Announcements:

• Next midterm on Thursday (3/15). A sample exam is available on D2L under “Content”. Chapters 6–9 will be covered. The exam that is posted only included Chapters 5-8, so may want to also look at Exam 3 sample test.

• Will cover center of mass today Chap 9.1-5.

• Clickers need to be registered each semester – if registered last semester, then it doesn’t work. I am going to put up the latest sum of clicker scores by Friday. Please register if you haven’t done so.

http://www.colorado.edu/physics/phys1110/phys1110_sp12/
A block slides without friction down a ramp as shown. As the block slides down the speed _______ and the magnitude of the acceleration ___________.

A. increases increases
B. increases decreases
C. decreases increases
D. decreases decreases
E. Impossible to tell
A block slides without friction down a ramp as shown. As the block slides down, the speed _______ and the magnitude of the acceleration ____________.

A. increases  increases  
B. increases  decreases  
C. decreases  increases  
D. decreases  decreases  
E. Impossible to tell
Question 1

Suppose you are stranded on a frictionless lake – it could have been my backyard deck this morning.

Is there anything you can do to reach the shore? If so, then what?
Consider the following three situations involving a ball of mass $m$:

A. The ball, moving at velocity $\vec{v}$, is brought to rest.
B. The ball starts at rest and experiences a force that accelerates it to a final velocity $\vec{v}'$.
C. The ball, moving at velocity $\vec{v}$, bounces off a wall, and leaves with velocity $-\vec{v}$.

The situation where the ball has the largest magnitude change in momentum, $\Delta \vec{p}$, is:

A. Situation A
B. Situation B
C. Situation C
D. They are all the same
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C. **Situation C**
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**Situation A:** $\Delta\vec{p} = \vec{p}_2 - \vec{p}_1 = 0 - m\vec{v} = -m\vec{v}$

**Situation B:** $\Delta\vec{p} = \vec{p}_2 - \vec{p}_1 = m\vec{v} - 0 = m\vec{v}$

**Situation C:** $\Delta\vec{p} = \vec{p}_2 - \vec{p}_1 = -m\vec{v} - m\vec{v} = -2m\vec{v}$
Generally set gravitational potential energy zero-point at bottom of swing.

When pendulum is pulled back by an angle $\theta$, gravitational potential energy is

$$U_{\text{grav}} = mgy = mg(L - L \cos \theta) = mgL(1 - \cos \theta)$$

When released, the potential energy is converted to kinetic energy and back to potential energy and so on.

This is a good example of conservation of energy.
Ballistic pendulum

Bullet ($m_b$) shot into pendulum catcher ($m_c$).

This is a completely inelastic collision.

\[ m_b v_b = (m_b + m_c) v_{bc} \]

Catcher+projectile rise up to a height converting kinetic energy to gravitational potential energy.

\[ \frac{1}{2} (m_b + m_c) v_{bc}^2 = (m_b + m_c) g L (1 - \cos \theta) \]

\[ v_b = \frac{m_b + m_c}{m_b} \sqrt{2 g L (1 - \cos \theta)} \]

\[ v_{bc} = \sqrt{2 g L (1 - \cos \theta)} \]
Clicker question 3  
Set frequency to BA

A person is throwing small balls with speed $v$ at a large, slow moving truck. The person has two balls of mass $m$, one of rubber and the other of putty. The rubber ball bounces back, while the ball of putty sticks to the truck. Which ball imparts the largest impulse to the truck?

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B. the putty ball  
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Impulse momentum theorem relates impulse to change in momentum $\vec{J} = \Delta \vec{p}$

The impulse given to the rubber ball is $2mv$. The impulse given to the putty ball is $mv$

By conservation of momentum or Newton’s 3$^{rd}$ law, the same impulse must have been given to the truck.
Center of mass

Suppose we have a system composed of a bunch of particles. We define the center-of-mass of the system as a point with coordinates given by

\[x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum m_i x_i}{\sum m_i}\]

\[y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum m_i y_i}{\sum m_i}\]

Can combine into

\[\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum m_i \vec{r}_i}{\sum m_i}\]

It is the average position of the particles (weighted by mass)

If solid bodies, then sums become integrals, namely

\[x_{cm} = \frac{1}{M} \int x \cdot dm \quad y_{cm} = \frac{1}{M} \int y \cdot dm\]
Center of Mass Example

Four floor tiles are laid out in an L-pattern as shown. The origin of the x-y axes is at the center of the lower left tile. What is the x-coordinate of the center of mass? What is the y-coordinate of CoM?

\[ x(cm) = \frac{(m_1 x_1 + m_2 x + m_3 x_3 + m_4 x_4)}{m_{total}} \]

Numbering the squares 1, 2, and 3 walking down, and #4 is the one on the right, gives

\[ x(cm) = \frac{(m*0 + m*0 + m*0 + m*a)}{4m} = \frac{a}{4}. \]

\[ y(cm) = \frac{(m_1 y_1 + m_2 y + m_3 y_3 + m_4 y_4)}{m_{total}} \]

Numbering the squares 1, 2, and 3 walking down, (and #4 is the one on the right), gives

\[ y(cm) = \frac{(m*2a + m*a + m*0 + m*0)}{4m} = \frac{3a}{4}. \]
Another Center of Mass Example

Where is the center of mass of the triangle?

All sides have the same length so all of the angles must be $60^\circ$.

Define our coordinate system as:

Location of top point:

\[
x_{\text{top}} = 0.5 \text{ m} \quad y_{\text{top}} = 1 \text{ m} \cdot \sin 60^\circ = 0.866 \text{ m}
\]

\[
x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{1 \text{ kg} \cdot 0 + 1 \text{ kg} \cdot 1 \text{ m} + 3 \text{ kg} \cdot 0.5 \text{ m}}{1 \text{ kg} + 1 \text{ kg} + 3 \text{ kg}} = \frac{2.5 \text{ kg} \cdot \text{m}}{5 \text{ kg}} = 0.5 \text{ m}
\]

\[
y_{\text{cm}} = \frac{m_1 y_1 + m_2 y_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{1 \text{ kg} \cdot 0 + 1 \text{ kg} \cdot 0 + 3 \text{ kg} \cdot 0.866 \text{ m}}{1 \text{ kg} + 1 \text{ kg} + 3 \text{ kg}} = \frac{2.6 \text{ kg} \cdot \text{m}}{5 \text{ kg}} = 0.52 \text{ m}
\]
The center of mass is

\[ \vec{r}_{\text{cm}} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \ldots}{M} \]

where \( M \) is the total mass.

The center of mass velocity is:

\[ \vec{v}_{\text{cm}} = \frac{d\vec{r}_{\text{cm}}}{dt} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots}{M} = \frac{\vec{P}}{M} \]

where \( \vec{P} \) is the total momentum of the system \( \vec{P} = M\vec{v}_{\text{cm}} \).

Note this means if the total momentum is constant then the center of mass velocity is constant.
Fireworks – timed to explode at apex
Center of mass motion

The center of mass acceleration is
\[ \ddot{a}_\text{cm} = \frac{d\dot{v}_\text{cm}}{dt} = \frac{m_1\ddot{a}_1 + m_2\ddot{a}_2 + \ldots}{M} \]

or
\[ M\ddot{a}_\text{cm} = m_1\ddot{a}_1 + m_2\ddot{a}_2 + \ldots \]

but we know that the net force acting on \( m_1 \) must equal \( m_1\ddot{a}_1 \) and the same for all the other masses so

\[ \sum F_1 = m_1\ddot{a}_1, \quad \sum F_2 = m_2\ddot{a}_2, \text{ and so on.} \]

Can also separate forces by external and internal forces

\[ \sum F = \sum F_{\text{ext}} + \sum F_{\text{int}} = M\ddot{a}_\text{cm} \]

By Newton’s 3\(^{rd}\) law, all of the internal forces must add to 0

\[ \sum F_{\text{ext}} = M\ddot{a}_\text{cm} \]
Note the Parabolic Movement of CoM
CoM- Diver from Helicopter
What does this buy us?

The equations $\vec{P} = M\vec{v}_{\text{cm}}$ and $\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{cm}}$ tell us that if we have an extended object, like the moon, we can treat it like a point object located at the center of mass.

To determine the motion, we don’t need to worry about the forces acting on each atom in an object as long as we know the center of mass.
What does this buy us?

Can also look at our conservation of momentum problems in a slightly different way.

Consider two equal mass carts on an air track initially connected by a compressed spring and then let go.

Using conservation of momentum: Initial momentum is 0 so the final momentum must be 0 and therefore the carts have equal magnitude momenta in opposite directions.

Using center of mass: Center of mass is in between the two carts, call it $x_{cm}$ and there is no external force so any time in the future $\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = x_{cm}$
Two blocks of masses $M$ and $2M$ are held against a massless compressed spring within a box of mass $3M$ and length $4L$ whose center is at $x=0$. All surfaces are frictionless. After the blocks are released they are each a distance $L$ from the ends of the box when they lose contact with the spring. Show that the position of the center of the box shifts by $L/6$ after the blocks collide and stick with it.

$CoM = \frac{-M \cdot L + 0 + 2M \cdot L}{6M}$

$CoM = \frac{L}{6}$
First the spring part – cons. of momentum $mv_1 = 2mv_2$ or $v_2 = 1/2v_1$. So $v_1$ is faster and gets to the wall of the box in $t_1 = L/v_1$ and collides and sticks to the wall.

Second, when block $m$, hits the wall of the box the box starts moving with speed given by $mv_1 = (m+3m)v_{\text{box}}$, so $v_{\text{box}} = 1/4v_1$.

When the first block hits the wall the 2m block has only moved a distance $L/2$, so still has to cover $L/2$ distance.

Third, the 2M block gets to the wall in a time given by $t_2 = t_1 + 1/2L / (v_2 + v_{\text{box}}) = t_1 + 2/3L/v_1$. The box stops.

Finally, distance travelled by box is given by $d_{\text{box}} = v_{\text{box}} (t_2 - t_1) = 1/4v_1 (2/3L / v_1) = L/6$
Center of Mass

Compute new center of mass for movement within the box.

\[ CoM = \frac{\sum M_i \cdot x_i}{\sum M_i} = \frac{-(2L + L/6) - 3ML/6 + 2M(2L - L/6)}{6M} \]

Center of Mass is unchanged because it is an isolated system!
Friday Lecture

You may want to read about rockets and the rocket equations before lecture on Friday.