^2\alpha \).

Acceleration along outside of pulley equals acceleration of 2 kg weight because connected by rope so \( a = a_{\tan} = R\alpha \).

\[
RT = \frac{1}{2} MR^2 \frac{a_{\tan}}{R} \quad \text{becomes} \quad T = \frac{1}{2} Ma
\]
A 2 kg weight is attached to the end of a rope coiled around a pulley with mass of 6 kg and radius of 0.1 m.

What is the acceleration of the weight?

We have two equations \( T = mg - ma \) and \( T = \frac{1}{2} Ma \)

\[
mg - ma = \frac{1}{2} Ma
\]

rearranges to

\[
2mg = Ma + 2ma
\]

Solving gives

\[
a = \frac{2mg}{M + 2m} = \frac{2 \cdot 2 \text{ kg} \cdot 10 \text{ m/s}^2}{6 \text{ kg} + 2 \cdot 2 \text{ kg}} = \frac{40 \text{ N}}{10 \text{ kg}} = 4 \text{ m/s}^2
\]
Clicker question 4  
Set frequency to BA

For the 2 kg weight attached to 6 kg pulley with radius of 0.10 m, what is the kinetic energy of the weight after dropping for 1 second. Remember \( a = 4 \text{ m/s}^2 \) and \( K = \frac{1}{2}mv^2 \), \( K = \frac{1}{2}I\omega^2 \), and \( I_{\text{disk}} = \frac{1}{2}MR^2 \).

A. 2 J  
B. 4 J  
C. 8 J  
D. 16 J  
E. 32 J
For the 2 kg weight attached to 6 kg pulley with radius of 0.10 m, what is the kinetic energy of the **weight** after dropping for 1 second. Remember $a=4 \text{ m/s}^2$ and $K=\frac{1}{2}mv^2$, $K=\frac{1}{2}I\omega^2$, and $I_{\text{disk}}=\frac{1}{2}MR^2$.

A. 2 J  
B. 4 J  
C. 8 J  
D. 16 J  
E. 32 J

**After 1 second**

$v = v_0 + at = 0 + 4 \text{ m/s}^2 \cdot 1 \text{ s} = 4 \text{ m/s}$

**Kinetic energy for the weight is**

$K = \frac{1}{2}mv^2 = \frac{1}{2} 2 \text{ kg} \cdot (4 \text{ m/s})^2 = 16 \text{ J}$