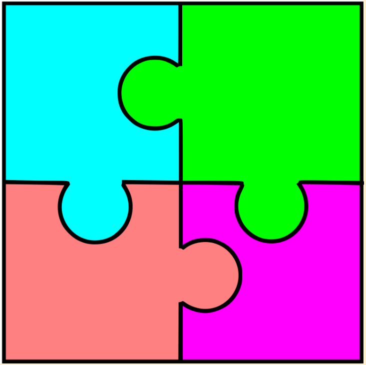
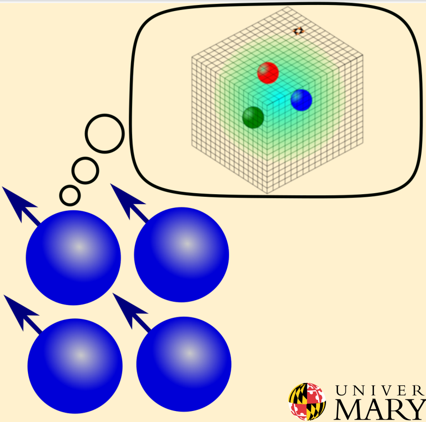
Pieces of the Puzzle:

Reaching QCD on Quantum Computers

Hank Lamm





UNIVERSITY OF MARYLAND

Thanks to the NuQS team!

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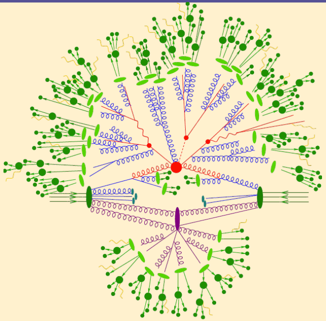
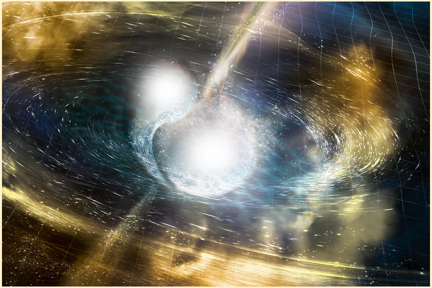
WASHINGTON, DC

Outline

1. Quantum Leap
2. Digitization
3. Initialization
4. Propagation
5. Evaluation
6. Conclusions

Finite-Density and Real-Time QFT have sign problems





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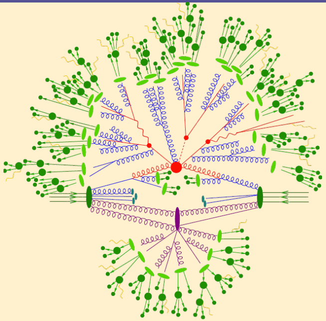

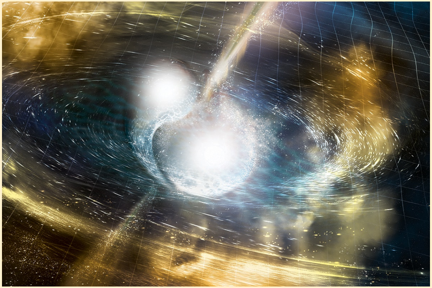

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}\phi \; e^{-iS_I} \mathcal{O} e^{-S_R}}{\int \mathcal{D}\phi \; e^{-S_R}} \frac{\int \mathcal{D}\phi \; e^{-S_R}}{\int \mathcal{D}\phi \; e^{-S_R} e^{-iS_I}} = \frac{\left\langle \mathcal{O} e^{-iS_I} \right\rangle_{S_R}}{\langle \sigma \rangle_{S_R}}$$

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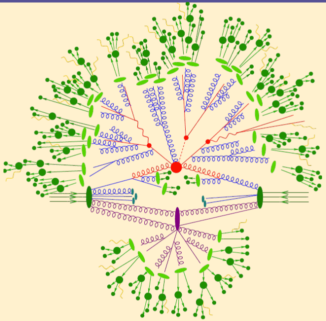

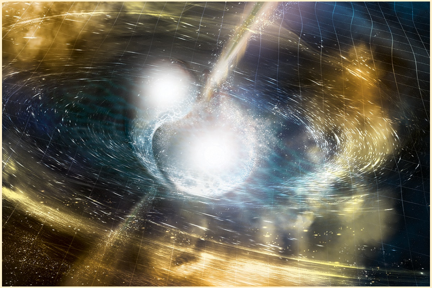

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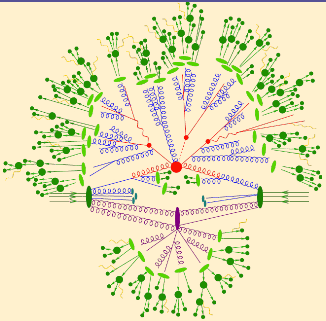

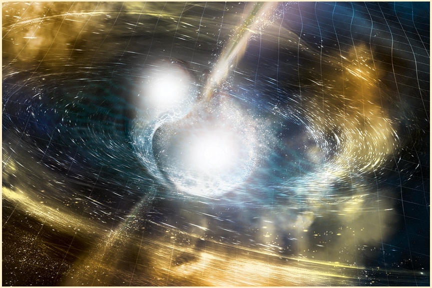

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- For real t: $\langle \sigma \rangle_{S_R} = 0$
- For $\mu \neq 0$: Need $\propto \langle \sigma \rangle_{S_R}^{-2}$ configurations

If you want these...you need quantum computers

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Converting Bits to Qubits

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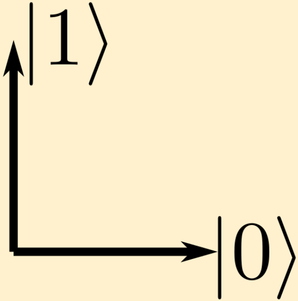
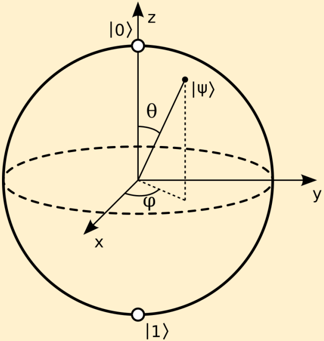
$$\{|0\rangle, |1\rangle\} \rightarrow \{a|0\rangle + b|1\rangle\}$$





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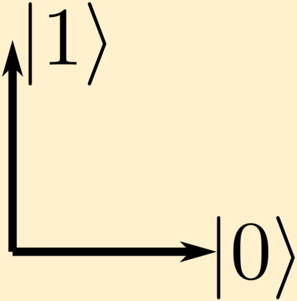

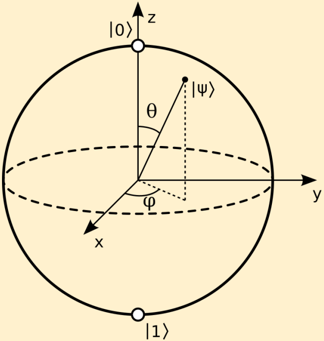

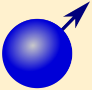

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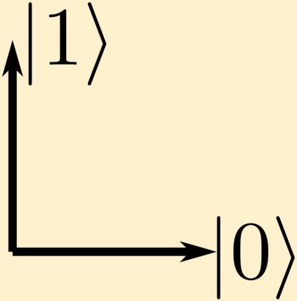

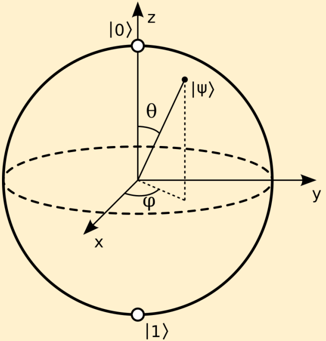

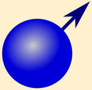

Digital QC provide entangled bits and gates, not field theories.

Lots of $$, Lots of Interest, Lots of Hype

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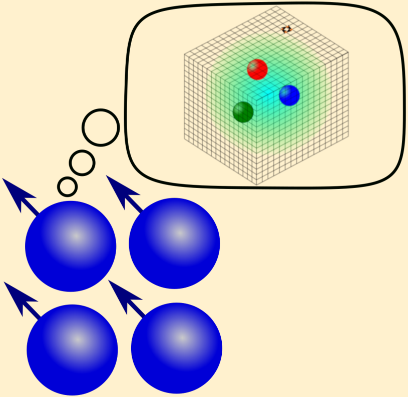
How can we use qubits for QFT analogous to LQCD



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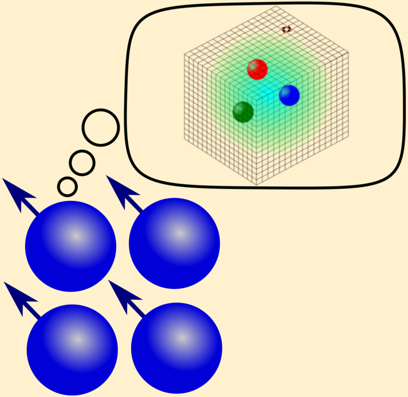

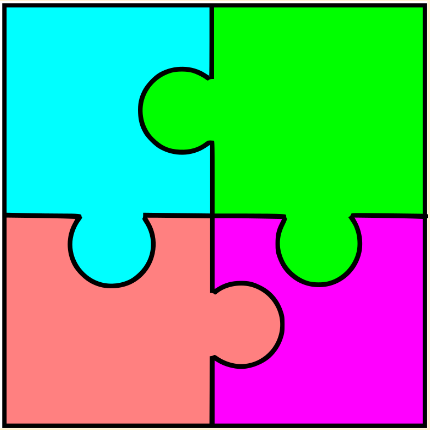

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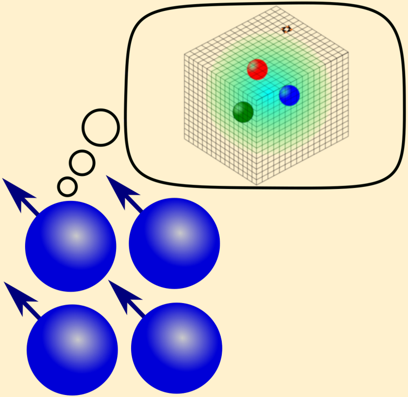

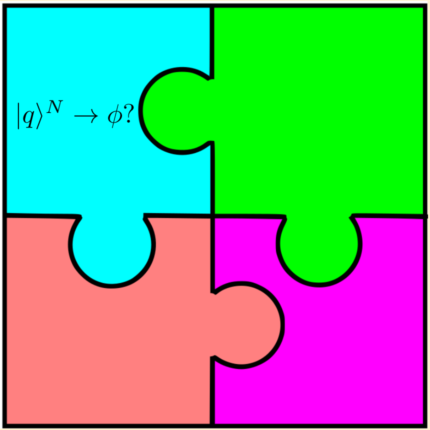

- Digitize: How are (continuous) fields represented as a register?

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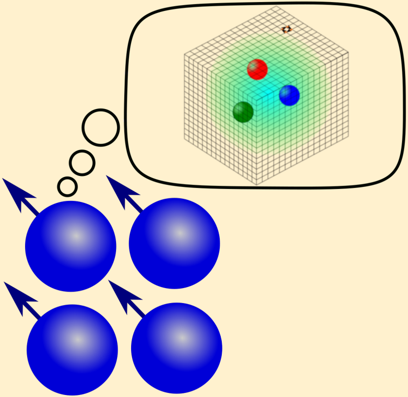

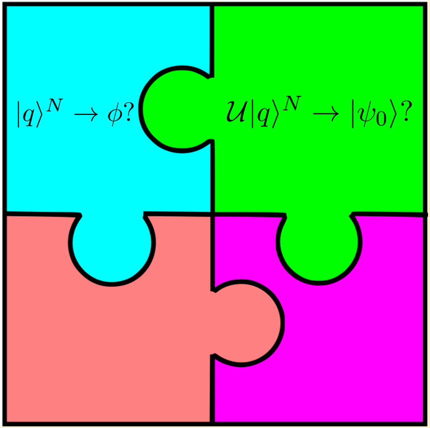

- Digitize: How are (continuous) fields represented as a register?
- Initalize: How can registers be set to a field configuration?

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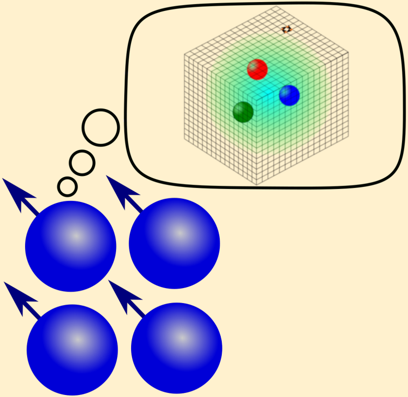

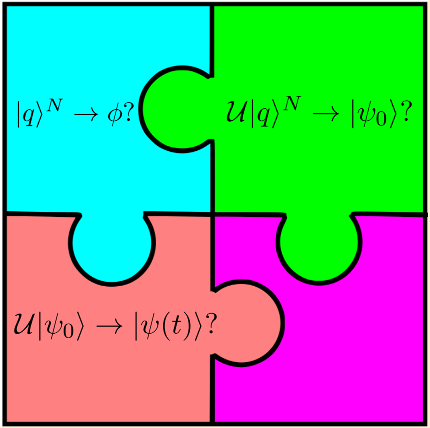

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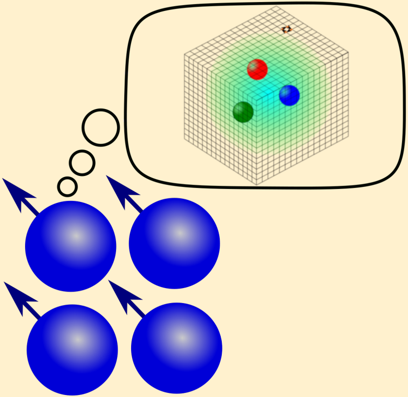

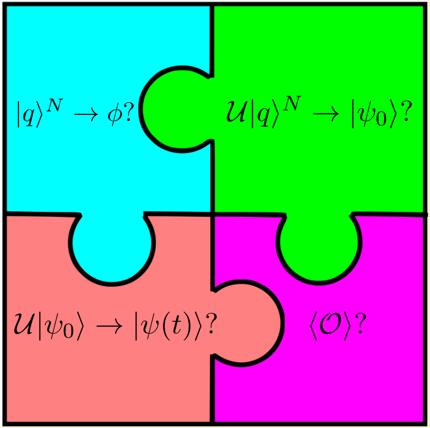

- Digitize: How are (continuous) fields represented as a register?
- Initalize: How can registers be set to a field configuration?
- Propagate: How can gates be combined to evolve states?
- Evaluate: How can observables of interest be computed?

Army you have, Army you might have, Army you want

	$N_{\|q\rangle} < 500$	$N_{\|q\rangle} \to \infty$
$N_{\mathcal{U}} \lesssim 100 N_{\|q\rangle}$	NISQ Noisy, Interm.	NESQ Noisy, Enorm.
$N_{\mathcal{U}} \to \infty$	FISQ Faithful, Interm.	FESQ Faithful, Enorm.

[1] Nam, Y., J.-S. Chen, N. C. Pisenti, K. Wright, C. Delaney, D. Maslov, K. R. Brown, S. Allen, J. M. Amini, J. Apisdorf, et al. In: *arXiv preprint arXiv:1902.10171* (2019).

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- Moore's law like behavior "could" render methods irrelevant.

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Can we efficently approximate elements of group?

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[3] Raychowdhury, I. and J. R. Stryker. In: (2018). arXiv: 1812.07554 [hep-lat].

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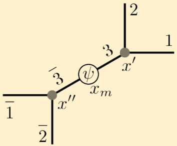

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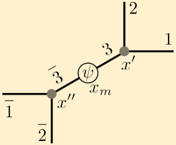

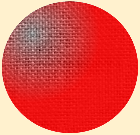

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- Discrete Subgroups (Accost Me)

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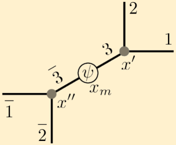

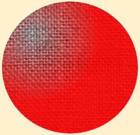

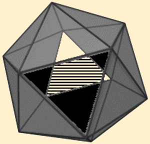

the binary icosahedral group

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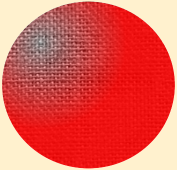
Fuzzy spheres can reproduce low-lying spectrum exactly[6]



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Fuzzy spheres can reproduce low-lying spectrum exactly[6]

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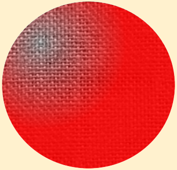

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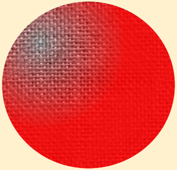

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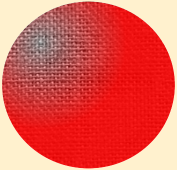

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- $$\psi(\mathbf{n}) = \psi_0 + \psi_i n_i + \frac{1}{2} \psi_{ij} n_i n_j + \dots . \quad (2)$$

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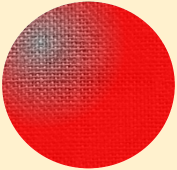

- Truncate the communting algebra of functions by a non-communting algebra
- The $O(3)$ sigma-model is defined by the Hamiltonian

$$\mathcal{H} = \sum_r \left[\frac{g^2}{2} \boldsymbol{\pi}(r)^2 + \frac{1}{2g^2 \Delta x^2} (\mathbf{n}(r+1) - \mathbf{n}(r))^2 \right], \quad (1)$$

- $$\psi(\mathbf{n}) = \psi_0 + \psi_i n_i + \frac{1}{2} \psi_{ij} n_i n_j + \dots . \quad (2)$$

- $$\Psi = \psi_0 \mathbb{1} + \psi_i \mathbb{J}_i + \frac{1}{2} \psi_{ij} \mathbb{J}_i \mathbb{J}_j + \dots , \quad (3)$$

  where $\mathbb{J}_i$, $i = 1, 2, 3$ are generators of $SU(2)$ in a given representation j

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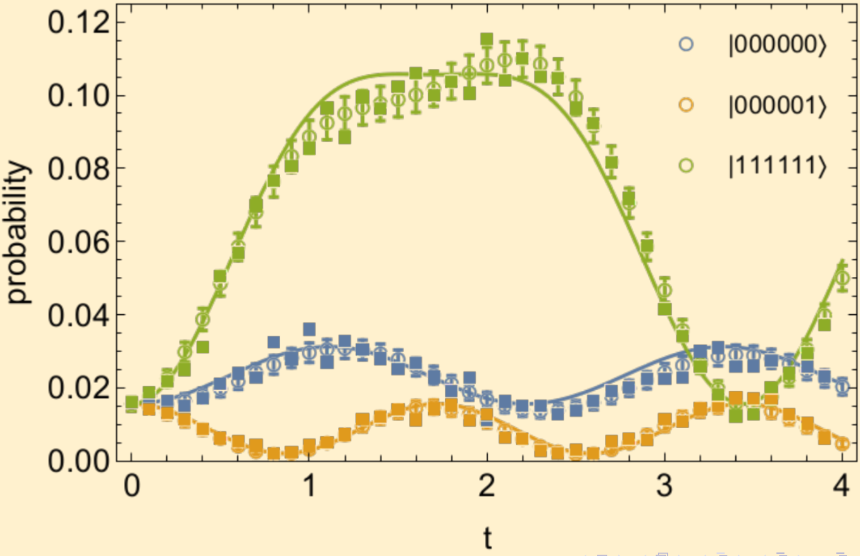
2 qubits per site, $12LT/\Delta t$ CNOT gates



Σ_{1080}: Largest Crystal-like Subgroup of $SU(3)$[7]

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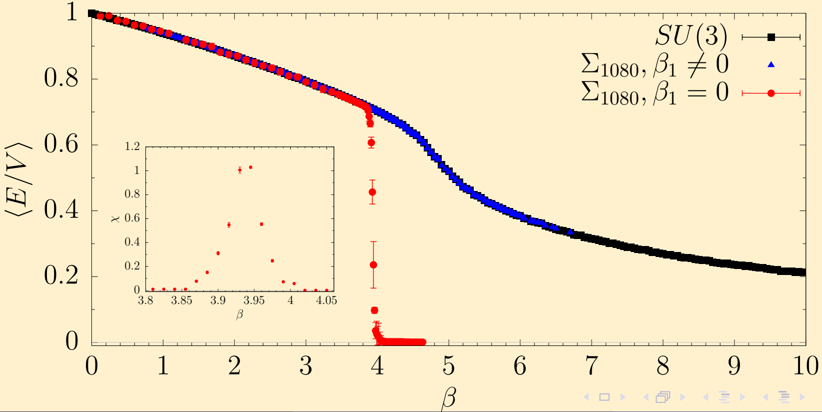
- But Wilson Action freezes at $\beta_c \approx 3.94(2)$ on 2^4!



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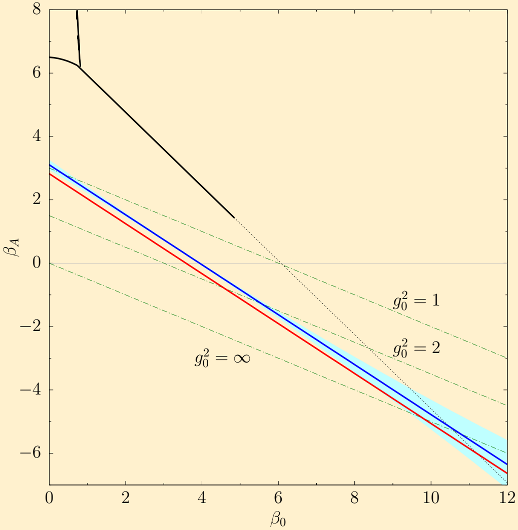

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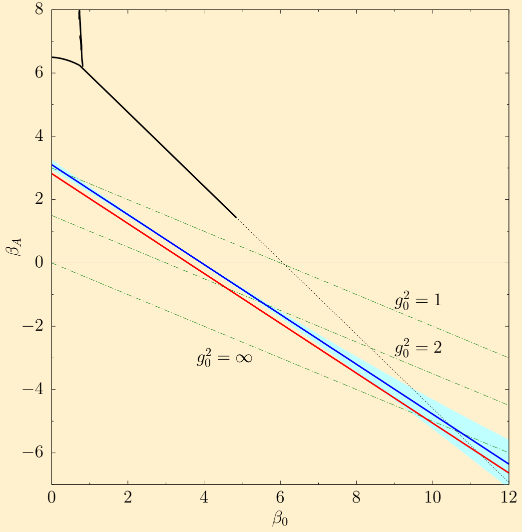

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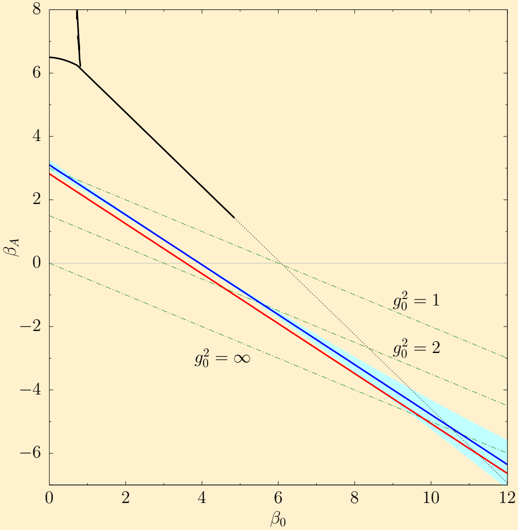

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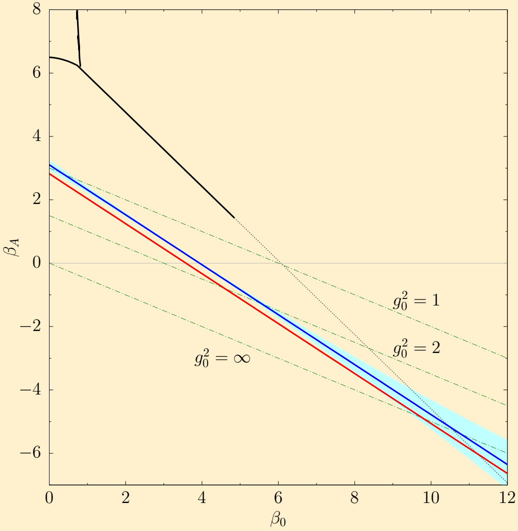

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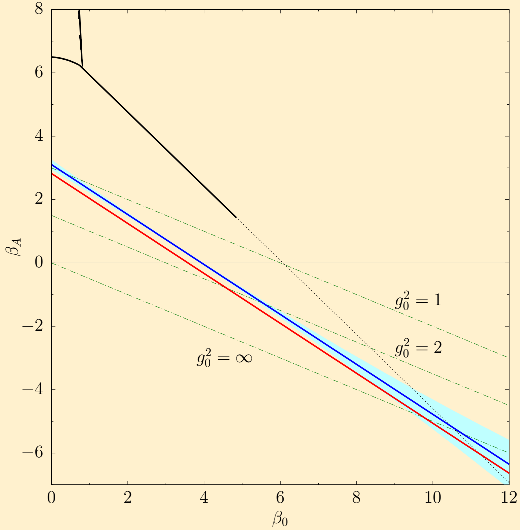

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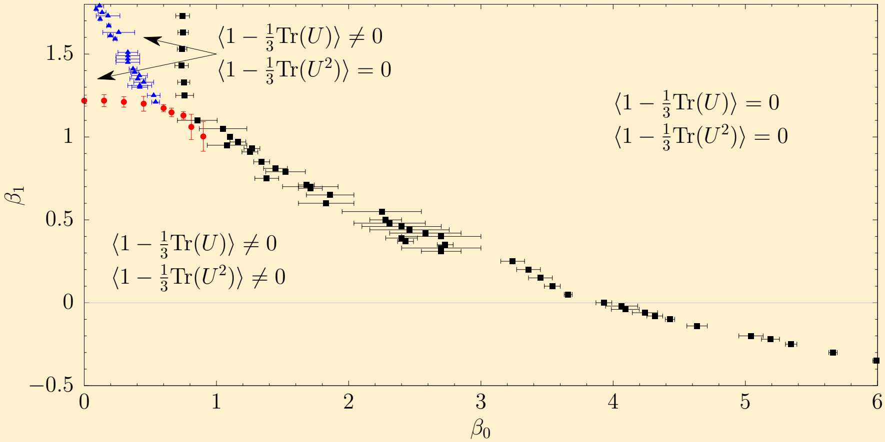

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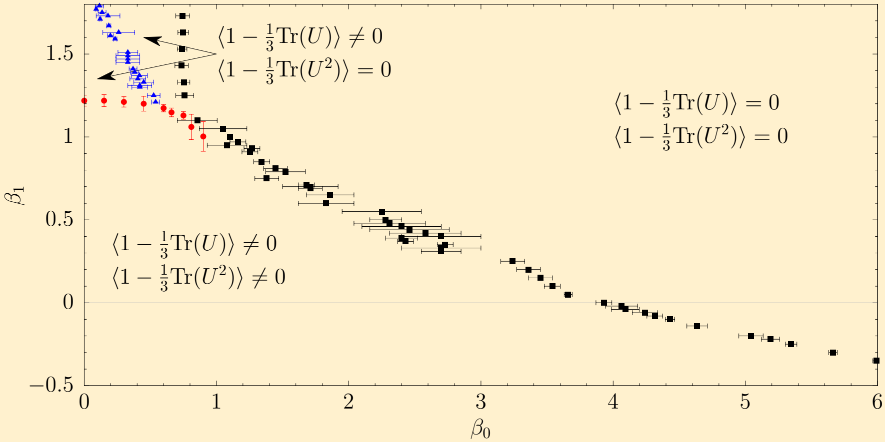

What are the states of strongly-coupled theories?

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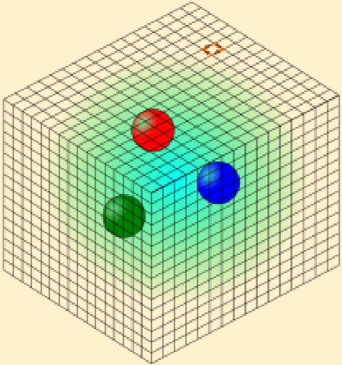
What is the proton state in terms of quarks and gluons?



$E\rho OQ$: A hybrid quantum-classical technique

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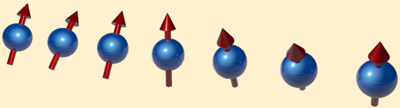

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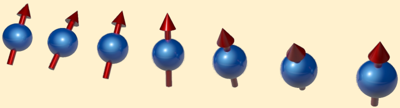

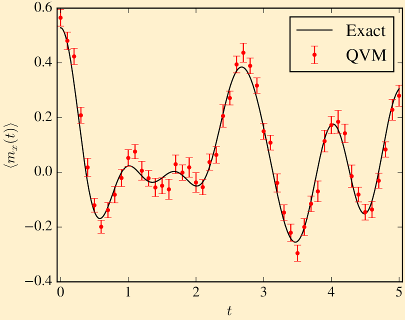

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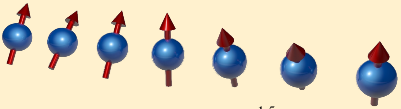

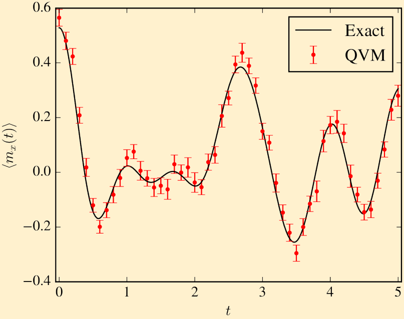

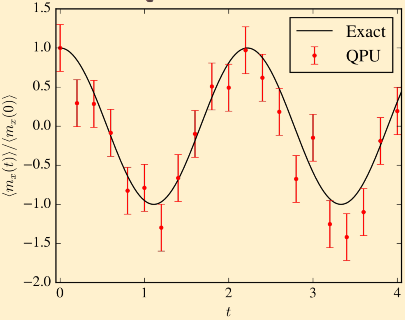

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Expectation value of a unitary operator U in a given state $|\Psi\rangle$. Introducing a single ancillary qubit, we construct a controlled unitary operator U_C, defined by

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Time evolving forward in time with $H_{\epsilon_1,\epsilon_2}$, and back with H_0 gives $C(\epsilon_1, \epsilon_2) \equiv \langle\Psi| \mathcal{U}(-t) \mathcal{U}_{\epsilon_1,\epsilon_2}(t) |\Psi\rangle$. Differentiating twice

$$-\left.\frac{\partial^2 C(\epsilon_1, \epsilon_2)}{\partial\epsilon_1 \partial\epsilon_2}\right|_{\epsilon_1=\epsilon_2=0} = \left\langle \mathcal{U}(-t) W_{\mu'\nu'}(x') \mathcal{U}(t) W_{\mu\nu}(x) \right\rangle \tag{8}$$

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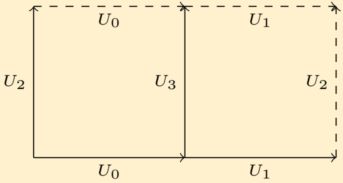
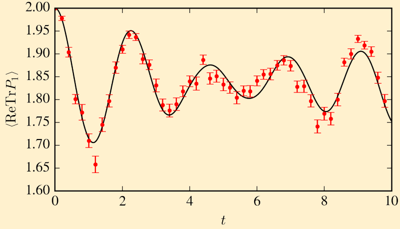
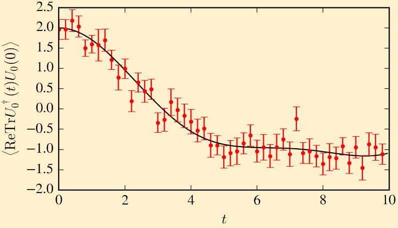


Four D_4 registers, and uses a total of 14 qubits: 12 for physical degrees of freedom, and 2 ancillary qubits. $t = 10$ with a Trotterization step of $\Delta t = 0.2$. In total, the quantum simulation entailed ~ 200 entangling gates per Trotterization time step.





How to obtain parton distribution functions?

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in K-S prochedure $\chi \propto \sigma_+$ and $\chi^{\dagger} \propto \sigma_-$ which can only be measured by decomposing into σ_x and σ_y measurements, so need 4 simulations where

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Simplfy to 1+1 Thirring, then the matrix element

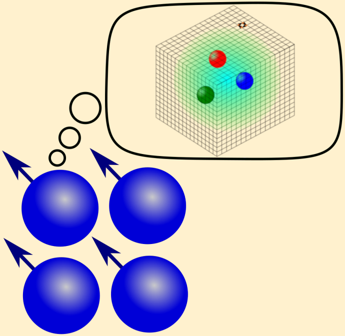
$$\langle P|\chi^{\dagger}(tn^{\mu})\chi(0)|P\rangle = \langle P|e^{iHt}\chi^{\dagger}(y)e^{-iHt}\chi(0)|P\rangle = \sum_{i,j=\{x,y\}} \frac{c_{ij}}{4}\langle P|U_{i,j}|P\rangle \tag{10}$$

in K-S prochedure $\chi \propto \sigma_+$ and $\chi^{\dagger} \propto \sigma_-$ which can only be measured by decomposing into σ_x and σ_y measurements, so need 4 simulations where

$$U_{i,j} = e^{iHt}\sigma_i e^{-iHt}\sigma_j \tag{11}$$

With this Hermitian construction, we can use the same U_C based procedure prevent collapse after first measurement σ_j at the cost of $2\times$ the measurements so 8 calculations per matrix element.

Ongoing Work of NuQS Collaboration



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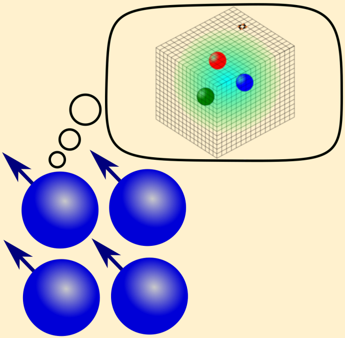

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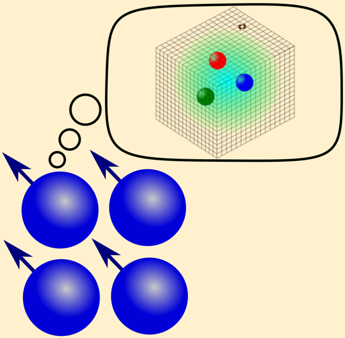

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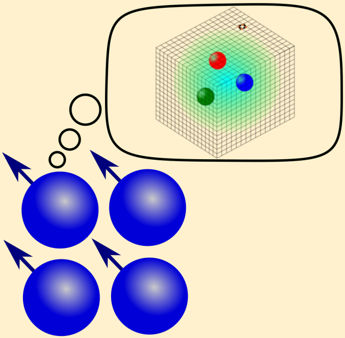

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- Initialize w/ Lattice Field Theory
  - Avoid the state specificiation?
- Evaluate Composite matrix elements
  - Parton Distribution Functions?