Walking, σ-particle vs. dilaton, complex CFT
(the sextet case study)

Lattice Higgs Collaboration (L$_{at}$HC)

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*Tantalizing big questions in the sextet case study:

Origin of near-conformal walking?

How light is the emergent 0++ scalar in chiral limit?

Need to extrapolate with IR EFT like σ-model, or dilaton, or …

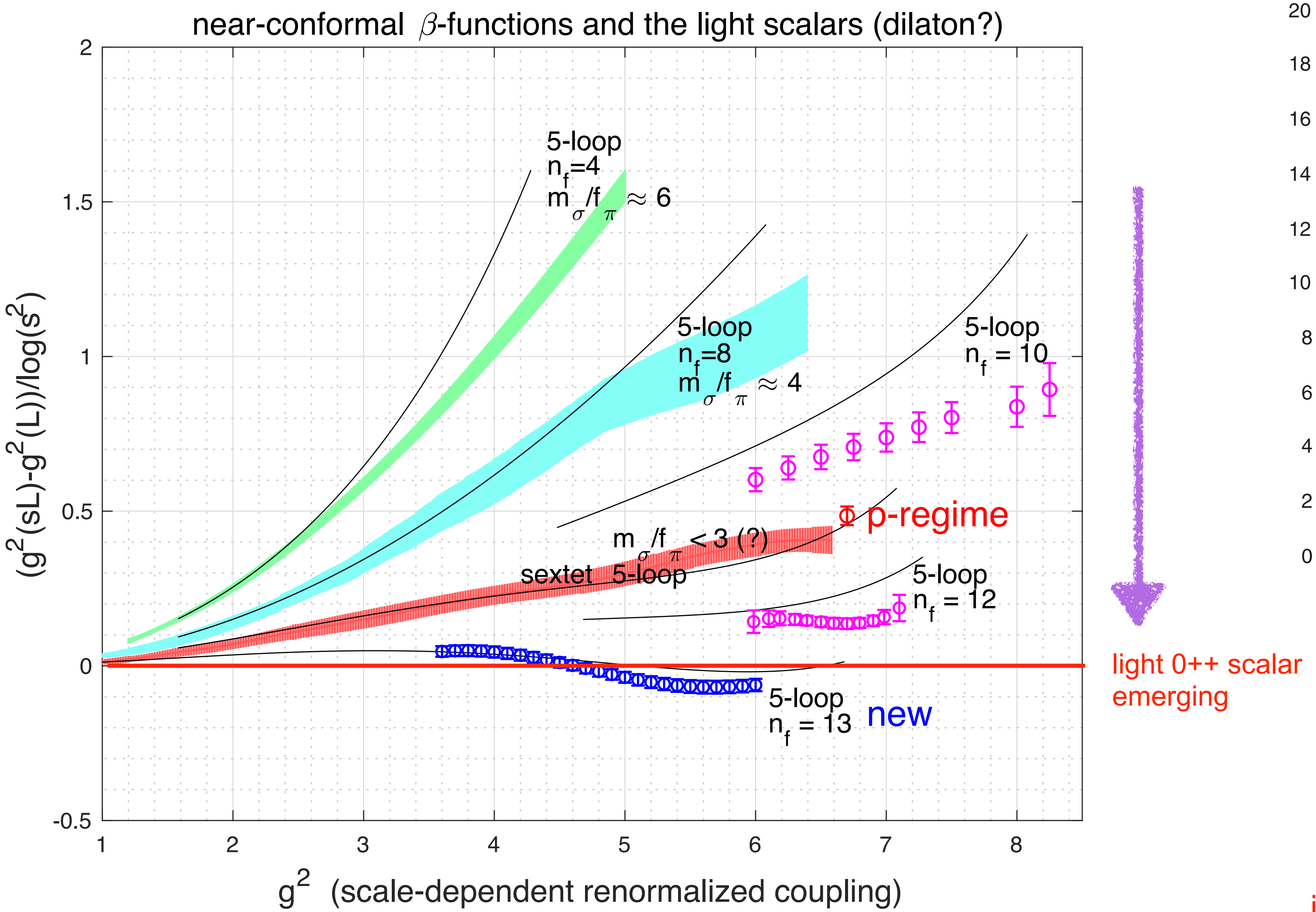
Need for new strategy of EFT in massless chiral limit

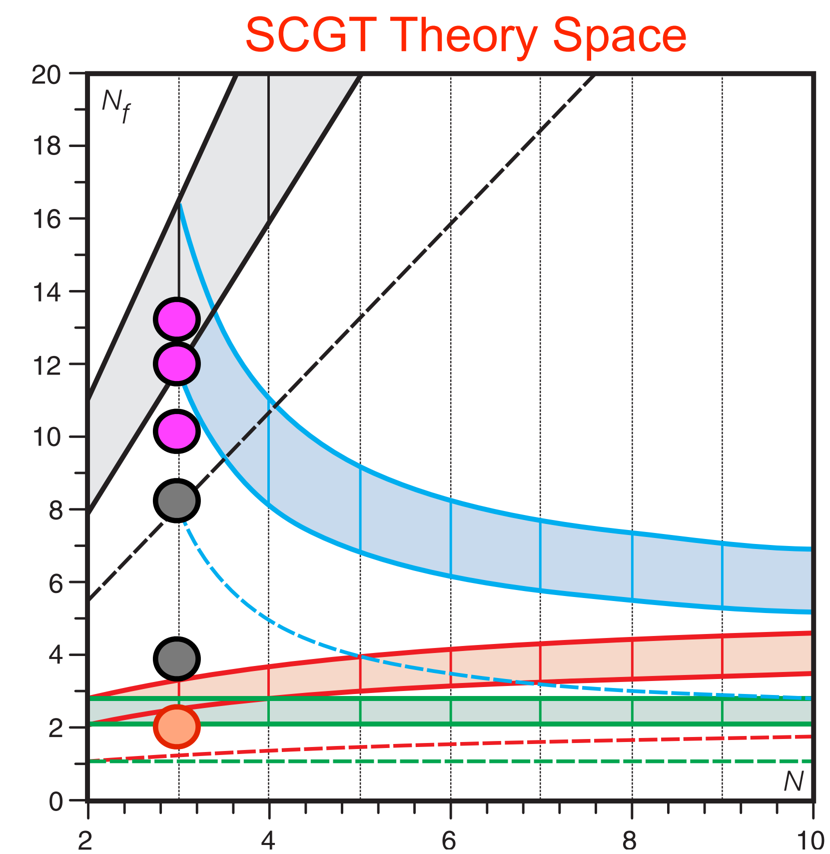
EFT in complex CFT walking paradigm ?

review of our recent results under these tantalizing questions →

*Tantalus, a king of ancient Phrygia in Greek mythology, made the mistake of gravely offending the gods. As a punishment, once dead the king was forced to stand in a pool of water, with fruit hanging just over his head. The water would recede every time the king tried to take a sip, and the fruit would lift away every time he reached to take a bite.

origin of the emergent light 0++ scalar (σ-particle or dilaton?)





to illustrate: sextet SU(3) color rep

one massless fermion doublet $\begin{bmatrix} u \\ d \end{bmatrix}$

χSB on Λ~TeV scale

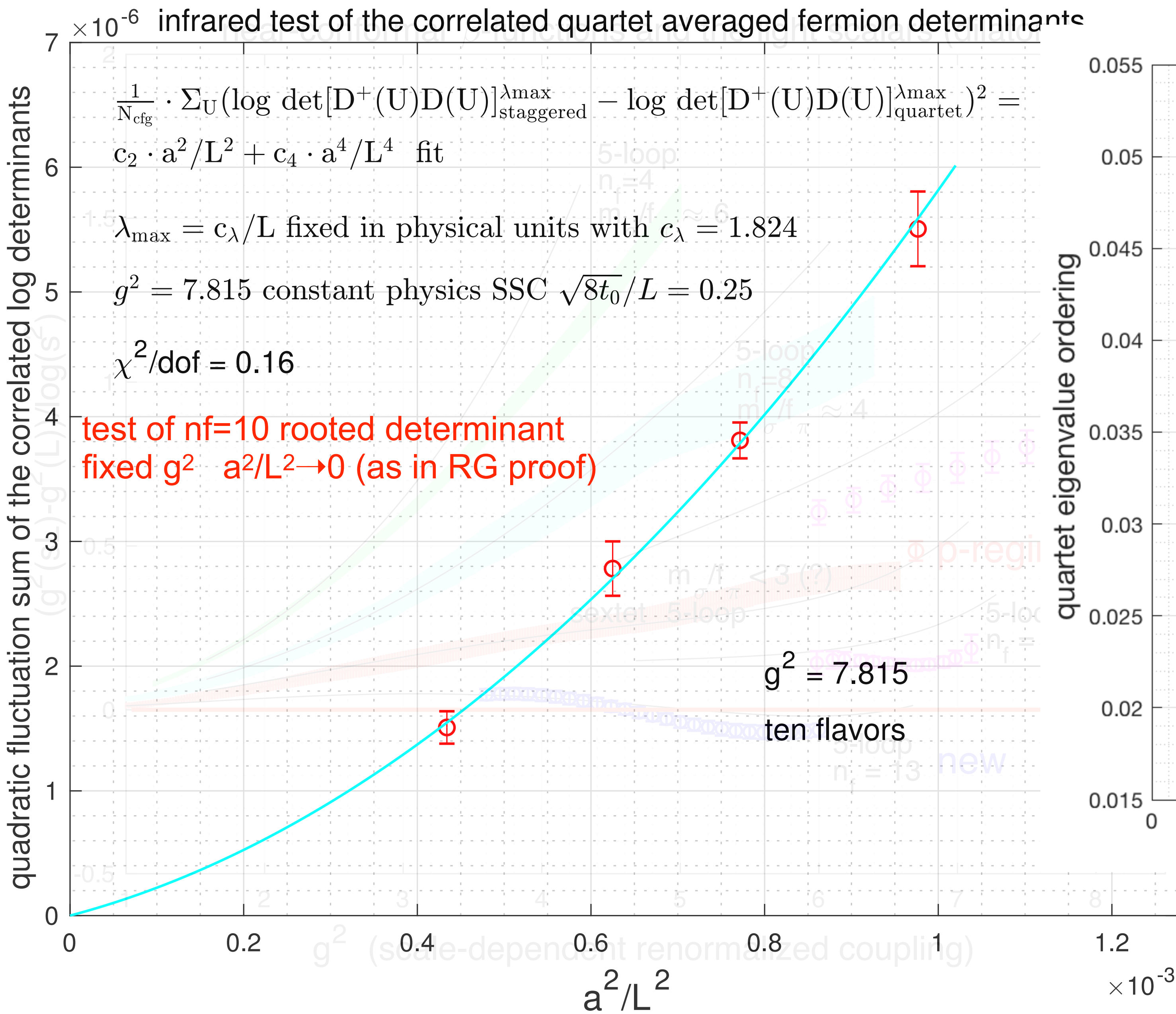
three Goldstone pions
become longitudinal
components of weak bosons

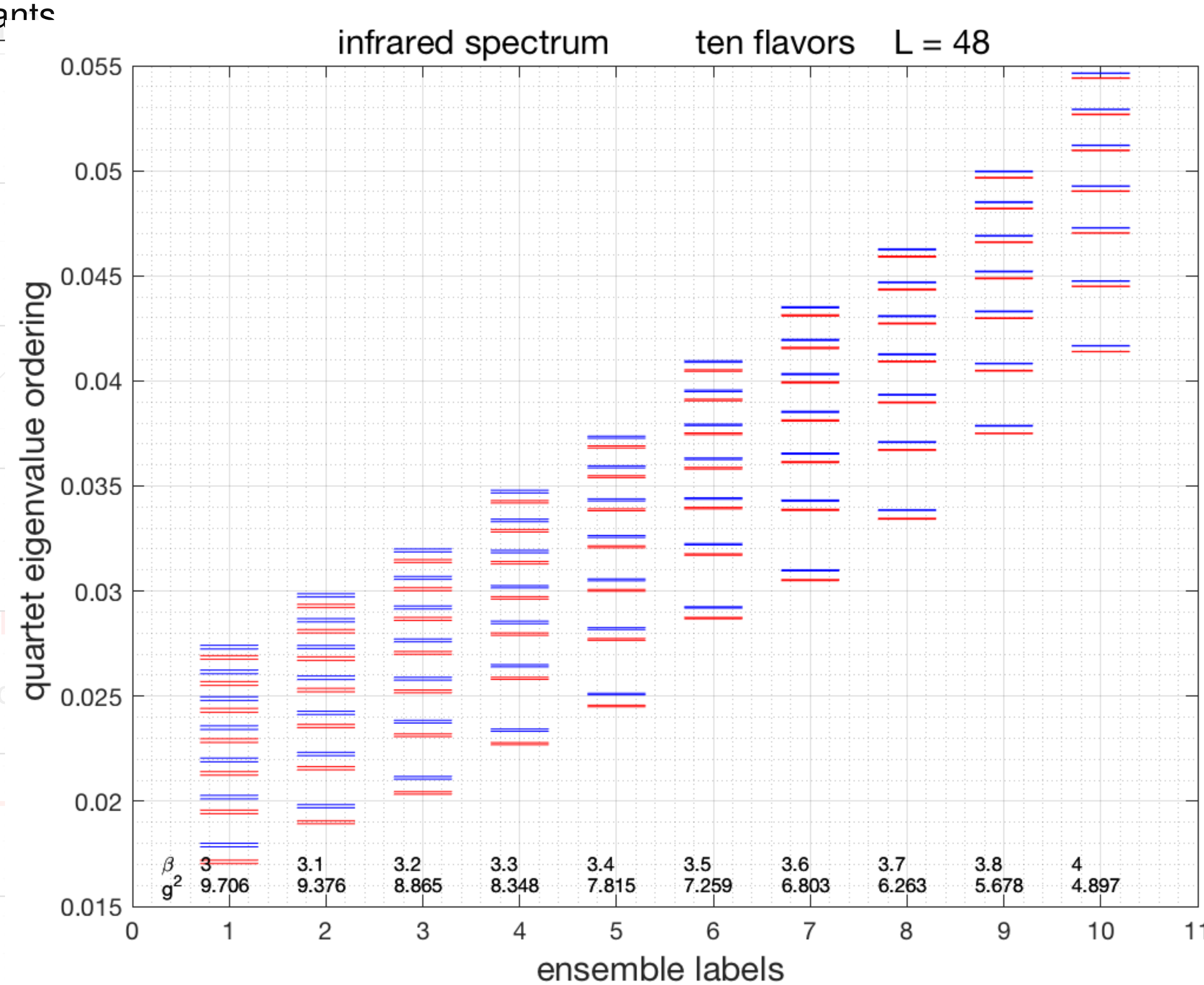
if Susskind and Weinberg only knew …

composite Higgs mechanism

- nf=10 β-function (no IRFP is found so far, contrary to earlier claims)
- nf=12 β-function controversy includes anomalous dimension
- nf=13 β-function LatHC reports nf=13 conformal IRFP (?)
- 5-loop β-function below CW: a pair of complex conjugate zeros (complex CFT help?)
- several models close to CW - focus in this talk on the sextet model

origin of the emergent light 0++ scalar (σ-particle or dilaton?)





- two zeros of 5-loop β-function collide between nf=12 and nf=13 and turn into complex conjugate pair of zeros
- sextet model closest to CW among near-conformal (except perhaps nf=12?) complex pair of CFT?

origin of the emergent light 0++ scalar (σ-particle, or dilaton, or …?)

simplest: light Higgs-like particle of linear σ-model in PT:

$SU(2)\otimes SU(2) \sim O(4)$ for sextet model

$$L = \frac{1}{2}\left(\partial_\mu \vec{\pi}\right)^2 + \frac{1}{2}\left(\partial_\mu \sigma\right)^2 - \frac{1}{2}\mu^2\left(\sigma^2 + \vec{\pi}^2\right) + \frac{1}{4} g\left(\sigma^2 + \vec{\pi}^2\right)^2 - \varepsilon\sigma$$

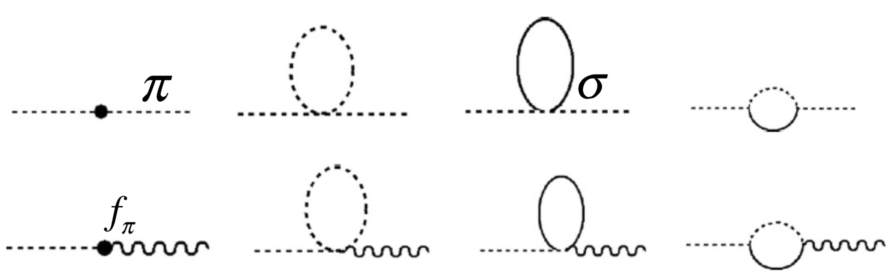
triviality analysis in $m_\sigma/f_\pi < 3$ range
$m_\pi = 0$ circa 1987-1988

$m_\sigma^2 \geq 3m_\pi^2$ tree level relation

$m_\sigma^2 \geq 2m_\pi^2$ with loop corrections

non-linear σ-model ➙ dilaton EFT:

p-regime data: 0++ is tracking the Goldstone pion with $m_\pi^2 \geq m_\sigma^2$, not like linear σ-model
New EFT is needed to extrapolate data to massless chiral limit



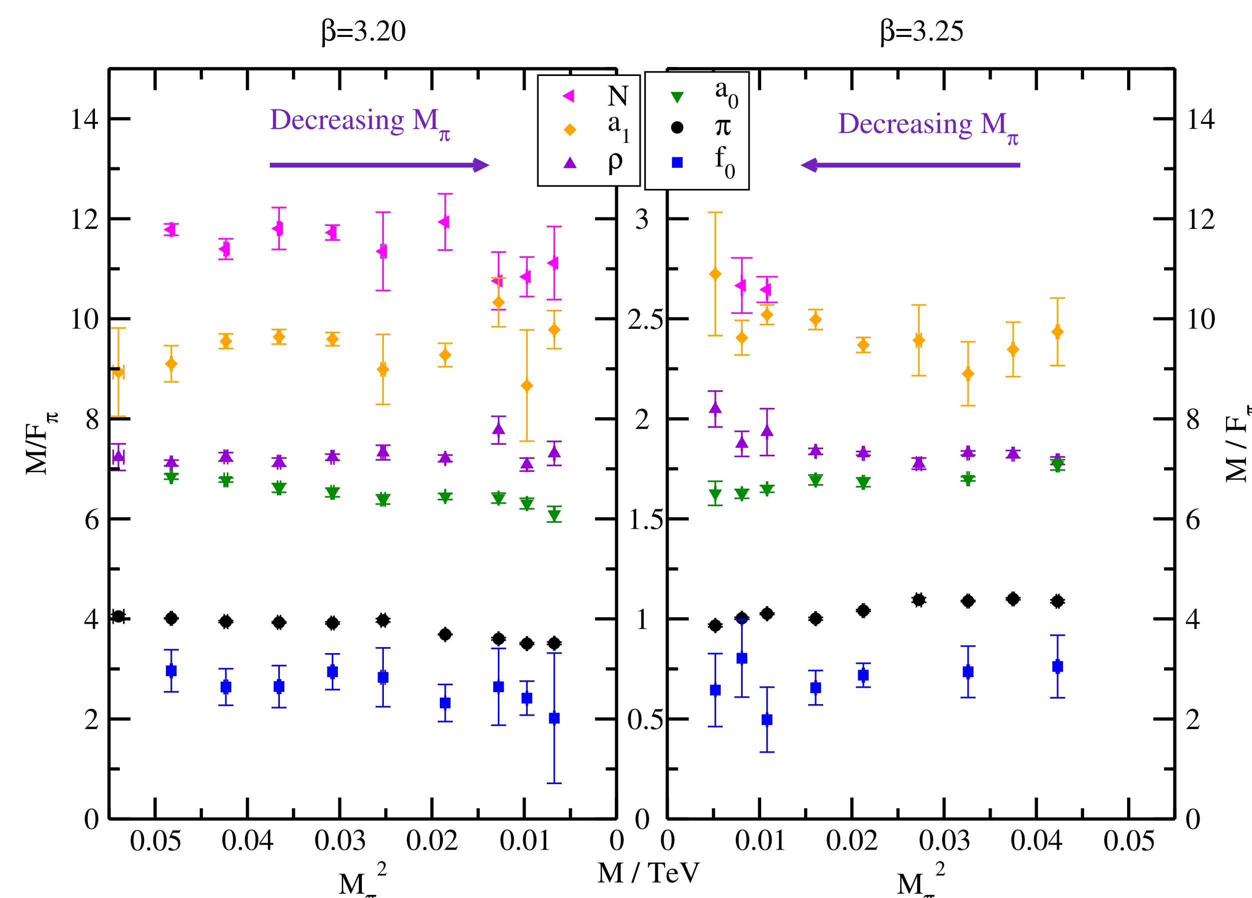
$$L = \frac{1}{2}\partial_\mu\sigma\,\partial_\mu\sigma - V(\sigma) + \frac{f_\pi^{\,2}}{4}\left(D_\mu\Sigma^\dagger D_\mu\Sigma\right)\cdot\left(1 + 2a\frac{\sigma}{f_\pi} + b\frac{\sigma^2}{f_\pi^{\,2}} + b_3\frac{\sigma^{\,3}}{f_\pi^{\,3}} + \ldots\right)$$

$\Sigma = e^{i\pi^a\tau^a/f_\pi}$ with τ^a Pauli matrices

$$V(\sigma) = \frac{1}{2}m_\sigma^2\cdot\sigma^{\,2} + d_3\left(\frac{m_\sigma^2}{2f_\pi}\right)\cdot\sigma^{\,3} + d_4\left(\frac{m_\sigma^2}{8f_\pi^{\,2}}\right)\cdot\sigma^{\,4} + \ldots$$

linear σ-model limit (SM): $a = b = d_3 = d_4 = 1$

dilaton EFT will require $a = b^2,\ b_3 = 0$ and special y(μ) in L



we analyze dilaton EFT first and test if non-linear σ-model limit exists without scale-dependent anomalous dimension y

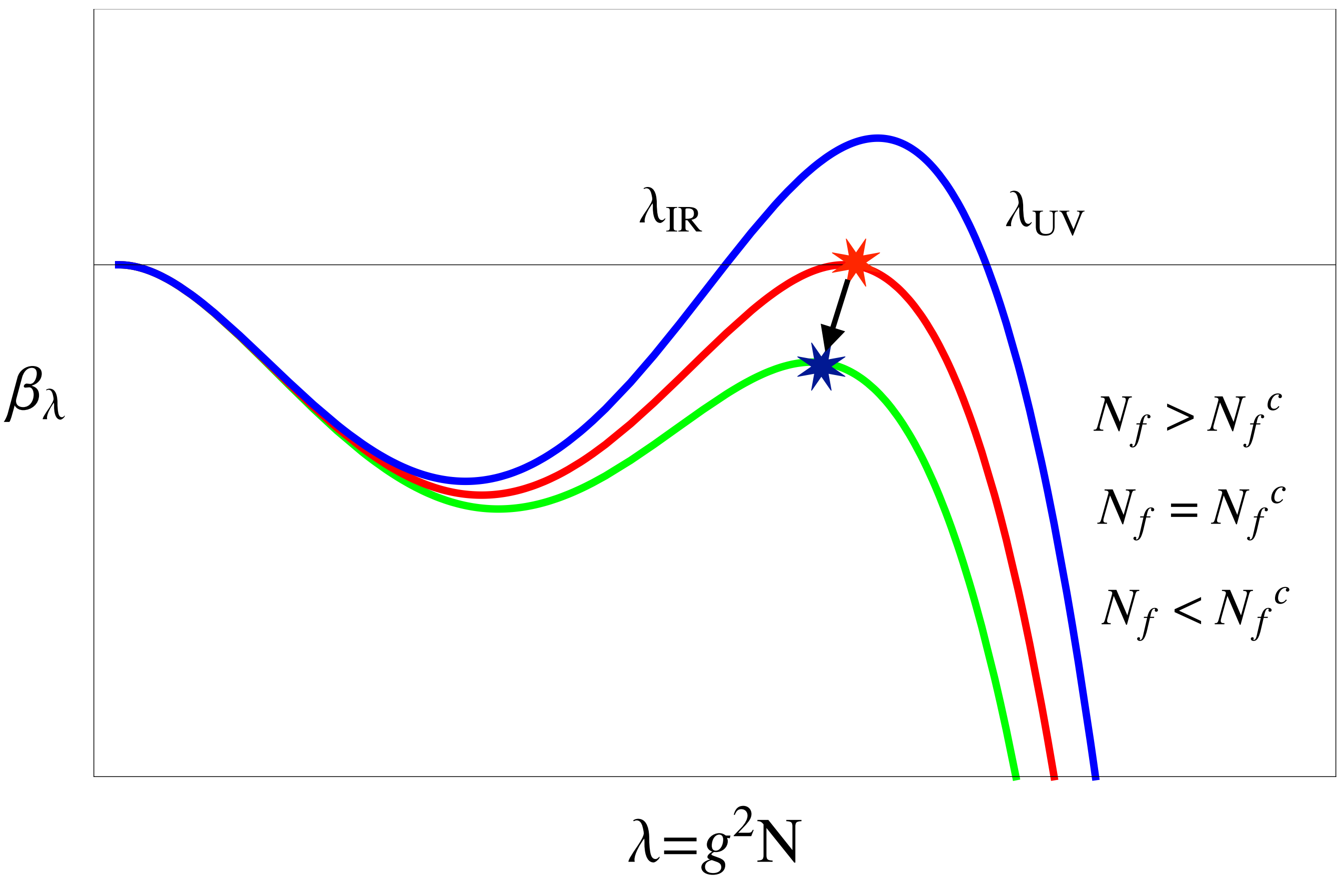
dilaton EFT with $\sigma(x)$ dilaton field and $\pi^a(x)$ Goldstone bosons

$$L = \frac{1}{2}\partial_\mu \chi \partial_\mu \chi - V_{\rm d}(\chi) + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 tr\left[\partial_\mu \Sigma^\dagger \partial_\mu \Sigma\right] - \frac{f_\pi^2 m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y tr\left(\Sigma + \Sigma^\dagger\right)$$

$y = 3 - \gamma$ where γ is the mass anomalous dimension

$\chi(x) = f_d\, e^{\sigma(x)/f_d}$ describes the dilaton field $\sigma(x)$

pion field $\Sigma = {\rm e}^{i\pi^a \tau^a / f_\pi}$ with τ^a Pauli matrices, tree level pion mass $m_\pi^2 = 2Bm$



- dilaton EFT has long history
- Golterman-Shamir expansion in $x = N_f/N$ variable
- Veneziano limit N ➛ ∞
- predicts walking around ✷ in p-regime (tree level) from expanding around CFT ✷
- based on scheme-dependent β-function ?
- flavor symmetry at ✷ is different from symmetry at ✷ ?

dilaton EFT with σ(x) dilaton field and π^a(x) Goldstone bosons

$$L=\frac{1}{2}\partial_\mu\chi\partial_\mu\chi-V_d(\chi)+\frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 tr\left[\partial_\mu\Sigma^\dagger\partial_\mu\Sigma\right]-\frac{f_\pi^2 m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y tr\left(\Sigma+\Sigma^\dagger\right)$$

Golterman-Shamir

Appelquist et al. notation

but we do our own IML analysis which is required for any conclusion!

$y=3-\gamma$ where γ is the mass anomalous dimension

$\chi(x)=f_d\, e^{\sigma(x)/f_d}$ describes the dilaton field $\sigma(x)$

pion field $\Sigma=e^{i\pi^a\tau^a/f_\pi}$ with τ^a Pauli matrices, tree level pion mass $m_\pi^2=2Bm$

$$V_\sigma=\frac{m_d^2}{2f_d^2}\left(\frac{\chi^2}{2}-\frac{f_d^2}{2}\right)^2$$ relevant deformation of IRFP theory

$$V_d=\frac{m_d^2}{16f_d^2}\chi^4\left(4\ln\frac{\chi}{f_d}-1\right)$$ nearly marginal deformation

Golterman-Shamir form

- f_π — Goldstone decay constant
- $m_\pi=2mB$ — Goldstone pions
- f_d — dilaton decay constant
- m_d — dilaton mass

$F_\pi,\, M_\pi,\, F_d,\, M_d$ with mass deformation

$$M_\pi^2\cdot F_\pi^{2-y}-2B_\pi\cdot f_\pi^{(2-y)}\cdot m=0$$ general V indep. scaling law

$$F_\pi^{(4-y)}\cdot(1-f_\pi^2/F_\pi^2)-2y\cdot n_f f_\pi^{(6-y)}B_\pi/m_d^2 f_d^2\cdot m=0 \quad \boxed{V'_\sigma(\chi=F_d)}$$

$$3F_\pi^2/M_\pi^2-f_\pi^2/M_\pi^2-2M_d^2/m_d^2\cdot f_\pi^2/M_\pi^2-y(y-1)n_f f_\pi^4/m_d^2 f_d^2=0 \quad \boxed{V''_\sigma(\chi=M_d)}$$

$$F_\pi^{(4-y)}\cdot\log(F_\pi/f_\pi)-y\cdot n_f f_\pi^{(6-y)}B_\pi\cdot m/m_d^2 f_d^2=0 \quad \boxed{V'_d(\chi=F_d)}$$

$$(F_\pi^2/M_\pi^2)\cdot(3\log(F_\pi/f_\pi)+1)-(M_d^2/m_d^2)\cdot(f_\pi^2/M_\pi^2)-y(y-1)n_f f_\pi^4/2m_d^2 f_d^2=0 \quad \boxed{V''_d(\chi=M_d)}$$

M_π, F_π, M_d input data at each m

$f_\pi,\, B,\, f_d,\, m_d,\, y$ fitted for all m

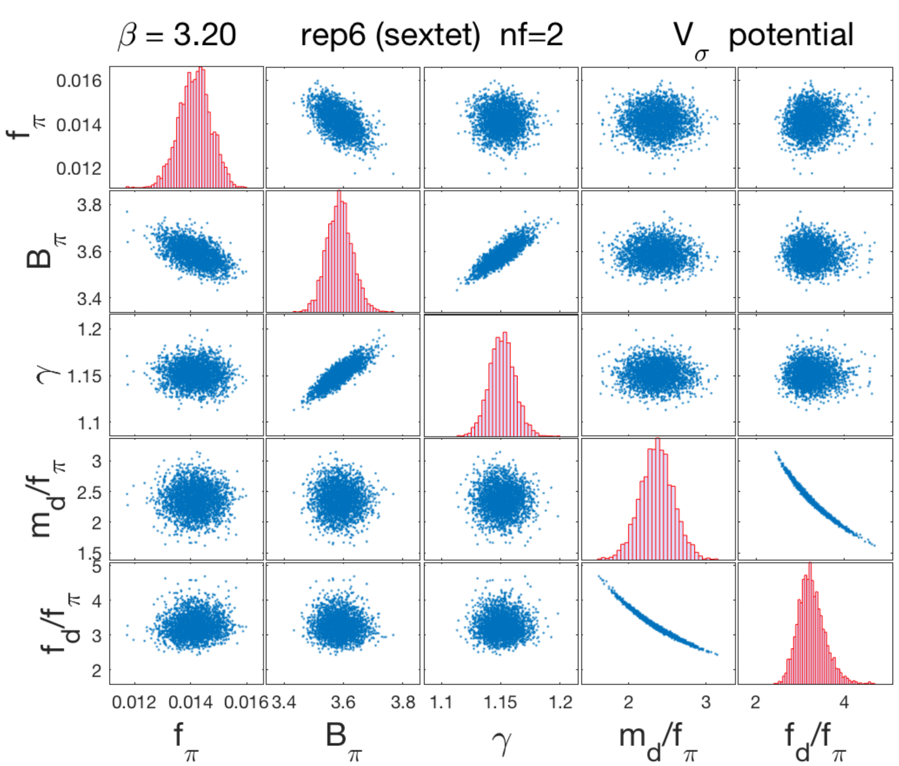
IML: Implicit Maximum Likelihood test

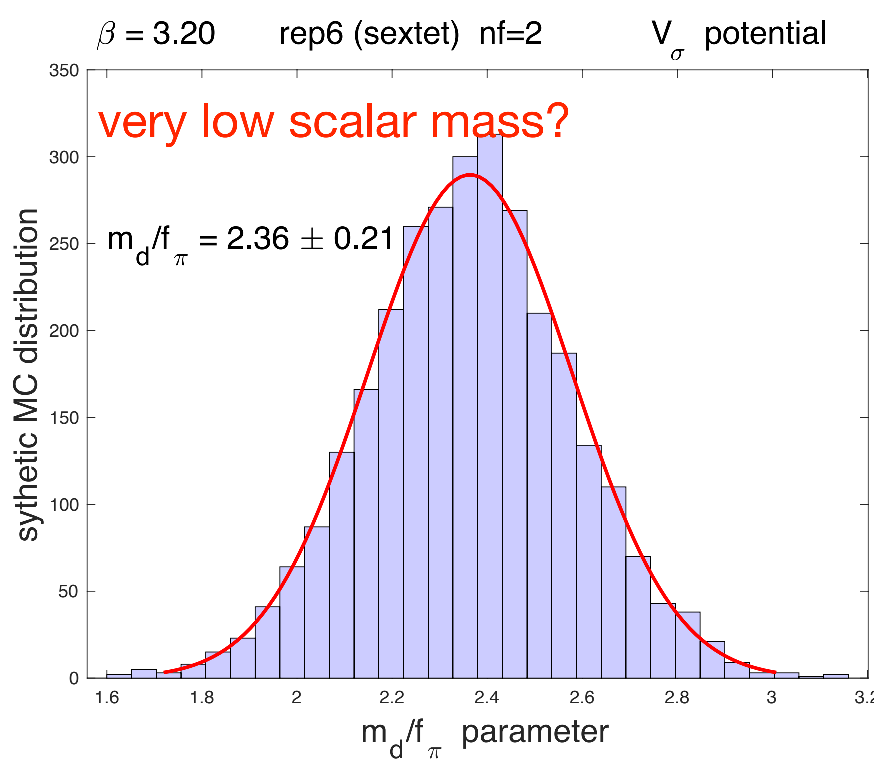
IML is very different from ML fitting

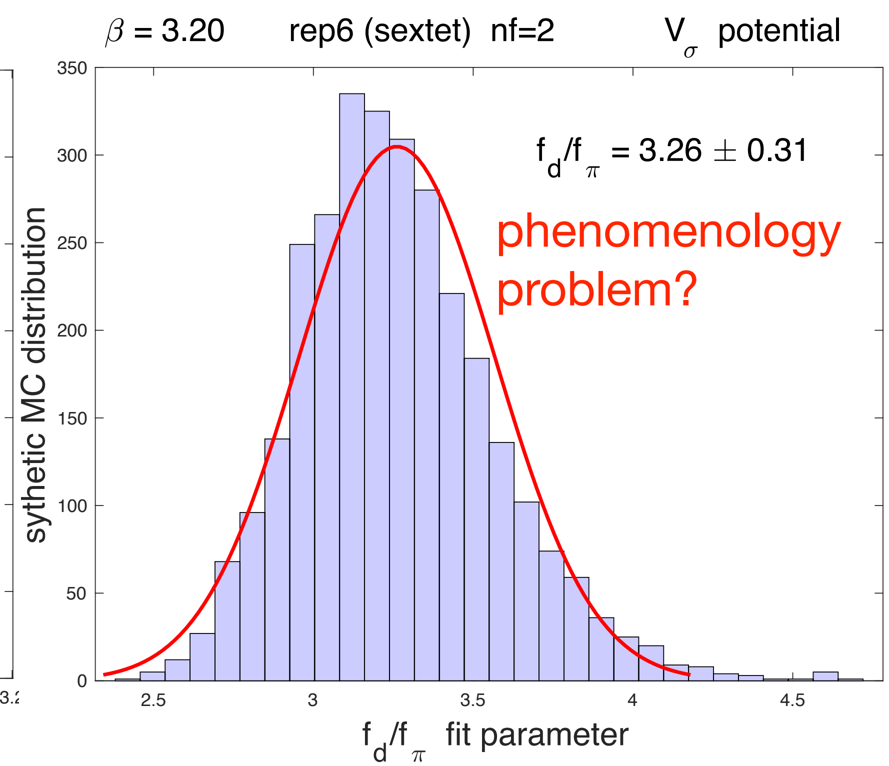
Perfect fits for V_σ!

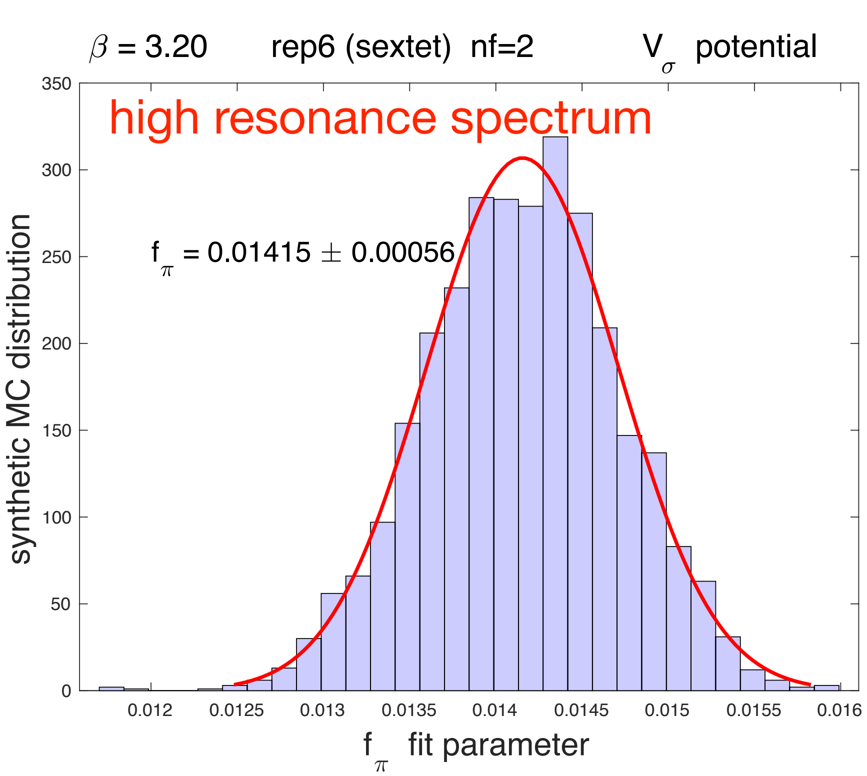
V_d fails!

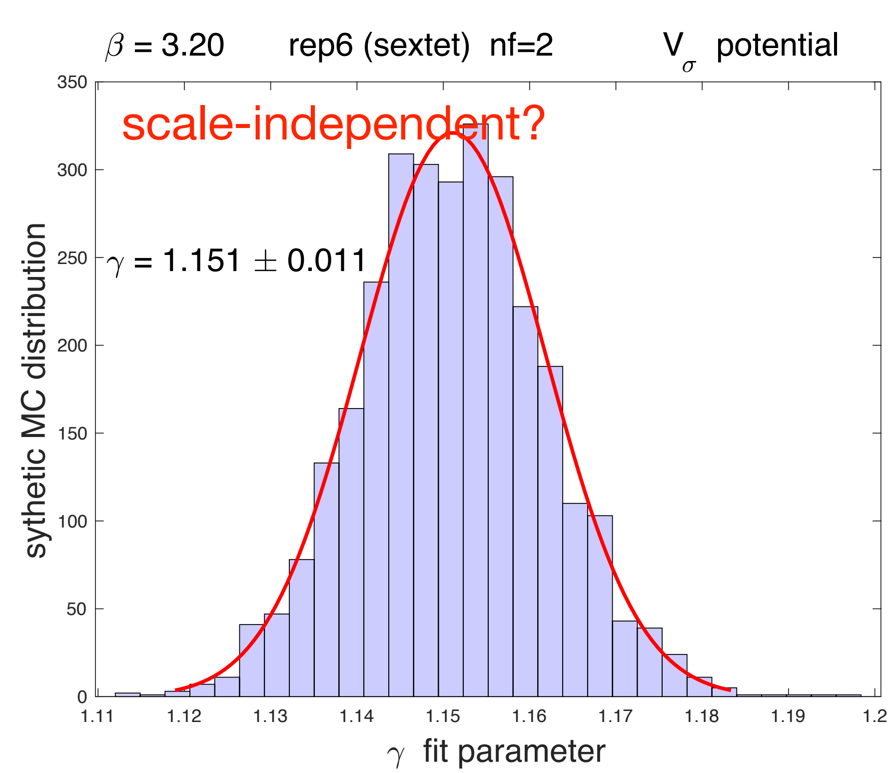
dilaton EFT with $\sigma(x)$ dilaton field and $\pi^a(x)$ Goldstone bosons (sextet model)











excellent sextet fits for V_σ!

also works at $N_f = 8$

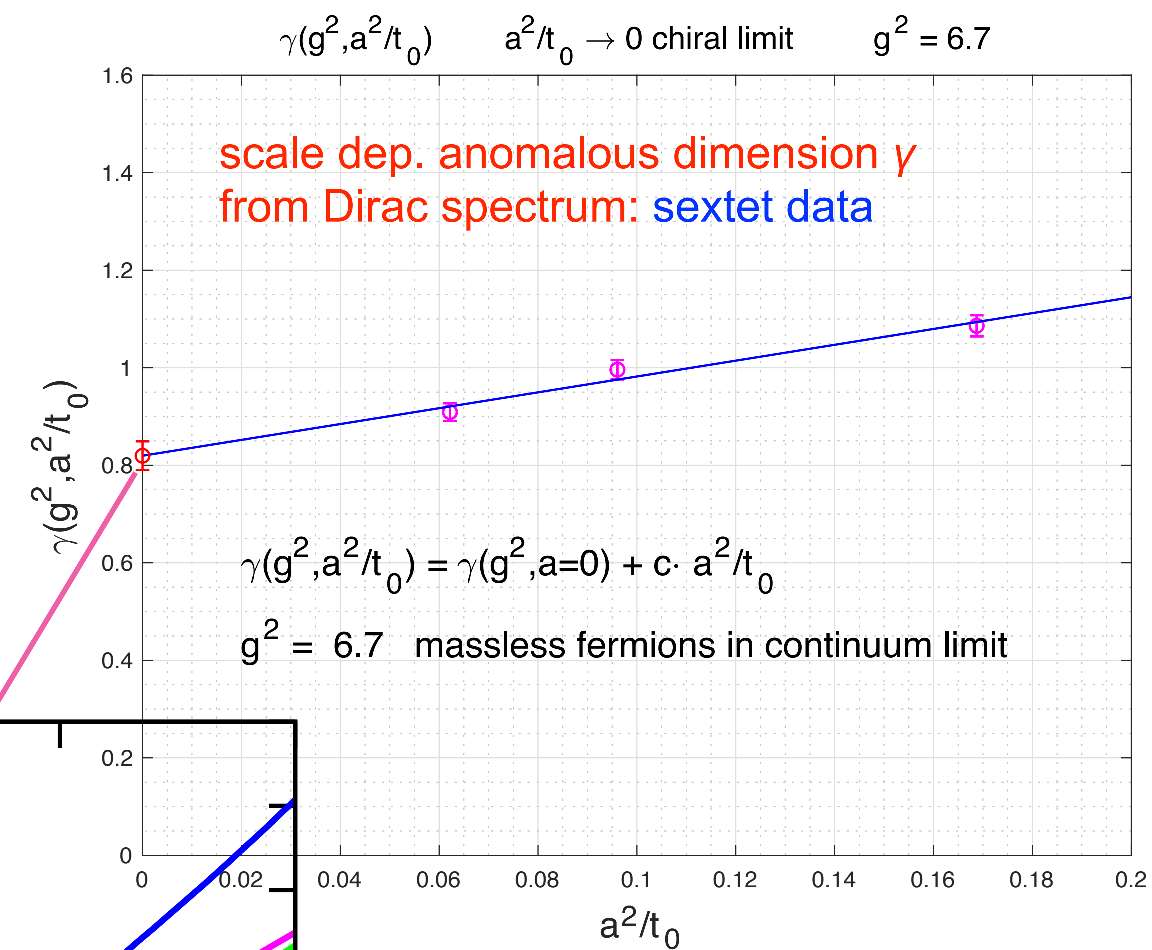
problem with Golterman-Shamir form of V_d

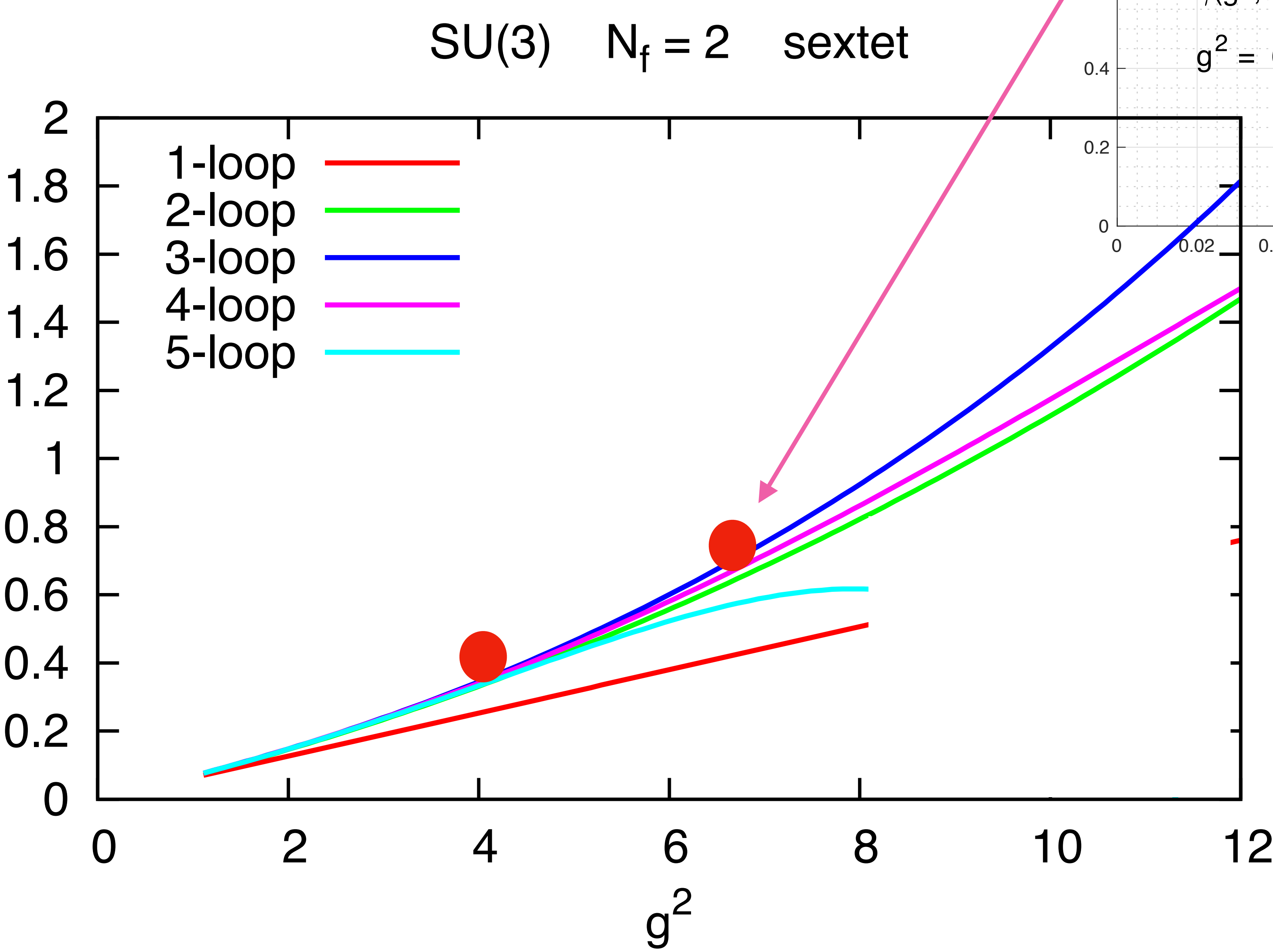
not consistent with sextet data, or $N_f = 8$ data!

and what is the meaning of the fixed $\boldsymbol{\gamma}$?

dilaton EFT with $\sigma(x)$ dilaton field and $\pi^a(x)$ Goldstone bosons (sextet model)

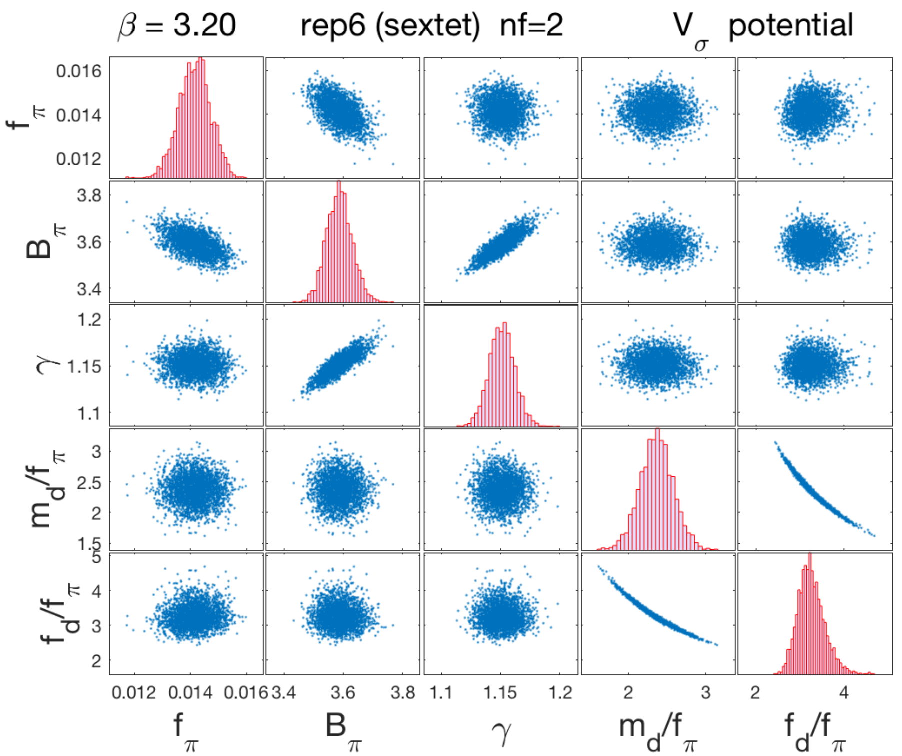
- Chebyshev expansion of mode number
- infinite volume limit from FSS
- m -> 0 chiral limit at fixed a
- a -> 0 continuum limit

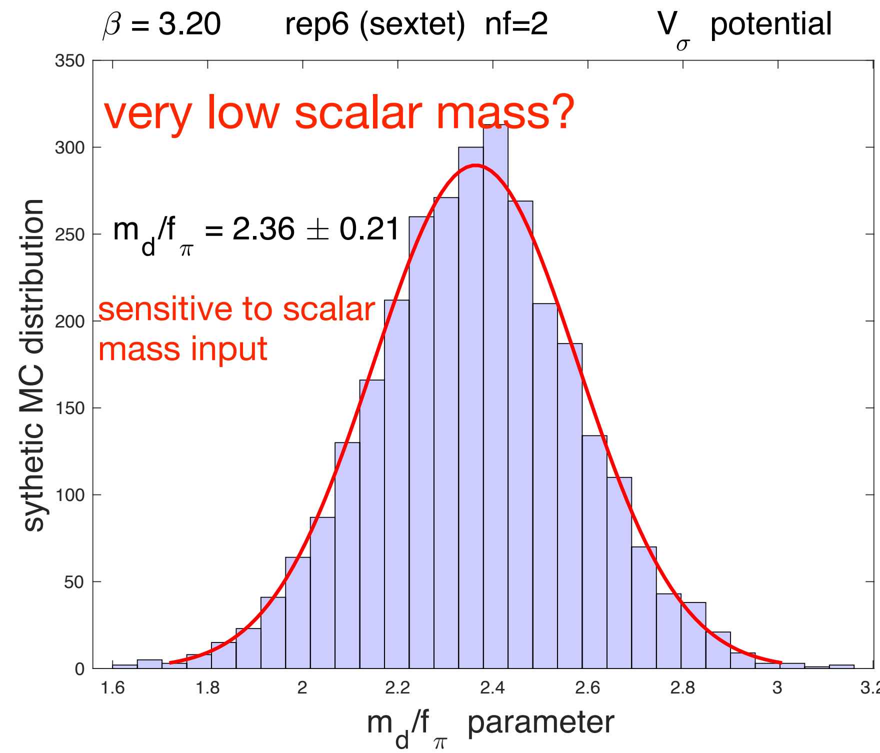


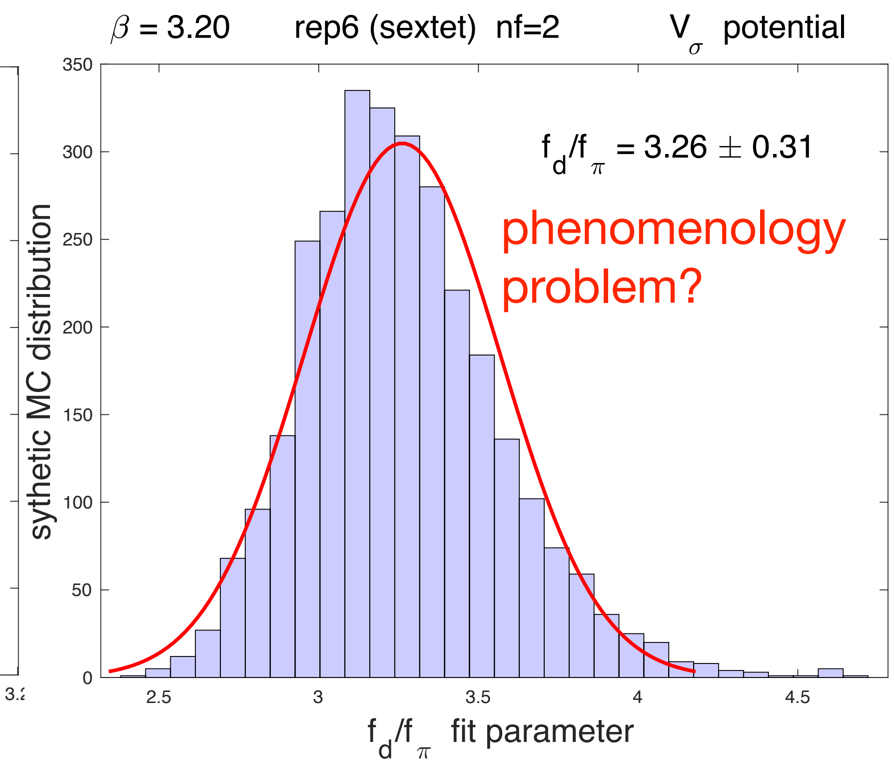


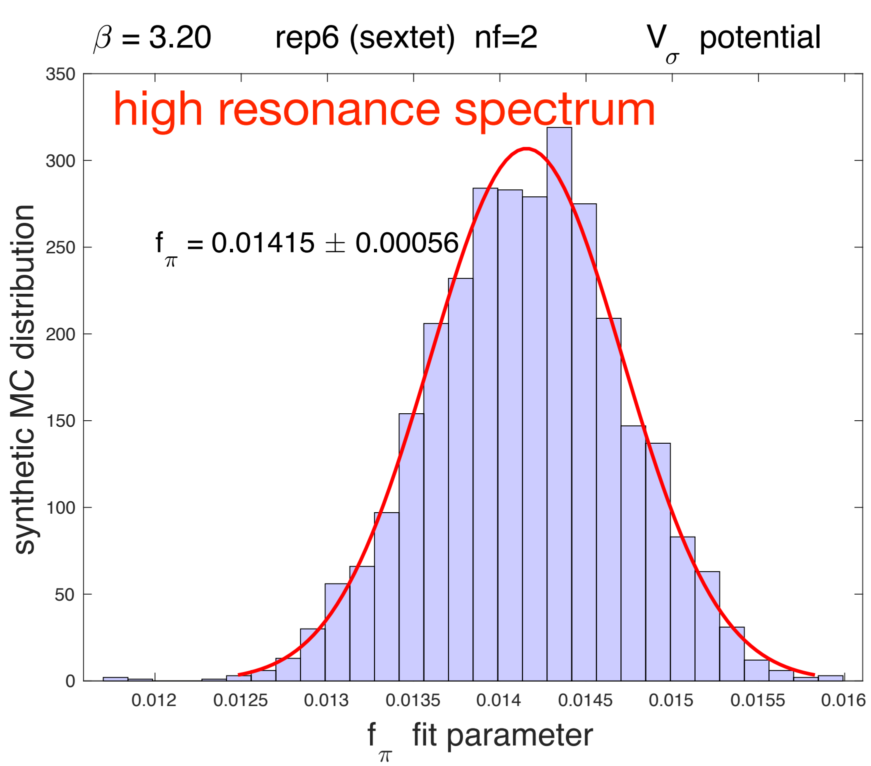
- needs model which predicts the drifting scale-dependence of the anomalous dimension $y=3-\gamma$
- in Lagrangian (and in fitting) requires theory for walking !

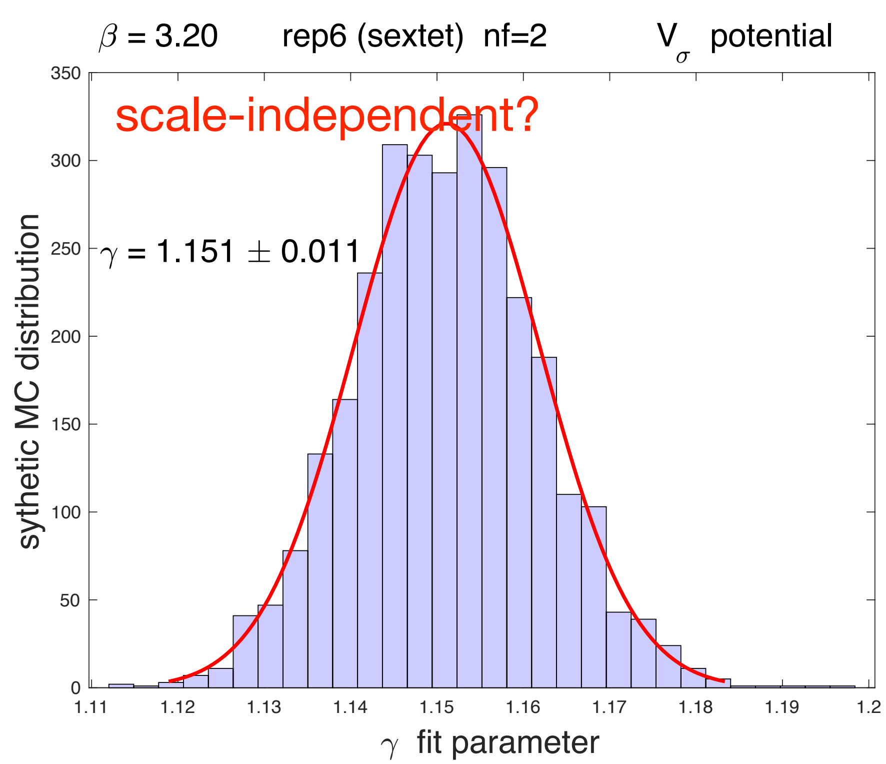
dilaton EFT with $\sigma(x)$ dilaton field and $\pi^a(x)$ Goldstone bosons (sextet model)











- the dilaton potential of Shamir-Golterman as tree level theory expanding around IRFP is not working for sextet data
- consistent LatHC analysis for two lattice spacing, with third under construction
- similar conclusion at nf=8 although single lattice spacing
- data is far above pion dominated chiral regime

to reach the chiral regime requires two orders of magnitude drop in fermion mass:
switch from p-regime to epsilon regime and related RMT

epsilon regime and RMT

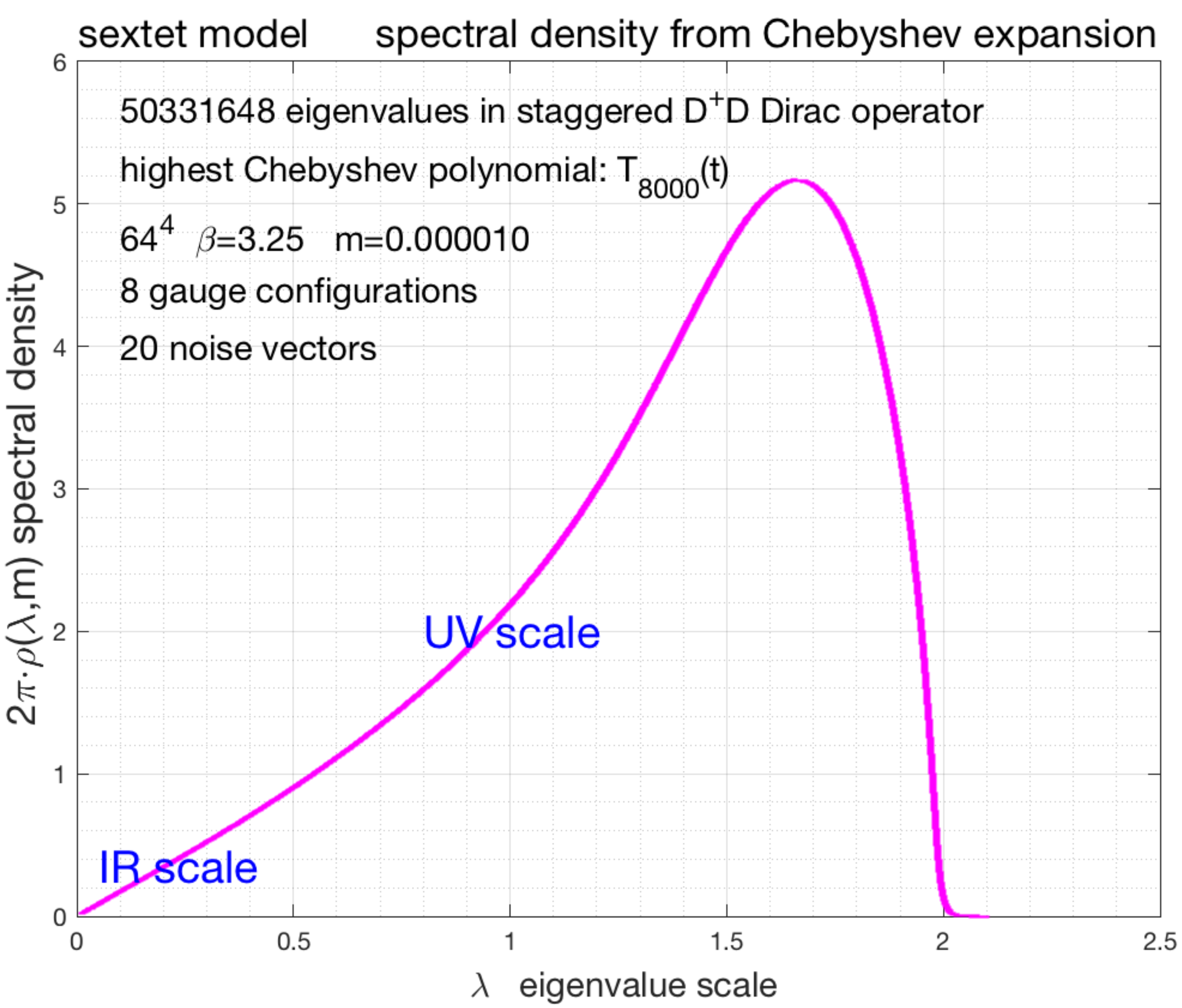
$$\mathcal{L}_\varepsilon = \frac{1}{2}\partial_\mu\chi\partial_\mu\chi - V_d(\chi) + \frac{m_\pi^2 f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y \mathrm{tr}\left[\Sigma_0+\Sigma_0^\dagger\right]$$

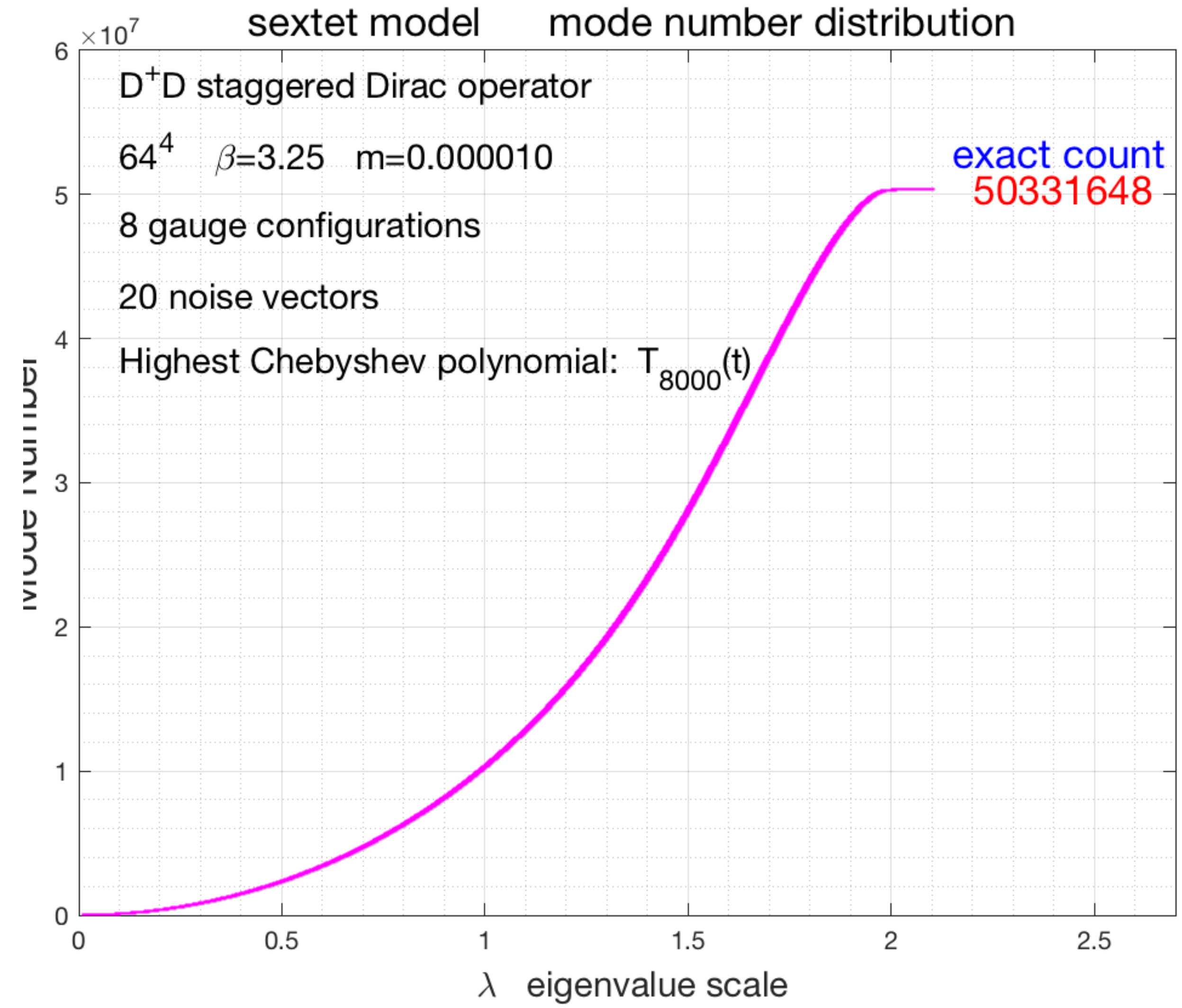
epsilon regime with very small fermion mass deformation

$$\mathcal{L}_\delta = \frac{1}{2}\partial_\mu\chi\partial_\mu\chi - V(\chi) + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 \mathrm{tr}\left[\partial_t\Sigma_0\,\partial_t\Sigma_0^\dagger\right]$$

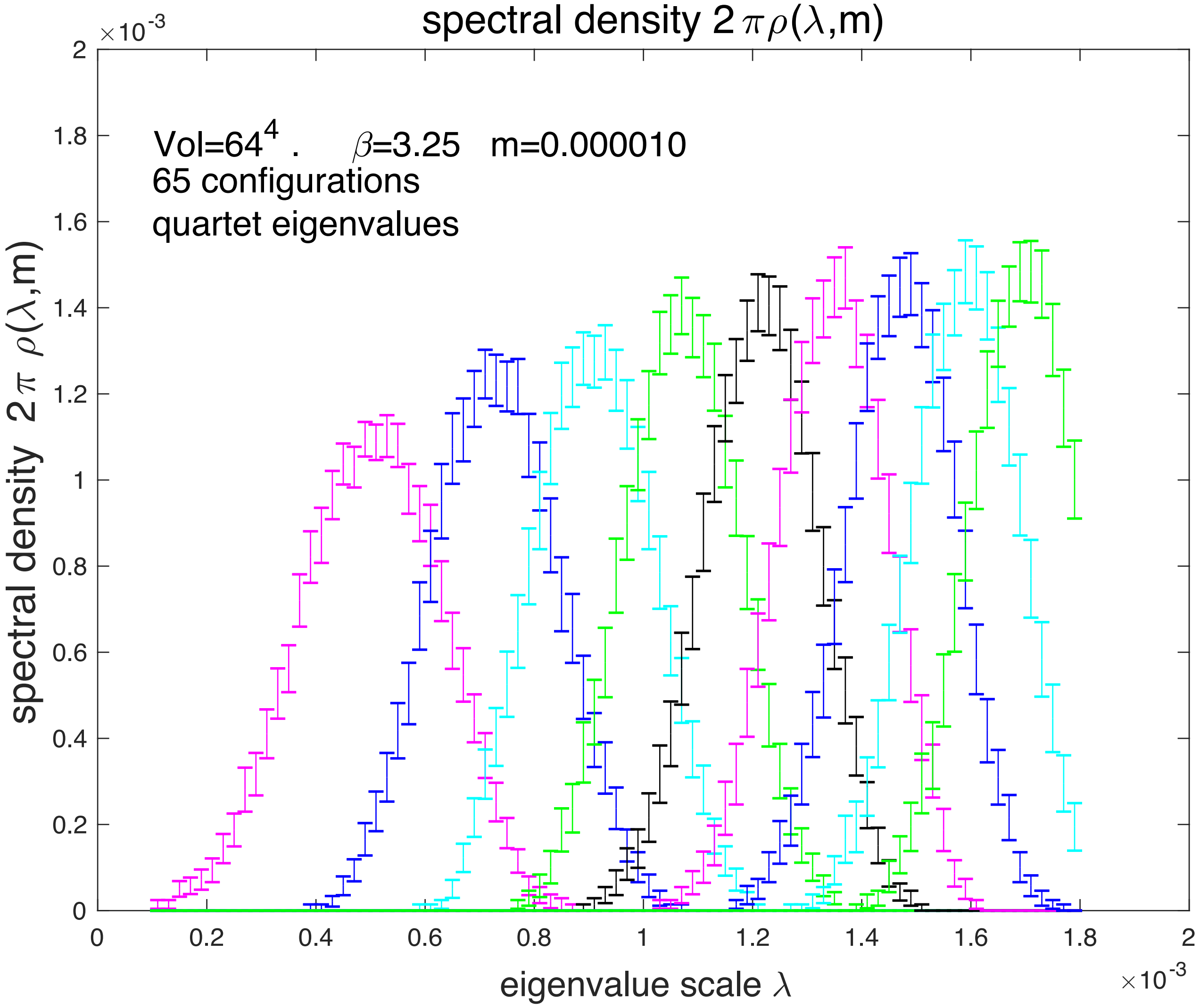
delta regime m=0
very small fermion mass deformation can be added

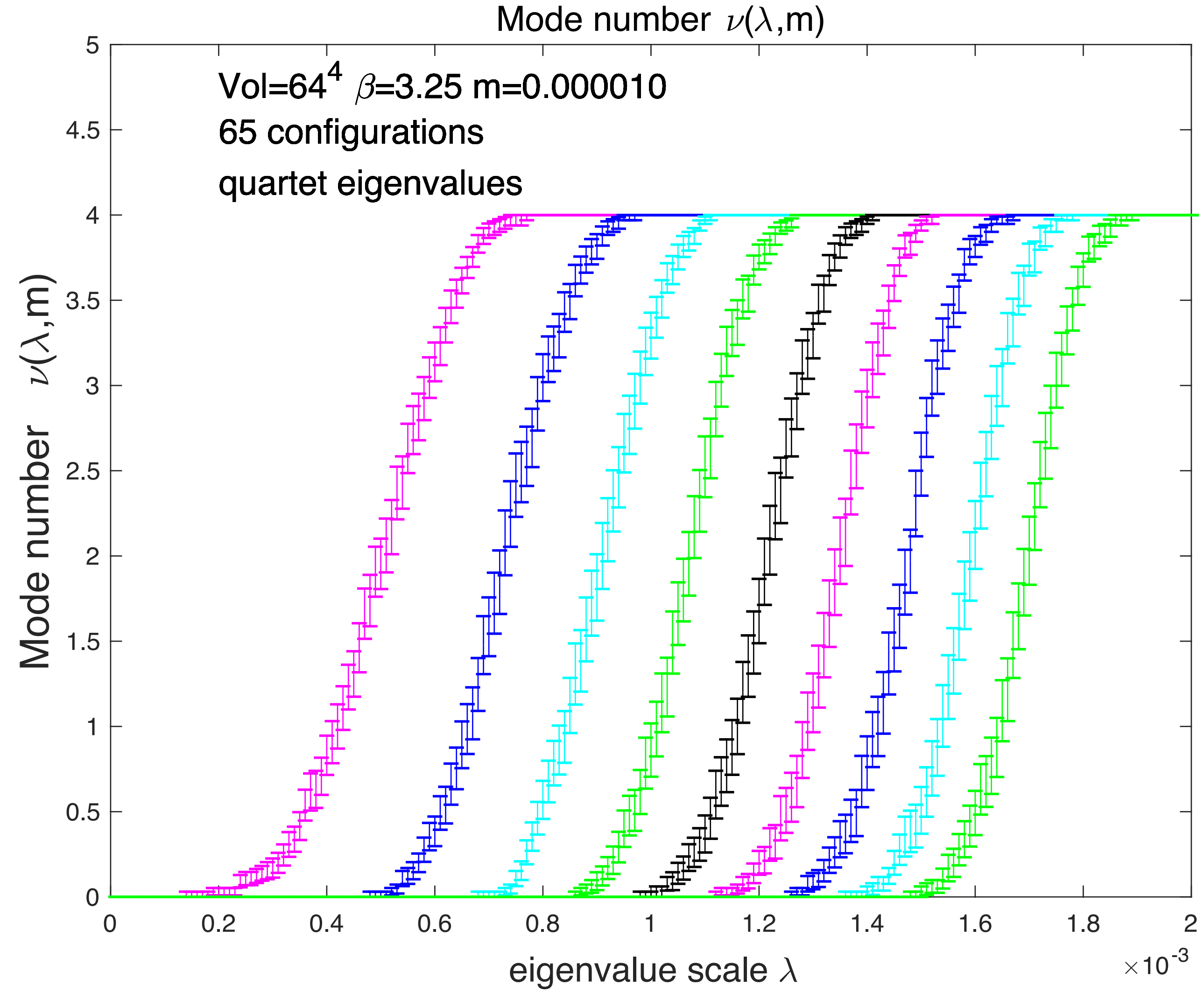
new ensembles at equivalent p-regime pion mass Mpi ~ 100 and volume size 64^4





epsilon regime and RMT





successful testing

ongoing analysis (preliminary results not shown)

walking and complex CFT

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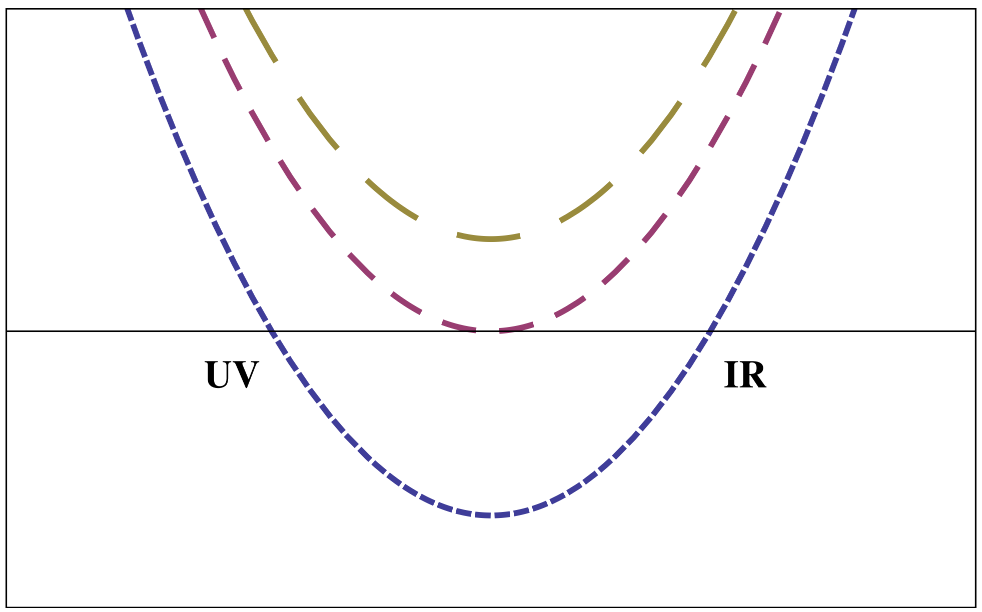
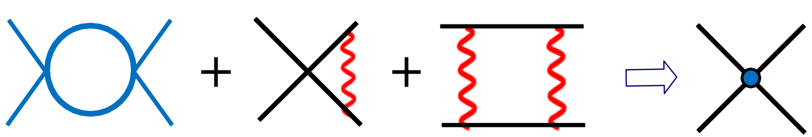




$$\mathcal{L}_{\text{CFT}} + \frac{f}{2}\mathcal{O}^{\dagger}_{ij}\mathcal{O}^{ij}$$ four-fermion deformation

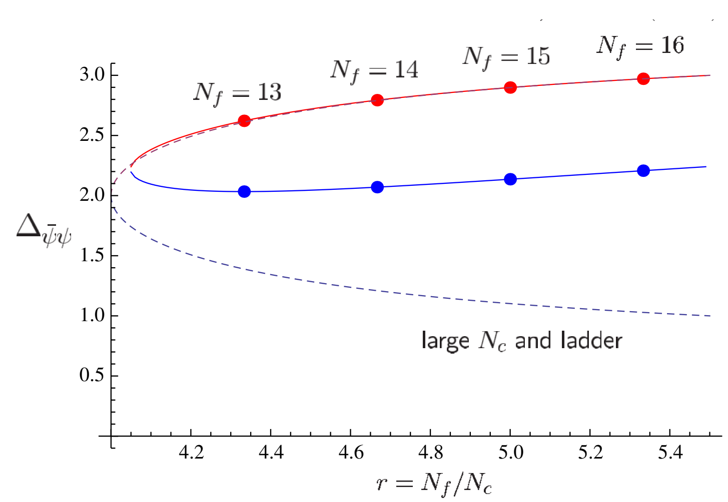
$$\Lambda\frac{d\bar{f}}{d\Lambda} = v\bar{f}^2 + (2\Delta - d)\bar{f} + a$$

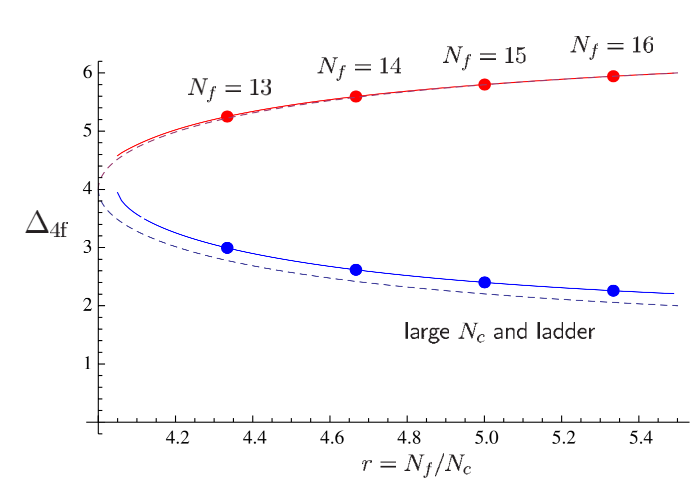
$$\beta'_{\bar{f}}|_{\pm} = \pm 2\sqrt{D}$$

$$\Delta_{\pm} = \frac{d}{2} \pm \sqrt{D}$$

Luca Vecchi 2010 talks about complex CFT
built on Gies et al., Terao et al., Kaplan et al.…

walking and complex CFT new paradigm?
flavor symmetry group is the same for walker and the CFT! paradigm change





walking and complex CFT

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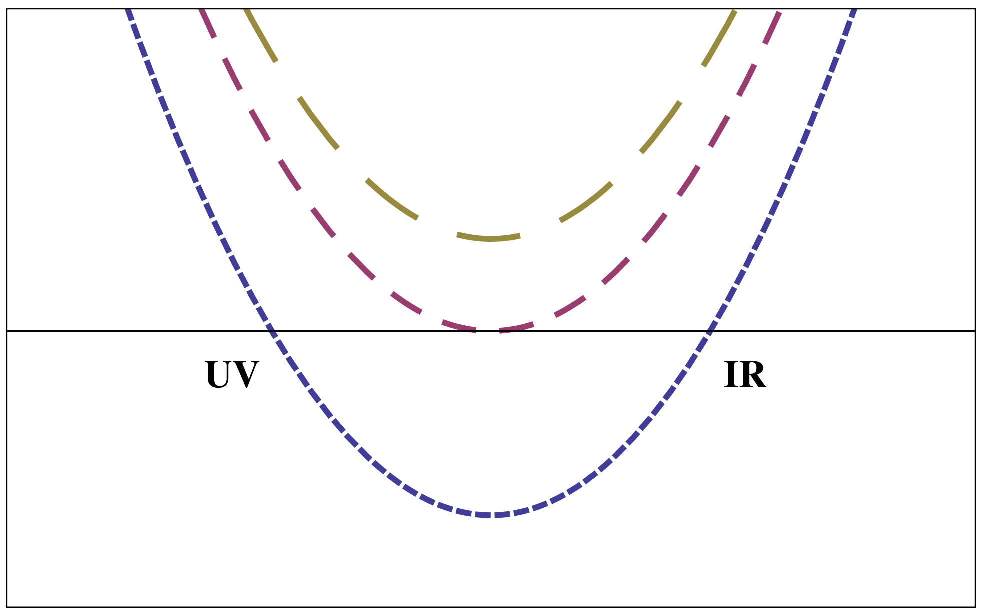

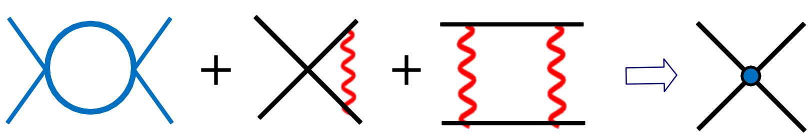

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we started work earlier on the realization of walking based on this idea

to distinguish near-conformal and conformal finite volume correlators (drifting scaling exponents distinguished from fixed conformal exponents).
we ran into difficulty not knowing the conformal exponents of the complex theory.

Gorbenko, Rychkov, Zan turned to a two-dimensional example (Potts model) for very detailed realization of working without apparently knowing about Vecchi

We ask now: can we realize the lattice Potts conformal field theories for continuous Q (flavor)

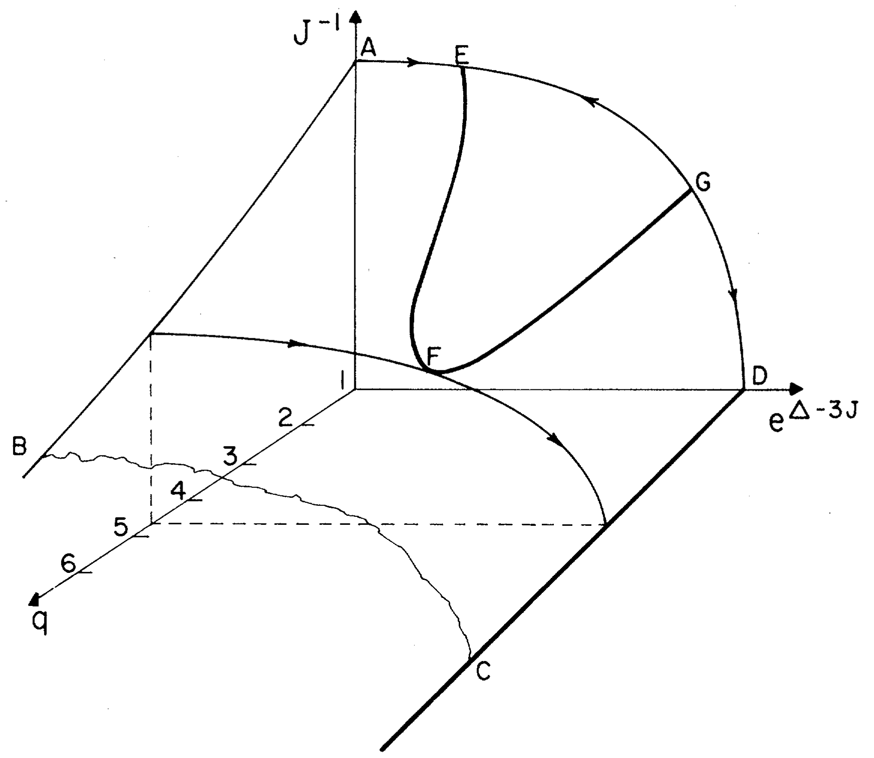
walking and complex CFT

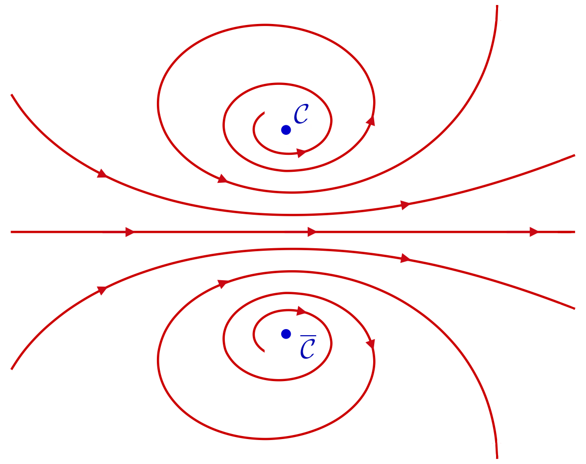
Potts model Q potts spin ~ flavor
described by CFT

Q=2 - 4 pair of real CFT with pair of zeros of the beta function works for continuous Q in cluster rep

Q > 4 complex CFT, like Q=5, 6, 7 …

What was identified before as Q=5 Potts is near-conformal and walking, supported by the complex IRFP pair, not critical in very well defined sense





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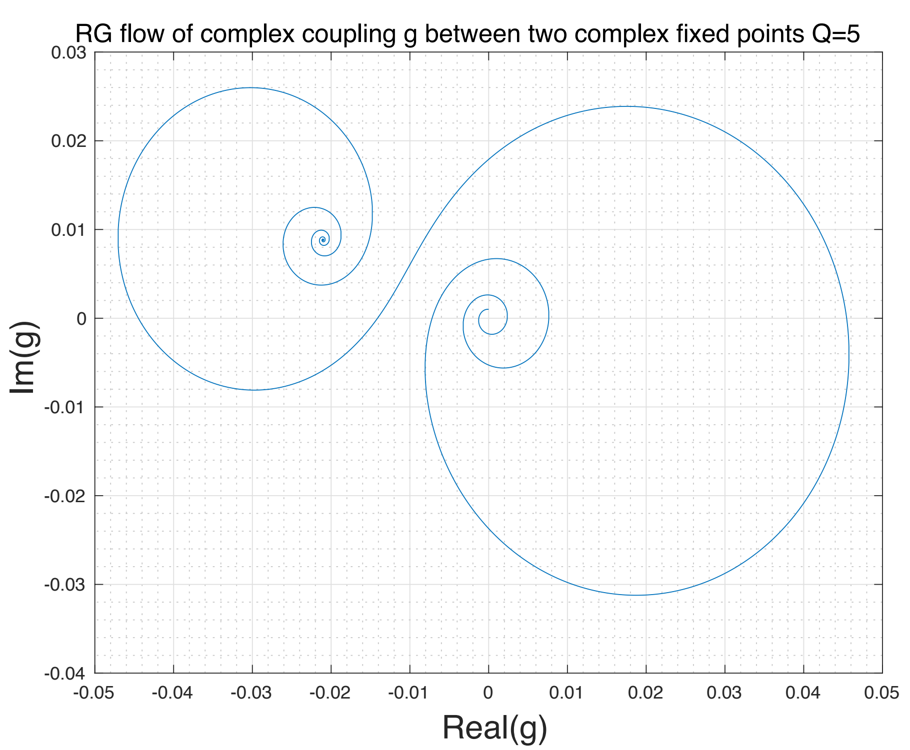

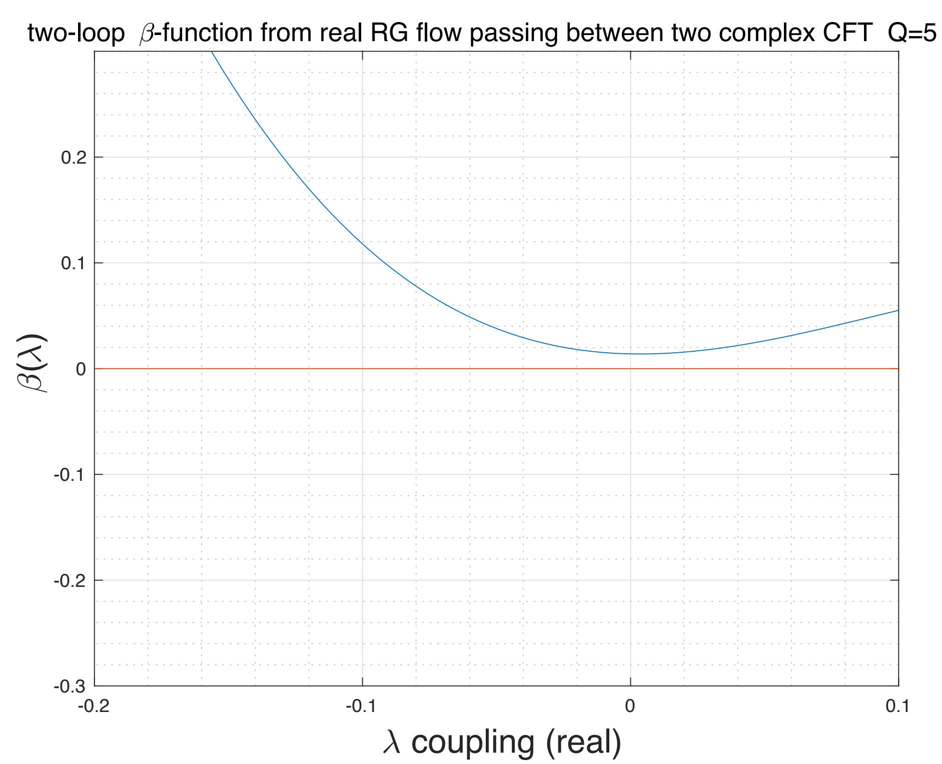

walking and complex CFT

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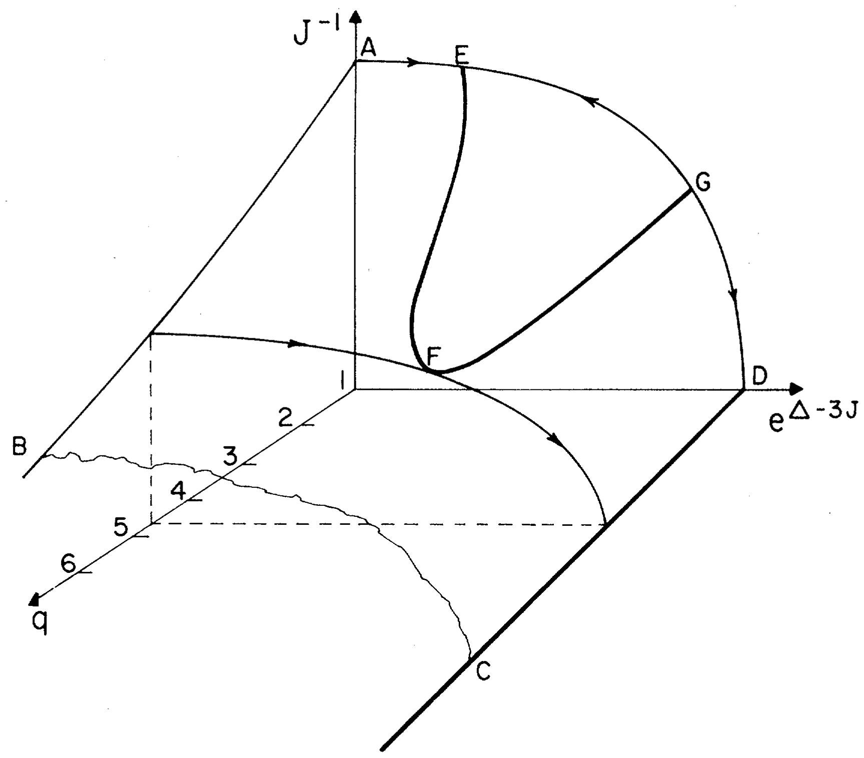
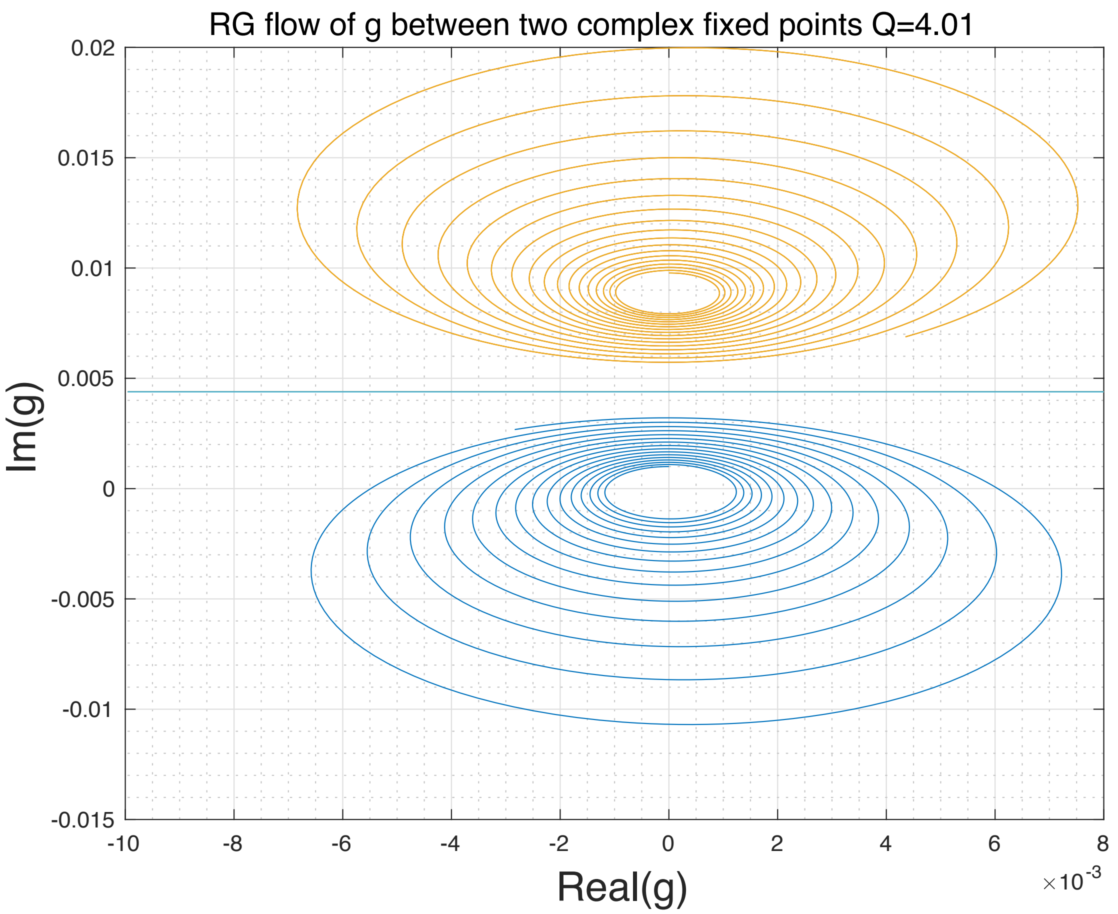
We use efficient Potts CUDA code for integer Q
Swendsen-Wang ~1 ns/spin

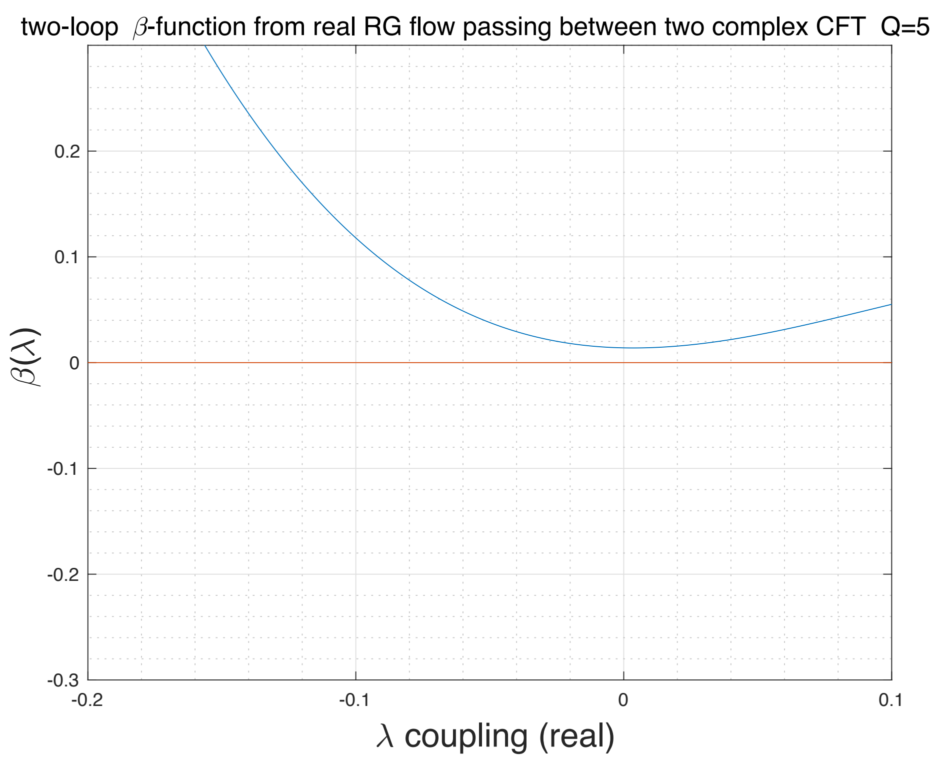
CPU cluster code for arbitrary Q

Tensor method and DMRG on Potts transfer matrix

working on lattice realization of the Potts complex CFT paradigm







Thank you