Computational strategies near conformality

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With Alexei Bazavov, Nouman Butt, Simon Catterall, Sam Foreman, Erik Gustafson, Yuzhi Liu, Philipp Preiss, Shan-Wen Tsai, Judah Unmuth-Yockey, Li-Ping Yang, Johannes Zeiher, and Jin Zhang

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Content of the talk

- RG flows in $SU(3)$ with N_f fundamental flavors
- Tensor tools: QC friends and competitors (RG)
- Symmetry preserving truncations (YM, arxiv:1903.01918)
- Quantum simulations experiments (analog): cold atoms, ions ...
- Quantum computations (digital): IBM, IonQ, Rigetti, ...
- Abelian Higgs model with cold atom ladders
- Benchmark for real time scattering (arXiv:1901.05944, PRD in press, with Erik Gustafson and Judah Unmuth-Yockey)
- Linear sigma model for $N_f \sim 10$
- Conclusions

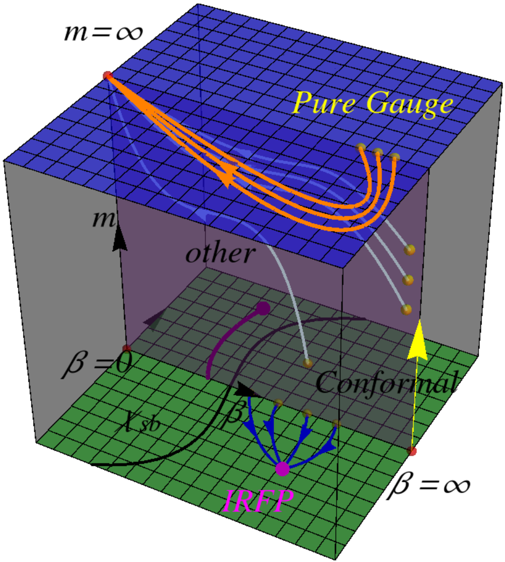
$SU(3)$ gauge group with N_f Dirac fundamentals

- 2-loop perturbative beta function: nontrivial zero for $N_f > 8$

N_f	α_c
9	5.23599
10	2.20761
11	1.2342
12	0.753982
13	0.467897
14	0.278017
15	0.1428
16	0.0416105
QED	$0.0073 \simeq 1/137$

- The one-loop beta function for $N_f = 12$ is one third of the one for $N_f = 3$. If a given change of coupling is achieved by changing the energy scale by a factor 10 for $N_f = 3$, then the scale needs to be changed by a factor 1000 for $N_f = 12$ to achieve the same change. This is a hint that our lattices are too small to completely capture the non-perturbative physics.

Schematic RG flows ($\beta = 6/g^2$), Yuzhi Liu et al. Lat13



RG flows from UV to IR ($N_f > N_{fc}$ and massless case)

1) Setting the initial conditions in the far UV

For a sufficiently high scale we can use the universal perturbative running/dimensional transmutation. The QCD analog is $\alpha_s(M_Z^2) \simeq 0.1$.

2) The intermediate scale

Using the reference scale in 1), we then reach a physical scale (say in TeV) where we are far from both fixed points. From a computational point of view, things look maximally nonlinear/multidimensional in both directions. It is challenging to capture the essential features with small lattices and one-dimensional RG flow approximations.

3) The deep IR scale (for $m = 0$ and attractive IRFP ($N_f > N_{fc}$))

As we continue, most of the irrelevant features get washed out and ultimately, the intermediate scale does not appear anymore. Conformal symmetry and chiral symmetry are unbroken and there is no confinement. Unlikely to be physically interesting on its own. However the connection to the effective theory of the massive case is interesting.

Breaking the conformal symmetry

There are two ways to break the conformal symmetry:

1. use $N_f < N_{fc}$ flavors
2. introduce a mass or several masses so that there are less than N_{fc} massless fermions

This drastically affects the third regime (deep IR) of the RG flows described before. At low energy we expect that chiral symmetry is broken and that confinement takes place. One expect that it is possible to build a effective theory inspired by our experience with ordinary QCD.

In the following, I discuss strategies to

1. do computations on large lattices (using Tensor RG or cold atom simulators)
2. build effective theories for massive theories with $N_f \sim 10$

Gapped, "confining", complex RG flows: 2D $O(N)$ models in the large-N limit

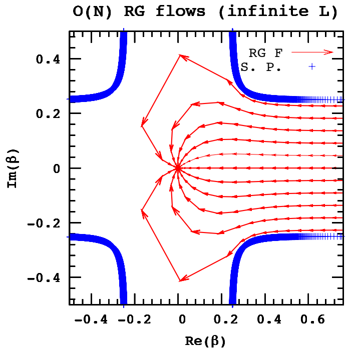
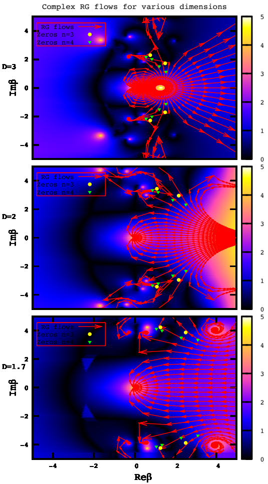


Figure: Complex RG I: $m_{gap} = \epsilon e^{i\theta}$ (small circle around 0), $\Lambda_{UV} \rightarrow \Lambda_{UV}/b$ go directly from weak coupling to strong coupling. The blending blue crosses are the β images of two lines of points located very close above and below the $[-8, 0]$ cut of $\beta(M^2)$ in the M^2 plane, Fisher's zeros stay outside of these lines (PRD 80 054020 and PRL 104 25160). Note: we now use $\beta \propto 1/g^2_{gauge}$.

More complex flows: Y. Liu, and Y. Meurice Lines of Fisher's zeros as separatrices for complex renormalization group flows Phys. Rev. D 83: 09008.



Computing with quantum devices (Feynman 82)?

- The number of transistors on a chip doubled almost every two years for more than 30 years
- At some point, the miniaturization involves quantum mechanics
- Capacitors are smaller but they are still on (charged) or off (uncharged)
- qubits: $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$ is a superposition of the two possibilities.
- Can we use quantum devices to explore large Hilbert spaces?
- Yes, if the interactions are localized (generalization of Trotter product formula, Lloyd 96)

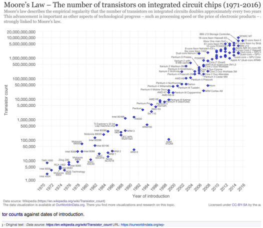


Figure: Moore's law, source: Wikipedia

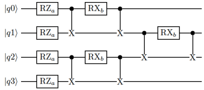


Figure 1: Circuit for 4 qubits with open boundary conditions

Figure: Quantum circuit for the quantum Ising model (E. Gustafson)

Golden dreams

Quantum simulators?

You can trap about 10^9 identical neutral atoms in an optical lattice fitting in about $1 mm^3$

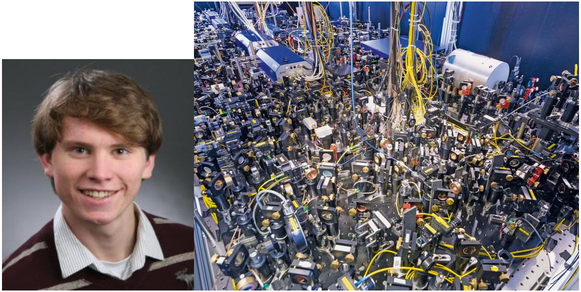


Figure: Left: Johannes Zeiher, a recent graduate from Immanuel Bloch's group can design ladder shaped optical lattices with nearest neighbor interactions. Right: an optical lattice experiment of Bloch's group.

QIS and QC: what do we want to do?

Problems where perturbation theory and classical sampling fail:

- Real-time evolution for QCD
- Jet Physics (crucial for the LHC program)
- Finite density QCD (sign problem)
- Near conformal systems require very large lattices
- Early cosmology
- Strong gravity

Strategy: many intermediate steps towards big goals

- High expectations for QC: new materials, fast optimization, security, ...
- Risk management: theoretical physics is a multifaceted landscape
- Lattice gauge theory lesson: big goals can be achieved with small steps
- Example of a big goal: ab-initio jet physics
- Examples of small steps: real-time evolution in 1+1 Ising model, 1+1 Abelian Higgs model, Schwinger model, 2+1 $U(1)$ gauge theory ,....
- Many possible paths: quantum simulations (trapped ions, cold atoms,...), quantum computations (IBM, Rigetti,...)
- Small systems are interesting: use Finite Size Scaling (data collapse, Luscher's formula,....)

Discretization for classically intractable problems

QC requires a complete discretization of QFT

- Discretization of space: lattice gauge theory formulation
- Discretization of field integration: tensor methods for compact fields (as in Wilson lattice gauge theory and nonlinear sigma models, the option followed here)
- Quantum computing (QC) methods for scattering in ϕ^4 (non-compact) theories are discussed by JLP (Jordan Lee Preskill)
- JLP argue that QC is necessary because of the asymptotic nature of perturbation theory (PT) in λ for ϕ^4 and propose to introduce a field cut (but this makes PT convergent! YM PRL 88 (2002))
- Non compact fields methods ($\lambda\phi^4$) see: Macridin, Spentzouris, Amundson, Harnik, PRA 98 042312 (2018) and Klco and Savage arXiv:1808.10378 ...

Tensor Renormalization Group (TRG)

- TRG: first implementation of Wilson program for lattice models with controllable approximations; no sign problems; truncation methods need to be optimized
- Models we considered: Ising model, $O(2)$, $O(3)$, principal chiral models, gauge models (Ising, $U(1)$ and $SU(2)$))
- Used for quantum simulators, measurements of entanglement entropy, central charge, Polyakov's loop ...
- Our group: PRB 87 064422 (2013), PRD 88 056005 (2013), PRD 89 016008 (2014), PRA90 063603 (2014), PRD 92 076003 (2015), PRE 93 012138 (2016) , PRA 96 023603 (2017), PRD 96 034514 (2017), PRL 121 223201 (2018), PRD 98 094511 (2018)
- Basic references for tensor methods for Lagrangian models: Levin and Nave, PRL 99 120601 (2007), Z.C. Gu et al. PRB 79 085118 (2009), Z. Y. Xie et al., PRB 86 045139 (2012)
- Schwinger model/fermions/CP(N): Yuya Shimizu, Yoshinobu Kuramashi; Ryo Sakai, Shinji Takeda; Hikaru Kawauchi.

Important ideas of the tensor reformulation

- In most lattice simulations, the variables of integration are compact and character expansions (such as Fourier series) can be used to rewrite the partition function and average observables as discrete sums of contracted tensors.
- Example: the $O(2)$ model $e^{\beta\cos(\theta_i-\theta_j)} = \sum_{n_{ij}=-\infty}^{+\infty} e^{in_{ij}(\theta_i-\theta_j)} I_{n_{ij}}(\beta)$
- This reformulations have been used for RG blocking but they are also suitable for quantum computations/simulations when combined with truncations.
- Important features:
  - Truncations do not break global symmetries
  - Standard boundary conditions can be implemented
  - Matrix Product State ansatzs are exact

TRG blocking: simple and exact!

Character expansion for each link (Ising example):

$$\exp(\beta\sigma_1\sigma_2) = \cosh(\beta)(1 + \sqrt{\tanh(\beta)}\sigma_1\sqrt{\tanh(\beta)}\sigma_2) =$$
$$cosh(\beta) \sum_{n_{12}=0,1} (\sqrt{\tanh(\beta)}\sigma_1\sqrt{\tanh(\beta)}\sigma_2)^{n_{12}}.$$

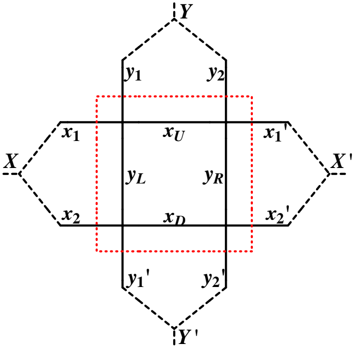
Regroup the four terms involving a given spin σ_i and sum over its two values ± 1. The results can be expressed in terms of a tensor: $T^{(i)}_{xx'yy'}$ which can be visualized as a cross attached to the site i with the four legs covering half of the four links attached to i. The horizontal indices $x,\ x'$ and vertical indices $y,\ y'$ take the values 0 and 1 as the index n_{12}.

$$T^{(i)}_{xx'yy'} = f_x f_{x'} f_y f_{y'} \delta\left(\mathrm{mod}[x + x' + y + y', 2]\right) \ ,$$

where $f_0 = 1$ and $f_1 = \sqrt{\tanh(\beta)}$. The delta symbol is 1 if $x + x' + y + y'$ is zero modulo 2 and zero otherwise.

TRG blocking (graphically)

Exact form of the partition function: $Z = (\cosh(\beta))^{2V} \mathrm{Tr} \prod_i T^{(i)}_{xx'yy'}$.
Tr mean contractions (sums over 0 and 1) over the links.
Reproduces the closed paths ("worms") of the HT expansion.
TRG blocking separates the degrees of freedom inside the block which are integrated over, from those kept to communicate with the neighboring blocks. Graphically :



TRG Blocking (formulas)

Blocking defines a new rank-4 tensor $T'_{XX'YY'}$ where each index now takes four values.

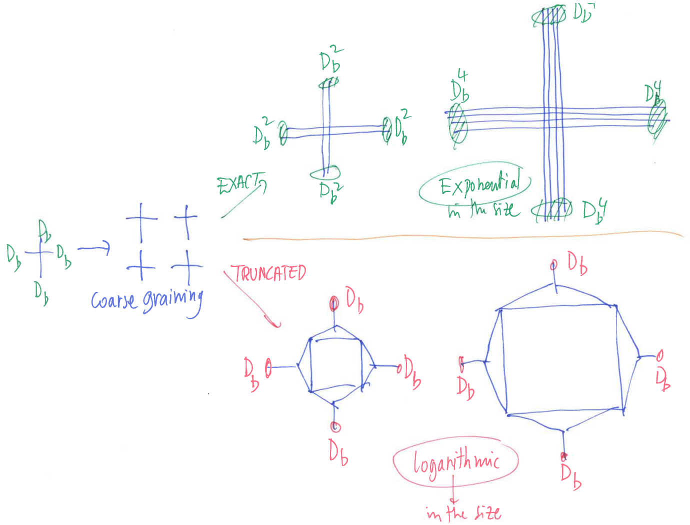
$$T'_{X(x_1,x_2)X'(x'_1,x'_2)Y(y_1,y_2)Y'(y'_1,y'_2)} = \sum_{x_U,x_D,x_R,x_L} T_{x_1x_Uy_1y_L} T_{x_Ux'_1y_2y_R} T_{x_Dx'_2y_Ry'_2} T_{x_2x_Dy_Ly'_1} ,$$

where $X(x_2, x_2)$ is a notation for the product states e. g. , $X(0,0) = 1,\ X(1,1) = 2,\ X(1,0) = 3,\ X(0,1) = 4$. The partition function can be written exactly as

$$Z = (\cosh(\beta))^{2V} \mathrm{Tr} \prod_{2i} T'^{(2i)}_{XX'YY'} ,$$

where $2i$ denotes the sites of the coarser lattice with twice the lattice spacing of the original lattice. Using a truncation in the number of “states" carried by the indices, we can write a fixed point equation.

TRG is a competitor for QC: CPU time $\propto \log(V)$ with no sign problems (both sides will benefit!)



Comparing with Onsager-Kaufman (with Haiyuan Zou, Yuzhi Liu and Alan Denbleyker et al., PRD 89)

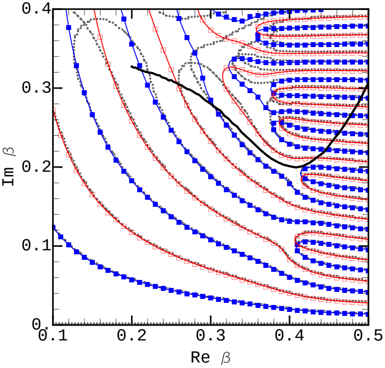


Figure: Zeros of Real (■) and Imaginary (□) part of the partition function of the Ising model at volume 8×8 from the HOTRG calculation with $D_s = 40$ are on the exact lines. Gray lines: MC reweighting solution. Thick Black curve: the "radius of confidence" for MC reweighting result, above this line, the error is large.

FAQ: Do truncation break global or local symmetries? No (Y.M. arXiv 1903.01918)

- Truncations of the tensorial sums are necessary, but do they break the symmetries of the model?
- In arXiv 1903.01918, we consider the tensor formulation of the non-linear O(2) sigma model and its gauged version (the compact Abelian Higgs model), on a D-dimensional cubic lattice, and show that tensorial truncations are compatible with the general identities derived from the symmetries of these models (selection rule ...).
- This selection rule is due to the quantum number selection rules at the sites and is independent of the particular values taken by the tensors. So if we set some of the tensor elements to zero as we do in a truncation, this does not affect the selection rule.
- The universal properties of these models can be reproduced with highly simplified formulations desirable for implementations with quantum computers or for quantum simulations experiments. Truncations are compatible with universality.

The O(2) model (Ising model with spin on a circle)

Integration measure: $\int \mathcal{D}\Phi = \prod_x \int_{-\pi}^{\pi} \frac{d\varphi_x}{2\pi}$

Action: $S[\Phi] = -\beta \sum_{x,i} \cos(\varphi_{x+\hat{i}} - \varphi_x)$ are invariant under

$$\varphi'_x = \varphi_x + \alpha$$

This implies that for a function f of N variables

$$\langle f(\varphi_{x_1}, \ldots, \varphi_{x_N}) \rangle = \langle f(\varphi_{x_1} + \alpha, \ldots, \varphi_{x_N} + \alpha) \rangle$$

Since f is 2π-periodic and can be expressed in terms Fourier modes

$$\left\langle \mathrm{e}^{(i(n_1\varphi_{x_1} + \ldots n_N\varphi_{x_N}))} \right\rangle = \mathrm{e}^{((n_1 + \ldots n_N)\alpha)} \left\langle \mathrm{e}^{(i(n_1\varphi_{x_1} + \ldots n_N\varphi_{x_N}))} \right\rangle$$

This implies that if $\sum_{n=1}^{N} n_i \neq 0, \text{ then } \left\langle \mathrm{e}^{(i(n_1\varphi_{x_1} + \cdots + n_N\varphi_{x_N}))} \right\rangle = 0.$
In arXiv 1903.01918, we show that this selection rule follows from local conservation laws encoded in the tensors.

Algebraic aspects (in one dimension)

In the Hamiltonian formalism, we introduce the angular momentum eigenstates which are also energy eigenstates

$$\hat{L}|n\rangle = n|n\rangle, \hat{H}|n\rangle = \frac{n^2}{2}|n\rangle$$

We assume that n can take any integer value from $-\infty$ to $+\infty$. As $\hat{H} = (1/2)\hat{L}^2$, it is obvious that $[\hat{L}, \hat{H}] = 0$.

The insertion of $\mathrm{e}^{i\varphi_x}$ in the path integral, translates into as operator $\widehat{\mathrm{e}^{i\varphi}}$ which raises the charge $\widehat{\mathrm{e}^{i\varphi}}|n\rangle = |n+1\rangle$, while its Hermitean conjugate lowers it $(\widehat{\mathrm{e}^{i\varphi}})^\dagger|n\rangle = |n-1\rangle$.

This implies the commutation relations

$$[L, \widehat{\mathrm{e}^{i\varphi}}] = \widehat{\mathrm{e}^{i\varphi}},\ [L, \widehat{\mathrm{e}^{i\varphi}}^\dagger] = -\widehat{\mathrm{e}^{i\varphi}}^\dagger, [\widehat{\mathrm{e}^{i\varphi}}, \widehat{\mathrm{e}^{i\varphi}}^\dagger] = 0.$$

Truncation effects on algebra

By truncation we mean that there exists some n_{max} for which

$$\widehat{e^{i\varphi}}|n_{max}\rangle = 0, \text{and } (\widehat{e^{i\varphi}})^\dagger|-n_{max}\rangle = 0.$$

The only changes the commutation relations are

$$\begin{aligned} \langle n_{max}|[\widehat{e^{i\varphi}}, \widehat{e^{i\varphi}}^\dagger]|n_{max}\rangle &= 1, \\ \langle -n_{max}|[\widehat{e^{i\varphi}}, \widehat{e^{i\varphi}}^\dagger]|-n_{max}\rangle &= -1, \end{aligned} \quad (1)$$

instead of 0. The truncation only affects matrix elements involving the $\widehat{e^{i\varphi}}$ operators but does not contradict that: If $\sum_{n=1}^{N} n_i \neq 0$, then $\langle 0|(\widehat{e^{i\varphi}})^{n_1} \dots (\widehat{e^{i\varphi}})^{n_N}|0\rangle = 0$ (with $(\widehat{e^{i\varphi}})^{-n} \equiv (\widehat{e^{i\varphi}}^\dagger)^n$ for $n > 0$))
Note: similar questions appear in quantum links formulations (see R. Brower, The QCD Abacus, hep-lat/9711027)

TRG Formulation of 3D Z_2 Gauge Theory

$$Z = \sum_{\{\sigma\}} \exp\left(\beta \sum_P \sigma_{12}\sigma_{23}\sigma_{34}\sigma_{41}\right),$$

For each plaquette the weight is

$$\sum_{n=0,1} (\sqrt[4]{\tanh(\beta)}\sigma_{12}\sqrt[4]{\tanh(\beta)}\sigma_{23}\sqrt[4]{\tanh(\beta)}\sigma_{34}\sqrt[4]{\tanh(\beta)}\sigma_{41})^n.$$

Regrouping the factors with a given σ_l and summing over ± 1 we obtain a tensor attached to this link

$$A^{(l)}_{n_1 n_2 n_3 n_4} = \left(\sqrt[4]{\tanh\beta}\right)^{n_1+n_2+n_3+n_4} \times \delta\left(\mathrm{mod}[n_1+n_2+n_3+n_4, 2]\right).$$

A and B tensors

The four links attached to a given plaquette p must carry the same index 0 or 1. For this purpose we introduce a new tensor

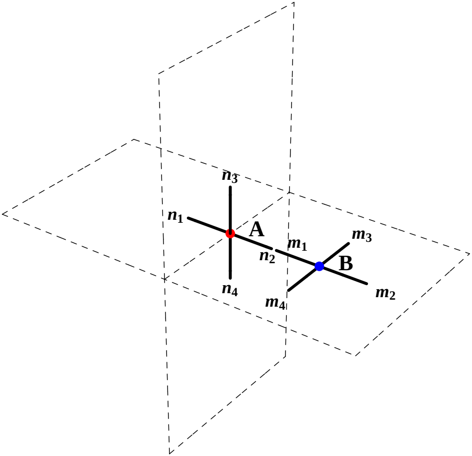
$$B^{(p)}_{m_1 m_2 m_3 m_4} = \delta(m_1, m_2, m_3, m_4) = \begin{cases} 1, & \text{all } m_i \text{ are the same} \\ 0, & \text{otherwise.} \end{cases}$$

The partition function can now be written as

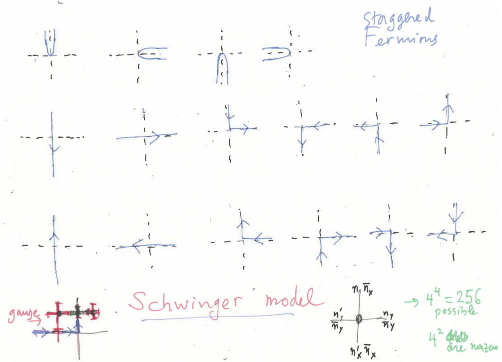
$$Z = (2\cosh\beta)^{3V} \mathrm{Tr} \prod_l A^{(l)}_{n_1 n_2 n_3 n_4} \prod_p B^{(p)}_{m_1 m_2 m_3 m_4},$$

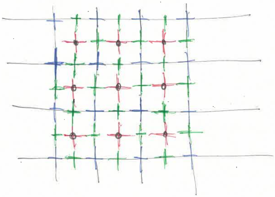
Note: one can move the tanh(β) from links to plaquettes

A and B tensors graphically



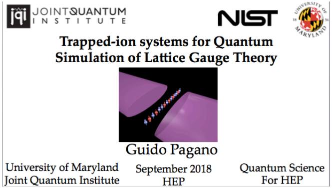
The Schwinger model (in progress with N. Butt, S. Catterall and J. Unmuth-Yockey)





No sign issue (to be confirmed)

Schwinger model with trapped ions



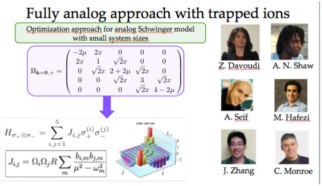
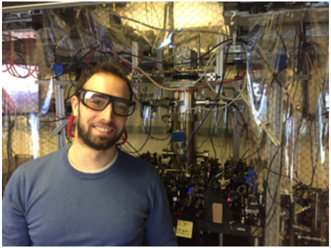


Figure: From Guido Pagano talk at Fermilab.



Dynamical gauge fields with cold atoms and molecules at UIUC

Overview of QIS and Quantum Emulation studies at University of Illinois

>> ***Prospects for studying dynamical gauge fields with cold atoms and molecules***

Bryce Gadway [Paul Kwiat, Virginia Lorenz, Brian DeMarco]
University of Illinois at Urbana-Champaign

Intersections between Nuclear Physics and Quantum Information (NPQI workshop)
Argonne National Laboratory – 2 pm, March 29, 2018

The Abelian Higgs model on a 1+1 space-time lattice

a.k.a. lattice scalar electrodynamics. Field content:

- Complex (charged) scalar field $\phi_x = |\phi_x| \mathrm{e}^{i\theta_x}$ on space-time sites x
- Abelian gauge fields $U_{x,\mu} = \exp iA_\mu(x)$ on the links from x to $x + \hat{\mu}$
- $F_{\mu\nu}$ appears when taking products of U's around an elementary square (plaquette) in the $\mu\nu$ plane
- Notation for the plaquette: $U_{x,\mu\nu} = \mathrm{e}^{i(A(x)_\mu + A(x+\hat{\mu})_\nu - A(x+\hat{\nu})_\mu - A(x)_\nu)}$
- $\beta_{pl.} = 1/e^2$ and κ is the hopping coefficient

$$\begin{aligned} S &= -\beta_{pl.} \sum_x \sum_{\nu<\mu} \mathrm{ReTr}\left[U_{x,\mu\nu}\right] + \lambda \sum_x \left(\phi_x^\dagger \phi_x - 1\right)^2 + \sum_x \phi_x^\dagger \phi_x \\ &- \kappa \sum_x \sum_{\nu=1}^{d} \left[\mathrm{e}^{\mu_{ch.}\delta(\nu,t)} \phi_x^\dagger U_{x,\nu} \phi_{x+\hat{\nu}} + \mathrm{e}^{-\mu_{ch.}\delta(\nu,t)} \phi_{x+\hat{\nu}}^\dagger U_{x,\nu}^\dagger \phi_x\right]. \end{aligned}$$

$$Z = \int D\phi^\dagger D\phi D U e^{-S}$$

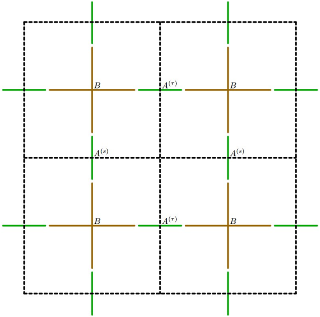
Unlike other approaches (Reznik, Zohar, Cirac, Lewenstein, Kuno,....) we will not try to implement the gauge field on the optical lattice.

Gauge-invariant tensor form: $Z = Tr[\prod T]$

(see PRD.88.056005 and PRD.92.076003)

$$Z = \propto \mathrm{Tr}\left[\prod_{h,v,\square} A^{(s)}_{m_{up} m_{down}} A^{(\tau)}_{m_{right} m_{left}} B^{(\square)}_{m_1 m_2 m_3 m_4}\right] .$$

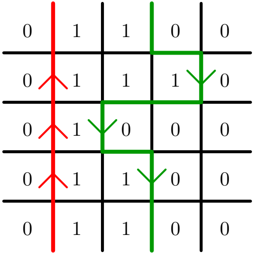
The traces are performed by contracting the indices as shown

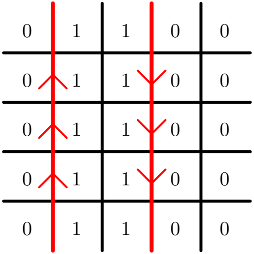


Polyakov loop: definition

Polyakov loop, a Wilson line wrapping around the Euclidean time direction: $\langle P_i \rangle = \langle \prod_j U_{(i,j),\tau} \rangle$ =exp($-F(\mathrm{single\ charge})/kT$); the order parameter for deconfinement.

With periodic boundary condition, the insertion of the Polyakov loop (red) forces the presence of a scalar current (green) in the opposite direction (left) or another Polyakov loop (right).





In the Hamiltonian formulation, we add $-\frac{\tilde{Y}}{2}(2(\bar{L}^z_{i^\star} - \bar{L}^z_{(i^\star+1)}) - 1)$ to H.

Universal functions (FSS): the Polyakov loop

arXiv:1803.11166 (Phys. Rev. Lett. 121, 223201) and
arXiv:1807.09186 (Phys. Rev. D 98, 094511)

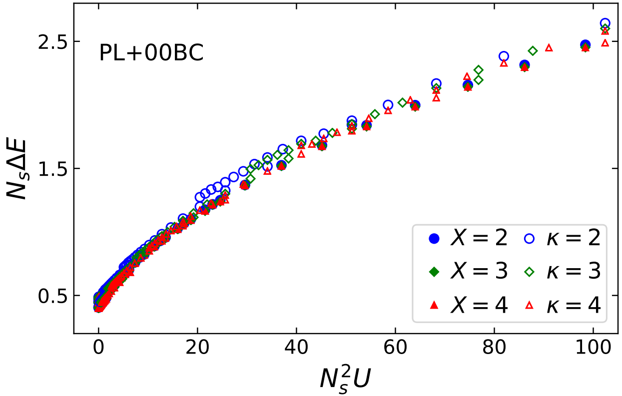


Figure: Data collapse of $N_s\Delta E$ defined from the insertion of the Polyakov loop, as a function of $N_s^2 U$, or $(N_s g)^2$ (collapse of 24 datasets). Numerical work by Judah Unmuth-Yockey and Jin Zhang.

Optical lattice implementation with a ladder

After taking the time continuum limit:

$$\bar{H} = \frac{\tilde{U}_g}{2}\sum_i \left(\bar{L}^z_{(i)}\right)^2 + \frac{\tilde{Y}}{2}\sum_i (\bar{L}^z_{(i)} - \bar{L}^z_{(i+1)})^2 - \tilde{X}\sum_i \bar{L}^x_{(i)}$$

5 states ladder with 9 rungs

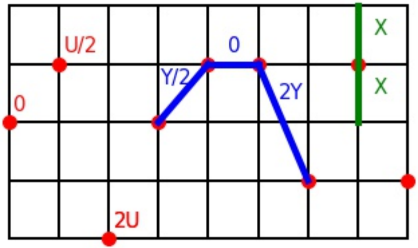


Figure: Ladder with one atom per rung: tunneling along the vertical direction, no tunneling in the the horizontal direction but short range attractive interactions. A parabolic potential is applied in the spin (vertical) direction.

Concrete Proposal

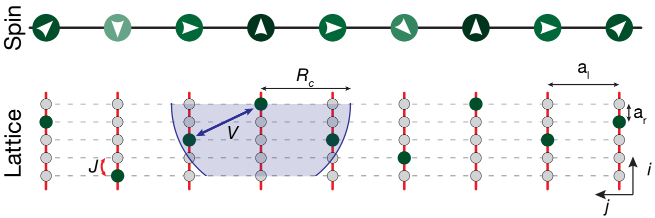
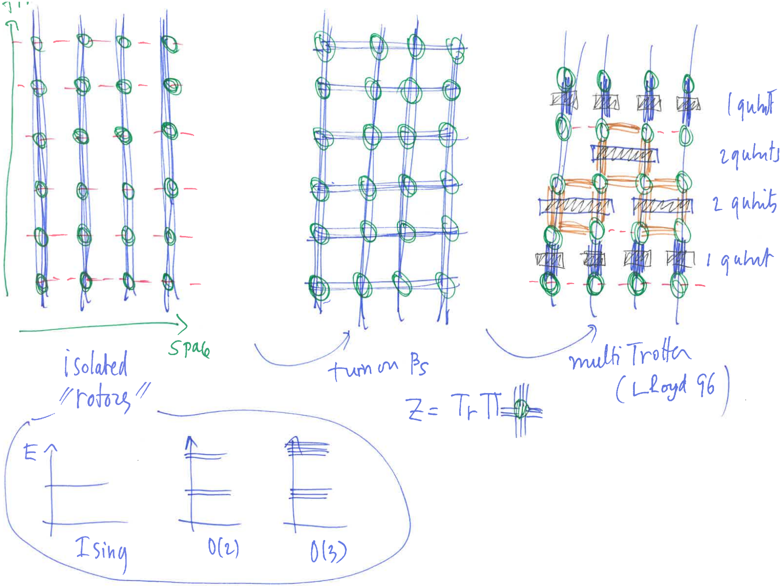


Figure: Multi-leg ladder implementation for spin-2. The upper part shows the possible m_z-projections. Below, we show the corresponding realization in a ladder within an optical lattice. The atoms (green disks) are allowed to hop within a rung with a strength J, while no hopping is allowed along the legs. The lattice constants along rung and legs are a_r and a_l respectively. Coupling between atoms in different rungs is implemented via an isotropic Rydberg-dressed interaction V with a cutoff distance R_c (marked by blue shading).

From tensors to circuits



Time evolution with QC: IBM and Rigetti

PHYSICAL REVIEW A **98**, 032331 (2018)

Quantum-classical computation of Schwinger model dynamics using quantum computers

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(Received 22 March 2018; published 28 September 2018)

We present a quantum-classical algorithm to study the dynamics of the two-spatial-site Schwinger model on IBM's quantum computers. Using rotational symmetries, total charge, and parity, the number of qubits needed to perform computation is reduced by a factor of ~5, removing exponentially large unphysical sectors from the Hilbert space. Our work opens an avenue for exploration of other lattice quantum field theories, such as quantum chromodynamics, where classical computation is used to find symmetry sectors in which the quantum computer evaluates the dynamics of quantum fluctuations.

PHYSICAL REVIEW LETTERS **121**, 170501 (2018)

Simulation of Nonequilibrium Dynamics on a Quantum Computer

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(Received 21 June 2018; revised manuscript received 6 September 2018; published 22 October 2018)

We present a hybrid quantum-classical algorithm for the time evolution of out-of-equilibrium thermal states. The method depends on classically computing a sparse approximation to the density matrix and, then, time-evolving each matrix element via the quantum computer. For this exploratory study, we investigate a time-dependent Ising model with five spins on the Rigetti Forest quantum virtual machine and a one spin system on the Rigetti 8Q-Agave quantum processor.

Quantum circuit for the quantum Ising model

Quantum circuit with 3 Trotter steps (arXiv:1901.05944 E. Gustafson, YM and J. Unmuth-Yockey)

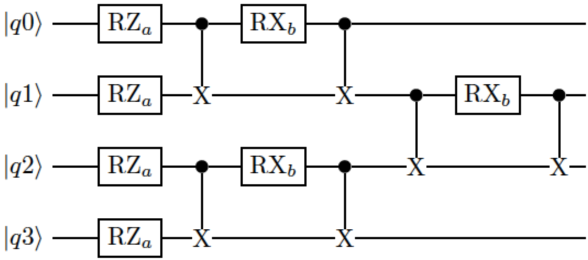


Figure 1: Circuit for 4 qubits with open boundary conditions

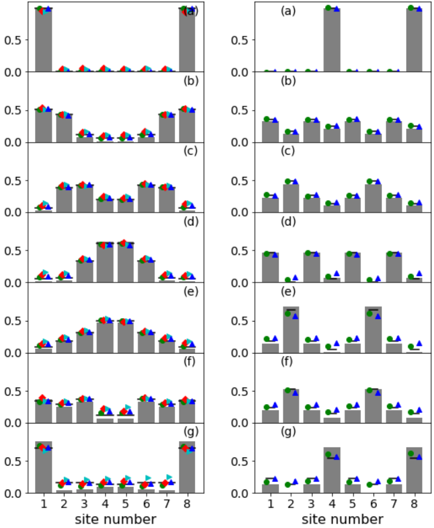
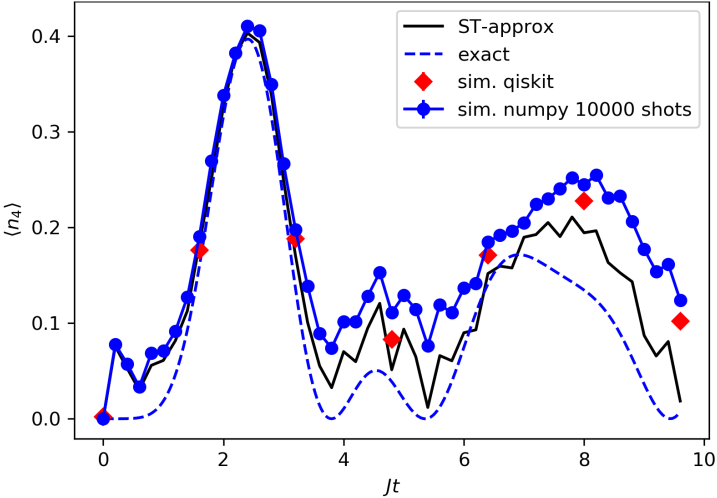


Figure: Evolution of two-particle initial states with OBC (Left) and PBC (Right). Simulations with QISKIT and numpy for current trapped ions or near future superconducting qubits (arXiv:1901.05944).

Syst. and stat. errors (1901.05944, PRD in press, see Erik Gustafson's talk)



Effective description with a linear sigma model?

- In lattice QCD, chiral perturbation theory provides useful parametrizations of the dependence on the mass, volume and lattice spacing; the σ is considered heavy compared to the light Goldstone bosons (pions).
- Recent lattice calculations indicate that as one adds more light flavors, it seems that the σ is lighter and cannot be integrated (need of linear theory?): Y. Aoki et al. (LatKMI), Phys. Rev. D89;T. Appelquist et al. (Lattice Strong Dynamics), Phys. Rev. D 93; Y. Aoki et al. (LatKMI), Phys. Rev. D96; A. D. Gasbarro and G. T. Fleming, PoS LATTICE2016, Z. Fodor et al. PoS LATTICE2014; E. Rinaldi et al. LatKMI, Lattice 2017; J. Kuti's talk.
- The axial anomaly seems to be a significant effect for the linear sigma model: J. Schechter and Y. Ueda, Phys. Rev. D 3, 2874 (1971); C. Rosenzweig, J. Schechter, and C. G. Trahern, Phys. Rev. D 21, 3388 (1980); G. ' t Hooft, Phys. Rept. 142, 357 (1986); Y. Meurice, Mod. Phys. Lett. A2, 699 (1987); A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 77, 094004 (2008).

PHYSICAL REVIEW D VOLUME 21, NUMBER 12 15 JUNE 1980

Is the effective Lagrangian for quantum chromodynamics a σ model?

C. Rosenzweig, J. Schechter, and C. G. Trahern
Physics Department, Syracuse University, Syracuse, New York 13210
(Received 5 September 1979)

A fully interacting effective chiral Lagrangian obeying the anomalous axial-baryon-current conservation law is constructed. This Lagrangian is a generalization of one implied by the 1/N approximation. In a

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Modern Physics Letters A, Vol. 2, No. 9 (1987) 699–706

BREAKING THE AXIAL U(1) DOES NOT ENHANCE SECOND CLASS DECAYS*

Y. MEURICE
High Energy Physics Division, Argonne National Laboratory, Argonne, IL 60439, USA

Revised 30 June 1987

We consider the description of the a_0 meson within a linear sigma model involving different types of explicit breaking of the axial U(1). Those new terms do not enhance f, confirming the

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FERMILAB-Pub-18-683-

Instanton Triggered Chiral Symmetry Breaking, the U(1) Problem and a Possible Solution to the Strong CP Problem

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(Dated: December 20, 2018)

We argue that instantons play a crucial role in triggering the spontaneous breaking of chiral symmetry in the physics of the three light quarks in quantum chromodynamics. However, instantons may also play an essential role in our understanding of the suppression of CP violation in the strong

$N_f = 2$ and 3

For N_f=2, a term used to describe the axial anomaly (($det\phi + det\phi^\dagger$)) provides a mass term with alternate signs:

$$V_a|_{N_f=2} \propto (\eta'^2 - \sigma^2 + \mathbf{a_0^2} - \boldsymbol{\pi}^2).$$

The $N_f = 2$ model was used by 't Hooft to explain the role that the instantons play in the spectrum because if we replace the effective bosonic degrees of freedom by their quark content ($\mathbf{a_0} \sim \bar{\psi}\tau\psi$ etc ..), we recognize a term of the 't Hooft determinant appearing in his instanton calculation.

For 3 flavors with equal masses and an anomaly term $\chi(det\phi + det\phi^\dagger)$ gives a mass spectrum (Y.M. MPLA 2 699, 1987)

$M^2_{\eta'} - M^2_\pi = (3/2)\chi f_\pi$

$M^2_\sigma - M^2_\pi = (3/2)\lambda_1 f^2_\pi - (1/2)\chi f_\pi$

$M^2_a - M^2_\pi = \lambda_2 f^2_\pi + \chi f_\pi$

Linear σ model with N_f flavors (Y. M. PRD 96, 114507)

- Approximate effective theory for a $SU(3)$ local gauge theory with N_f Dirac fermions (hyperquarks) in the fundamental representation and with equal masses (2 masses: D. Floor and E. Gustafson).
- The low energy fields are $2N_f^2$ bosons (σ, $\mathbf{a_0}$, η' and $\boldsymbol{\pi}$)
- The Lagrangian has three parts:
    - a renormalizable $U(N_f)_L \bigotimes U(N_f)_R$ invariant part
    - a $SU(N_f)_V$ invariant mass term
    - a term representing the effects of the axial anomaly.
- We calculated the tree-level spectrum for arbitrary N_f
- Using lattice results (LatKMI), we found combinations of the meson masses that vary slowly with the hyperquark mass and N_f
- The anomaly term plays a leading role in the mass spectrum
- $M_\sigma^2 \simeq (2/N_f - C_\sigma)M_{\eta'}^2$ in the chiral limit
- Lattice measurements of $M_{\eta'}^2$ and the approximate constants C_σ could help locating the boundary of the conformal window where $M_\sigma^2 = 0$ for $N_{fc} \simeq 2/C_\sigma$.

The meson fields ϕ_{ij}

- ϕ: $N_f \times N_f$ matrix of effective fields ϕ_{ij} transforming like $\bar{\psi}_{Rj}\psi_{Li}$ with the summation over the color indices implicit.
- Under a general transformation of $U(N_f)_L \bigotimes U(N_f)_R$,

$$\phi \rightarrow U_L \phi U_R^\dagger \ .$$

- We use a basis of $N_f \times N_f$ Hermitian matrices Γ^α such that

$$Tr(\Gamma^\alpha \Gamma^\beta) = (1/2)\delta^{\alpha\beta},$$

  to express ϕ in terms of N_f^2 scalars (0^+ in J^P notation), denoted S_α, and N_f^2 pseudoscalars (0^-), denoted P_α:

$$\phi = (S_\alpha + iP_\alpha)\Gamma^\alpha,$$

  with a summation over $\alpha = 0, 1, \dots N_f^2 - 1$. We use the convention that $\Gamma_0 = \mathbf{1}/\sqrt{2N_f}$ while the remaining $N_f^2 - 1$ matrices are traceless.
- References: J. Schechter et al., PRD 3 2874; 't Hooft, Phys. Rep. 142 357;Y. M., MPL A2 699 and refs. therein

The effective Lagrangian and its symmetries

- The diagonal subgroup $U(N_f)_V$ defined by the elements of $U(N_f)_L \bigotimes U(N_f)_R$ such that $U_L = U_R$.
- under $U(N_f)_V$, S_0 and P_0 are singlets denoted σ and η' respectively while the remaining components transform like the adjoint representation and are denoted $\mathbf{a_0}$ and π respectively.
- Effective Lagrangian: canonical kinetic term and 3 potential terms

$$\mathcal{L} = Tr\partial_\mu\phi\partial^\mu\phi^\dagger - V_0 - V_a - V_m,$$

- V_0 is the most general $U(N_f)_L \bigotimes U(N_f)_R$ invariant renormalizable expression:

$$\begin{aligned} V_0 \quad &\equiv \quad -\mu^2 Tr(\phi^\dagger\phi) + (1/2)(\lambda_\sigma - \lambda_{a0})(Tr(\phi^\dagger\phi))^2 \\ &\quad\quad + (N_f/2)\lambda_{a0} Tr((\phi^\dagger\phi)^2). \end{aligned}$$

  The use of $\lambda_\sigma - \lambda_{a0}$ will become clear when we write the mass formulas.
- The first two terms of V_0 and the kinetic term have a larger symmetry group $O(2N_f^2)$.

Mass and Axial anomaly terms

- Mass term which is the same for the N_f flavors:

$$V_m \equiv -(b/\sqrt{2N_f})(Tr\phi + Tr\phi^\dagger) = -b\sigma.$$

It is invariant under $SU(N_f)_V$.

- Anomaly term (for hyperquarks in the fundamental representation):

$$V_a \equiv -2(2N_f)^{N_f/2-2}X(det\phi + det\phi^\dagger),$$

is invariant under $SU(N_f)_L \bigotimes SU(N_f)_R$ but breaks the axial $U(1)_A$. The prefactor $2(2N_f)^{N_f/2-2}$ is chosen in order to make the expression of the spectrum as simple as possible. The parameter X has a mass dimension $4 - N_f$.

Chiral symmetry breaking

We assume that chiral symmetry is spontaneously broken by a $SU(N_f)$-invariant vacuum expectation value (v.e.v.):

$$\langle \phi_{ij} \rangle = v\delta_{ij}/\sqrt{2N_f}.$$

This amounts to say that $\langle \sigma \rangle = v$ while the other v.e.v.s are zero. We impose that

$$\partial V/\partial \phi|_{\langle \phi \rangle} = 0.$$

Thanks to the simple form of the v.e.v.s, these N_f^2 equations reduce to a single one:

$$-\mu^2 v + (1/2)\lambda_\sigma v^3 - (X/N_f)v^{N_f-1} = b.$$

In the chiral limit, this description breaks down as we go across the boundary of the conformal windows (other degrees of freedom).

Pions

For the pions, we have:

$$M_\pi^2 = -\mu^2 + (1/2)\lambda_\sigma v^2 - (X/N_f)v^{N_f-2}.$$

Note: when $X = 0$, $(1/2)\lambda_\sigma v^2 > M_\pi^2$
Using the minimization condition

$$-\mu^2 v + (1/2)\lambda_\sigma v^3 - (X/N_f)v^{N_f-1} = b,$$

this can be recast in the form

$$M_\pi^2 v = b.$$

When $b = 0$, $M_\pi^2 v = 0$ and $v \neq 0$ implies that in this chiral limit, the pions are exactly massless Nambu Goldstone bosons.
The v.e.v. v is related to the pion decay constant in the following way:

$$f_\pi = \sqrt{2/N_f}v.$$

The spectrum

The other results for the spectrum can be written in a compact way:

$$
\begin{aligned}
M^2_{\eta'} - M^2_{\pi} &= Xv^{N_f-2} \\
M^2_{\sigma} - M^2_{\pi} &= \lambda_\sigma v^2 - (1-2/N_f)Xv^{N_f-2} \\
M^2_{a0} - M^2_{\pi} &= \lambda_{a0} v^2 + (2/N_f)Xv^{N_f-2}.
\end{aligned}
$$

In the chiral limit ($b = 0$), this reduces to

$$
\begin{aligned}
M^2_{\sigma} &= \lambda_\sigma v^2 - (1-2/N_f)M^2_{\eta'} \\
M^2_{a0} &= \lambda_{a0} v^2 + (2/N_f)M^2_{\eta'}.
\end{aligned}
$$

Note: when $X = 0$, $(1/2)\lambda_\sigma v^2 > M^2_\pi$ and $M^2_\sigma > 3M^2_\pi$

Dimensionless ratios

In order to allow comparisons of numerical results at different lattice spacings, we introduce the dimensionless ratios:

$$R_\sigma \equiv \lambda_\sigma v^2/M^2_{\eta'},$$

and

$$R_{a_0} \equiv \lambda_{a0} v^2/M^2_{\eta'}.$$

We want to test the idea that these quantities vary slowly with the explicit breaking of chiral symmetry (due to the mass of the fermions m_f) and N_f. If it is the case, the mass formulas have simple approximate forms which could provide a nice intuitive picture. To make things completely clear, the ratios should be understood as functions of the spectroscopic data, namely

$$\begin{aligned} R_\sigma &= (M^2_\sigma - M^2_\pi)/M^2_{\eta'} + (1 - 2/N_f)(1 - M^2_\pi/M^2_{\eta'}) \\ R_{a_0} &= (M^2_{a_0} - M^2_\pi)/M^2_{\eta'} - (2/N_f)(1 - M^2_\pi/M^2_{\eta'}). \end{aligned}$$

Dimensionless ratios vs. $M_\pi / M_{eta'}$ with LatKMI data

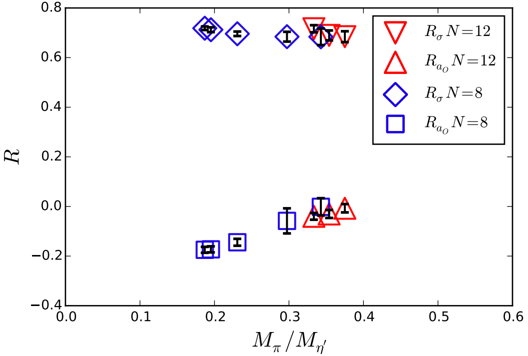

Figure: R_σ for N_f=8 (diamonds) and 12 (upside-down triangles) and R_{a_0} for N_f=8 (squares) and 12 (triangles), versus $M_\pi / M_{\eta'}$.

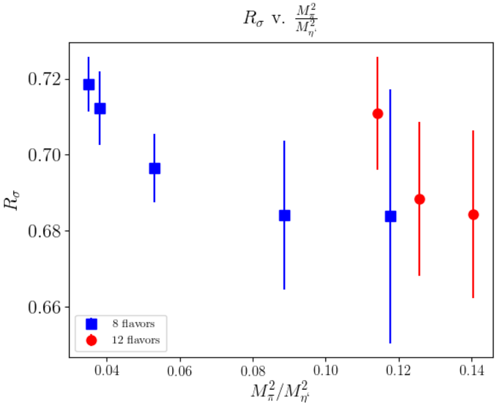

Figure: R_σ for N_f=8 (squares) and 12 (circles) versus $(M_\pi/M_{\eta'})^2$.

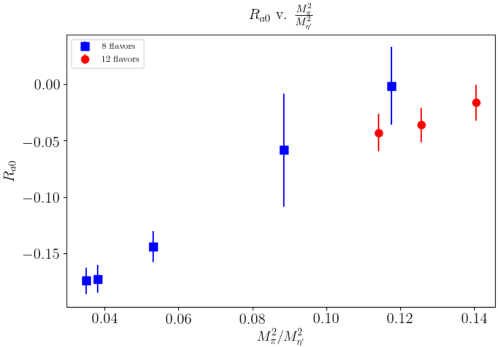

Figure: R_{a_0} for N_f=8 (squares) and 12 (circles) versus $(M_\pi/M_{\eta'})^2$.

Stability of the potential with negative λ_{a0}

The figures show that for small m_f, we have negative values of R_{a_0}. Can λ_{a0} be negative? The stability of the effective potential requires that V_0 stays positive for large values of the field ϕ. Using the symmetry $U(N_f)_L \bigotimes U(N_f)_R$ of V_0, we can diagonalize ϕ. $\phi^\dagger\phi$ is then a diagonal matrix with positive diagonal terms $|\alpha_i|^2$ and the sum of the two quartic terms of V_0 will remain positive when $\lambda_{a0} = -|\lambda_{a0}|$ is negative provided that

$$(1/2)(\lambda_\sigma + |\lambda_{a0}|)(\sum_{i=1}^{N_f} |\alpha_i|^2)^2 \geq (N_f/2)|\lambda_{a0}| \sum_{i=1}^{N_f} |\alpha_i|^4.$$

This inequality should remain valid for any choice of α_i. Considering the case where only one $|\alpha_i|$ becomes arbitrarily large, we get the requirement

$$\lambda_\sigma \geq (N_f - 1)|\lambda_{a0}|$$

which is sufficient to insure that the inequality is satisfied in general.

Two masses, D. Floor, E. Gustafson and Y. M., Phys. Rev. D 98, 094509 (2018).

Mass term with N_1 flavors of mass m_1 and N_2 flavors of mass m_2,

$$V_m \equiv -(Tr\mathcal{M}\phi + h.c.) = -b_0 S_0 - b_8 S_8$$

. The matrix $\mathcal{M}$ can be written as $b_0\Gamma^0 + b_8\Gamma^8$ and V_m is invariant under $SU(N_1)_V \bigotimes SU(N_2)_V$. We assume that this vector symmetry is not broken spontaneously and that the vacuum expectation of ϕ has the form:

$$\langle\phi\rangle = \frac{1}{\sqrt{2N_f}}.\begin{pmatrix} v_1\mathbf{1}_{N_1\times N_1} & 0 \\ 0 & v_2\mathbf{1}_{N_2\times N_2}. \end{pmatrix} \quad (2)$$

This means that $\langle S_0\rangle = v_0$ and $\langle S_8\rangle = v_8$ or equivalently

$$\langle\phi\rangle = v_0\Gamma^0 + v_8\Gamma^8.$$

The transformation between the two expressions is

$$\begin{aligned} v_1 &= v_0 + \sqrt{N_2/N_1}v_8 \quad (3) \\ v_2 &= v_0 - \sqrt{N_1/N_2}v_8, \quad (4) \end{aligned}$$

Mass inversion in the a_0 sector

$$\begin{aligned}\Delta_\sigma &\equiv M^2_\sigma - M^2_\pi \\ \Delta_{a0} &\equiv M^2_{a0} - M^2_\pi \\ \Delta_{\eta'} &\equiv M^2_{\eta'} - M^2_\pi,\end{aligned}$$

$$M^2_{\pi_{hh}} - M^2_{\pi_{ll}} \simeq \frac{(v_2 - v_1)}{v}\Delta_{a0}.$$

$$M^2_{\pi_{lh}} - M^2_{\pi_{ll}} \simeq \frac{1}{2}(M^2_{\pi_{hh}} - M^2_{\pi_{ll}})$$

$$M^2_{a0_{hh}} - M^2_{a0_{ll}} \simeq \frac{(v_2 - v_1)}{v}(3\Delta_{a0} - \frac{8}{N_f}\Delta_{\eta'})$$

$$M^2_{a0_{lh}} - M^2_{a0_{ll}} \simeq \frac{1}{2}(M^2_{a0_{hh}} - M^2_{a0_{ll}})$$

Expected splittings for M^2_π: we need to have $v_2 > v_1$ because $\Delta_{a_0} > 0$, but then $M^2_{a0_{hh}} < M^2_{a0_{hl}} < M^2_{a0_{ll}}$ because $\frac{3\Delta_{a_0} - (8/N_f)\Delta_{\eta'}}{M^2_{\eta'}} < 0$ for all the LatKMI datasets

N_f	(am_f)	$\frac{\Delta_{a_0}}{M^2_{\eta'}}$	$\frac{3\Delta_{a_0}-(8/N_f)\Delta_{\eta'}}{M^2_{\eta'}}$
8	0.012	0.0584(74)	-0.669(71)
8	0.015	0.0644(78)	-0.724(81)
8	0.02	0.0885(95)	-0.640(70)
8	0.03	0.160(41)	-0.38(16)
8	0.04	0.214(28)	-0.22(11)
12	0.04	0.0854(94)	-0.225(52)
12	0.05	0.1079(65)	-0.163(51)
12	0.06	0.1124(73)	-0.261(65)

Table: Values $\frac{\Delta_{a_0}}{M^2_{\eta'}}$ and $\frac{3\Delta_{a_0}-(8/N_f)\Delta_{\eta'}}{M^2_{\eta'}}$ using the LatKMI data .

The reasons for the inversion are clear if you look at detailed formulas. Not seen for mass splitting used in R. C. Brower, A. Hasenfratz, C. Rebbi, E. Weinberg, and O. Witzel, Phys. Rev. D 93, 075028 (2016).

Radiative corrections (With J. Bijnens, in progress)

The reasons for the inversion are clear. Assuming $v_2 > v_1$ to get the standard ordering for the pseudoscalars, we see that $\lambda_{a_0} < 0$ makes $\lambda_{a_0} v_2^2$ more negative for $M^2_{a0_{hh}}$. In addition, the anomaly term for $M^2_{a0_{hh}}$ has larger powers of v_1 and lower powers of v_2 than $M^2_{a0_{ll}}$ and the coupling is positive so again it inverts the ordering. Can radiative corrections modify this picture?

$$\lambda_{\sigma\sigma\sigma\sigma} v^2 = -(1/8N_f)(\Delta_\sigma) + (1/3 - N_f/4 + N_f^2/24)\Delta_{\eta'}$$

3 *pages* ...

$$\delta M_\sigma^2 = 12\lambda_{\sigma\sigma\sigma\sigma}(M_\sigma^2/16\pi^2)(1 - \log(M_\sigma^2/\mu^2)) + \ldots$$

- Not clear that perturbation theory makes sense for lattice data
- Tree level formula may not be reliable
- Vacuum stability issues
- Work in progress

Conclusions

- Tensor Field Theory is a generic tool to discretize path integral formulations of lattice model with compact variables
- TRG: exact blocking, a friendly competitor to QC; we have all the basic blocks for QCD; sampling is also possible (Gattringer ...)
- Truncations respect symmetries
- TRG: gauge-invariant approach for the quantum simulation of gauge models.
- Finite size scaling: small systems are interesting
- Need for dedicated quantum simulations and computations facilities
- A better understanding of conformal (or near conformal) lattice gauge models is necessary before attempting model building
- The anomaly term plays a leading role in the mass spectrum
- Tree level effective linear models may not be reliable
- Thanks!

Acknowledgements:

This research was supported in part by the Dept. of Energy under Award Numbers DOE grants DE-SC0010113, and DE-SC0019139