Constraint Effective Potentials in Gauge-Higgs Unification models

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Motivation

Gauge-Higgs Unification

- origin of the potential responsible for the Brout-Englert-Higgs mechanism still unknown
- in Gauge-Higgs Unification (GHU) models the Higgs field is associated with some extra-dimensional components of a gauge field [Manton, 1979]
- one extra dimension: Higgs potential is zero at tree level and generated only through quantum effects
- 1-loop: Higgs mass is finite, fermions needed for SB through Hosotani mechanism [Hosotani, 1983].
- what happens non-perturbatively? $\longrightarrow$ lattice study.
- measure constraint effective potential [Kuti, Shen, 1988]
- also relevant for effective Polyakov loop actions

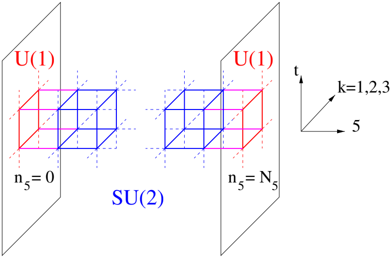
5D Orbifold Model Lattice Formulation

Wilson Gauge action for bulk gauge group $\mathrm{SU}(2)$:

$$S_W^{orb} = \frac{\beta_4}{2} \sum_{\mu,\nu} w \cdot \mathrm{tr}\,\{1 - U_{\mu\nu}\} + \frac{\beta_5}{2} \sum_{\mu} \mathrm{tr}\,\{1 - U_{\mu 5}\}$$

Boundary links satisfy $gUg^{-1} = U$ with $g = -i\sigma^3$

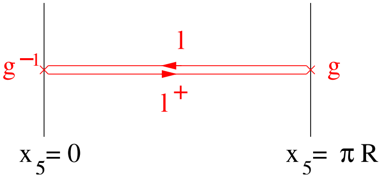
$$w = \begin{cases} \frac{1}{2} & \text{plaquette on boundary} \\ 1 & \text{otherwise} \end{cases}$$



5D Orbifold

- bare anisotropy is $\gamma = \sqrt{\beta_5/\beta_4} \simeq a_4/a_5$
- N_5 is number of *links* in the fifth dimension
- $N_5 a_5 = \pi R$, R radius of extra dimension

5D Orbifold Model Higgs operators



Scalar Polyakov loop (defined at $n = (n_\mu, 0)$)

$$p(n) \;=\; l(n)\; g\; l^\dagger(n)\; g^\dagger$$

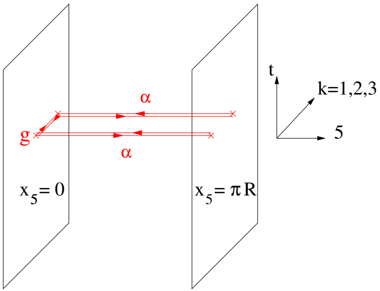
Higgs field

$$h(n) \;=\; [p(n) - p^\dagger(n), g]/(4N_5) \;\sim\; A_5^1\sigma^1 + A_5^2\sigma^2$$

Higgs operators

$$\mathcal{H}(n_0) = \sum_{n_1,n_2,n_3} \mathrm{tr}\,[hh^\dagger] \quad , \quad \mathcal{P}(n_0) = \sum_{n_1,n_2,n_3} \mathrm{tr}\,[p]$$

Lattice operators: Gauge boson



Gauge boson operators (defined at $n = (n_\mu, 0)$)

$$\mathcal{Z}(n_0, k) = \sum_{n_1,n_2,n_3} \mathrm{tr}\left[g\, U(n,k)\, \alpha(n + a_4\hat{k})\, U^\dagger(n,k)\, \alpha(n)\right]$$

$$\mathcal{Z}'(n_0, k) = \sum_{n_1,n_2,n_3} \mathrm{tr}\left[g\, U(n,k)\, l(n + a_4\hat{k})\, U^\dagger(n',k)\, l^\dagger(n)\right]$$

α is the $SU(2)$ projection of h; $n' = (n_\mu, N_5)$

Spontaneous symmetry breaking

Elitzur's theorem [Elitzur, 1975]

In the gauge-invariant lattice theory, spontaneous symmetry breaking (SSB) must originate from the breaking of a *global* symmetry.

Stick symmetry

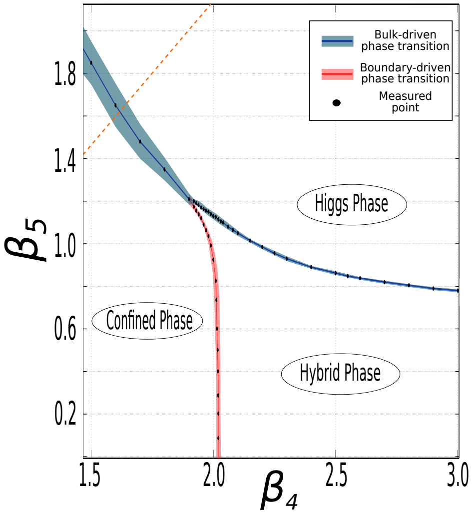
Global transformation [Ishiyama, Murata, So and Takenaga, 0911.4555]

$$\begin{cases} U(n_5=0,5) \to g_s^{-1}\, U(n_5=0,5) \\ U(n_5=0,\mu) \to g_s^{-1}\, U(n_5=0,\mu)\, g_s \end{cases} \quad \text{with } g_s = -i\sigma^1$$

Gauge boson operators $\mathcal{Z}$, $\mathcal{Z}'$ are odd under the stick symmetry, they are the order parameters of SSB

[Irges and Knechtli, arXiv:1312.3142]

5D Orbifold Model Phase Diagram



Notes

On the torus

- confined and Coulomb phase

On the orbifold

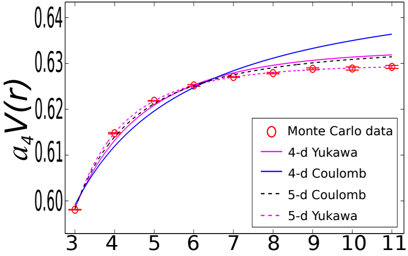
- One more phase: $U(1)$ gauge links deconfine separately
- No compactification observed at $\gamma > 1$
- Interesting physics is found at $\gamma < 1$

first order PTs [Alberti, Irges, Knechtli and Moir, 1506.06035]

Dimensional Reduction

Higgs phase

- The orbifold's $SU(2)$ bulk feels the fifth dimension

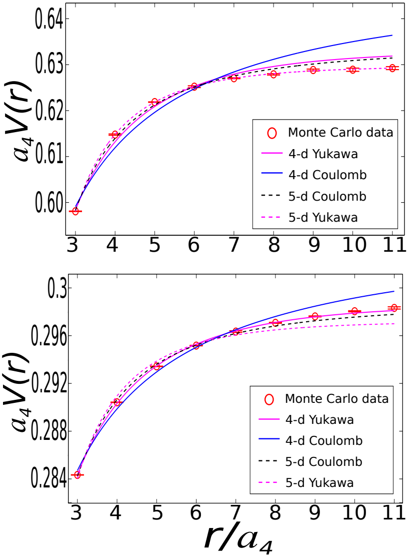


Dimensional Reduction

Higgs phase

- The orbifold's $SU(2)$ bulk feels the fifth dimension
- But near the PT the boundary remains four-dimensional
- The fitted Yukawa masses agree with the measured one

$\Rightarrow$ dimensional reduction via localization



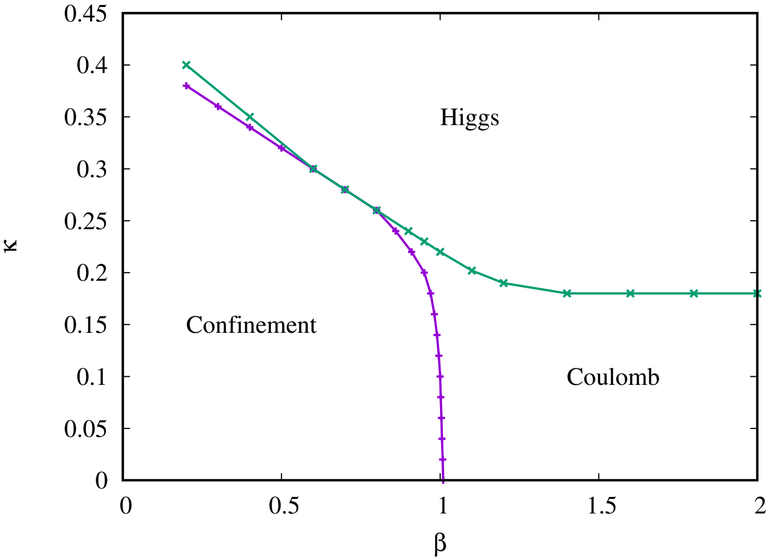
[Alberti, Irges, Knechtli and Moir, 1506.06035]

5D Orbifold Model Summary

Non-perturbative Gauge-Higgs Unification

- $\mathrm{SU}(2)$ pure gauge theory on a 5D orbifold has a Higgs phase with the Higgs mechanism realized as a quantum and bosonic effect
- a Standard-Model like spectrum can be reproduced
- cut-off effects appear to be small, excited state energies are 2–5 times larger than the ground states
- localization on the 4D boundaries is observed
- 5D Orbifold boundary resembles a 4D Abelian gauge-Higgs model (with $q = 2$)
- $$S_\phi[U,\phi] = \sum_x |\phi(x)|^2 + \lambda(|\phi(x)|^2 - 1)^2 - 2\kappa \sum_\mu \mathrm{Re}\left\{\phi^\dagger(x)[U_\mu(x)]^q \phi(x + a\hat{\mu})\right\}$$
- first study of U_Ω in a gauge theory...

4D Abelian Higgs Phase Diagram ($\lambda = 1.0$)



4D Abelian Higgs Unitary Gauge

$$\phi(x) = \rho(x)\exp i\varphi(x) \rightarrow \phi_1 = \rho\cos\varphi,\ \phi_2 = \rho\sin\varphi$$

$$\Rightarrow d\phi_1 d\phi_2 = |\det\begin{pmatrix}\cos\varphi & -\rho\sin\varphi \\ \sin\varphi & \rho\cos\varphi\end{pmatrix}| d\rho d\varphi = \rho d\rho d\varphi$$

$$S_\rho[V,\rho] = \sum_x \rho_x^2 + \lambda(\rho_x^2 - 1)^2$$

$$-2\kappa\rho_x \sum_\mu \rho_{x+\hat{\mu}} \mathrm{tr}\,(\underbrace{e^{-i\varphi_x} U_{x,\mu} e^{i\varphi_{x+\hat{\mu}}}}_{=V_{x,\mu}})$$

$$\prod_{x,\mu}\int_{SU(2)} dU_{x,\mu} \prod_x \int_{-\infty}^{\infty} d\phi(x)\ e^{-S(U,\phi)} \rightarrow$$

$$\prod_{x,\mu}\int_{SU(2)} dU_{x,\mu} \prod_x \int_0^{\infty} \rho(x) d\rho(x) \int_{U(1)} d\varphi(x)\ e^{-S(U,\rho,\varphi)} \propto$$

$$\prod_{x,\mu}\int_{SU(2)} dV_{x,\mu} \prod_x \int_0^{\infty} d\rho(x)\ e^{-S(V,\rho)+\ln\rho}$$

4D Abelian Higgs Constraint HMC

$$H[V,\rho] \;=\; S_\rho[V,\rho] - \ln\rho + \frac{1}{2}\sum_x \pi(x)^2 + \mu\left(\frac{1}{\Omega}\sum_x \rho(x) - \Phi\right)$$

constraint EOMs: [Fodor, Holland, Kuti, Nogradi, Schroeder, 0710.3151]

$$\dot\rho(x,t) \;=\; \frac{\partial H}{\partial \pi(x,t)} = \pi(x,t)$$

$$\dot\pi(x,t) \;=\; -\frac{\partial H}{\partial \rho(x,t)} = -\frac{\partial S_\rho}{\partial \rho(x,t)} + \frac{1}{\rho} - \frac{\mu}{\Omega}$$

time derivatives of the constraint to ensure it's unchanged

$$\frac{\partial}{\partial t}\left(\frac{1}{\Omega}\sum_x \rho(x) - \Phi\right) = \sum_x \dot\rho(x) = \sum_x \pi(x) = 0$$

$$\frac{\partial}{\partial t}\sum_x \pi(x) = \sum_x \dot\pi(x) = 0 \Rightarrow \mu = \sum_x\left(\frac{1}{\rho} - \frac{\partial S_\rho}{\partial \rho(x)}\right)$$

4D Abelian Higgs Constraint Effective Potential

the derivative of the potential with respect to Φ can be obtained from

$$e^{-\Omega U_\Omega(\Phi)} = \int \mathcal{D}\rho\mathcal{D}V\delta\left(\frac{1}{\Omega}\sum_x \rho(x) - \Phi\right) e^{-S+\ln(\rho)}$$

$$\propto -\int \mathcal{D}\rho\mathcal{D}V\mathcal{D}\pi e^{-H[\rho,V,\Phi]}$$

using a variable shift $\rho = \rho' + \Phi$ to avoid the δ-function in the first line [O'Raifeartaigh, Wipf, Yoneyama, 1986], or equivalently

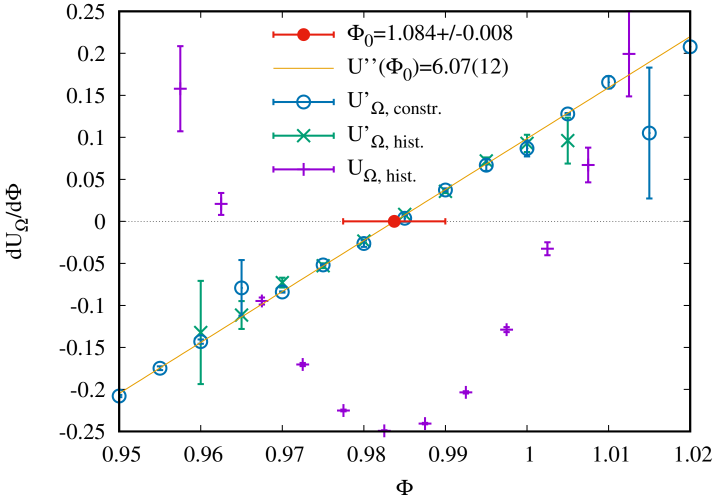
$$-\Omega U'_\Omega(\Phi)e^{-\Omega U_\Omega(\Phi)} = -\int \mathcal{D}\rho\mathcal{D}V\mathcal{D}\pi e^{-H[\rho,V,\Phi]}\frac{dH}{d\Phi}$$

$$\Rightarrow U'_\Omega(\Phi) = \frac{1}{\Omega}\left\langle\frac{dH}{d\Phi} = -\mu\right\rangle_\Phi = \frac{1}{\Omega}\left\langle\sum_x\left(\frac{\partial S_\rho}{\partial\rho(x)} - \frac{1}{\rho}\right)\right\rangle_\Phi$$

where $\langle\ldots\rangle_\Phi$ means the expectation value at fixed Φ

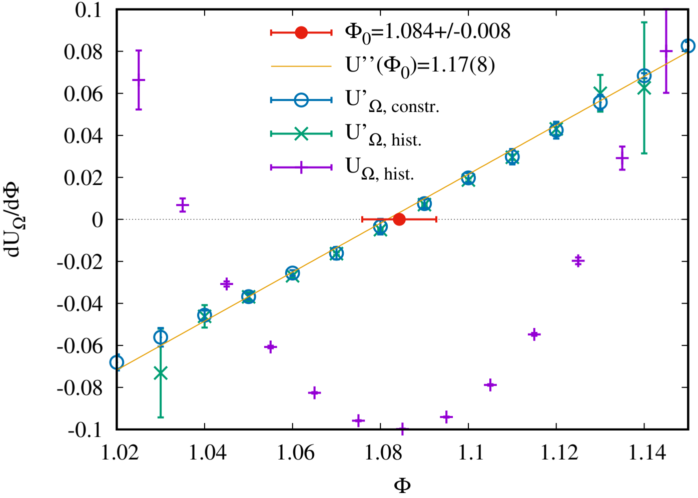
4D Abelian Higgs Effective Potential

$\beta = 0.6, \kappa = 0.3, \lambda = 1.0, am_H \approx 2.1(3)$



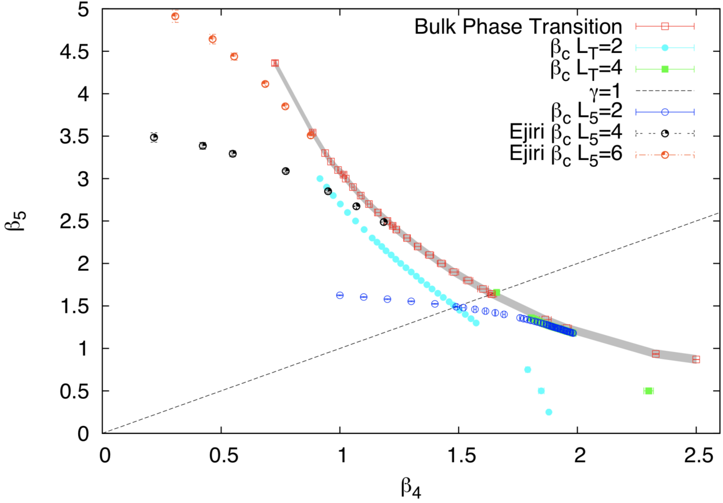
4D Abelian Higgs Effective Potential

$$\beta = 1.4, \kappa = 0.17, \lambda = 0.15, am_H \approx 0.9(2)$$



5D Torus Phase Diagram [Knechtli, Luz, Rago, arXiv:1110.4210]

first step towards 5D orbifold model we work on the torus



5D Torus Constraint HMC

new gauge-invariant, symplectic algorithm for constraint Hamiltonian of the Higgs operator given by the Polyakov loop along the fifth dimension $P_5 = \prod_{n=0}^{N_5}[U_5(x,n)] = e^{N_5 H}$

$$
\begin{aligned}
S_W^{tor} &= \frac{\beta_4}{2}\sum_{\mu,\nu} \mathrm{tr}\,\{1 - U_{\mu\nu}\} + \frac{\beta_5}{2}\sum_{\mu} \mathrm{tr}\,\{1 - U_{\mu 5}\} \\
H[U] &= S_W^{tor}[U] + \frac{1}{2}\sum_{\mathbf{x},\mu} \mathrm{tr}\,[\pi_\mu^2(\mathbf{x})] + \lambda(\frac{1}{2\Omega}\sum_x \mathrm{tr}\,P_5(x) - \Phi) \\
\dot{U}_5(x,n_5) &= \pi_5(x,n_5)U_5(x,n_5) \\
\dot{\pi}_5(x,n_5) &= -\frac{\partial S[U_5]}{\partial U_5(x,n_5)} + \frac{\lambda}{8\Omega}\mathrm{tr}\,[\ldots\sigma_i U_5(x,n_5)\ldots]\sigma^i
\end{aligned}
$$

following [Hairer, Lubich, Wanner, 2006] we use an extension of the so-called Rattle algorithm to general Hamiltonians for constraint systems and apply it to our problem

Newton-Störmer-Verlet-leapfrog method on the Torus

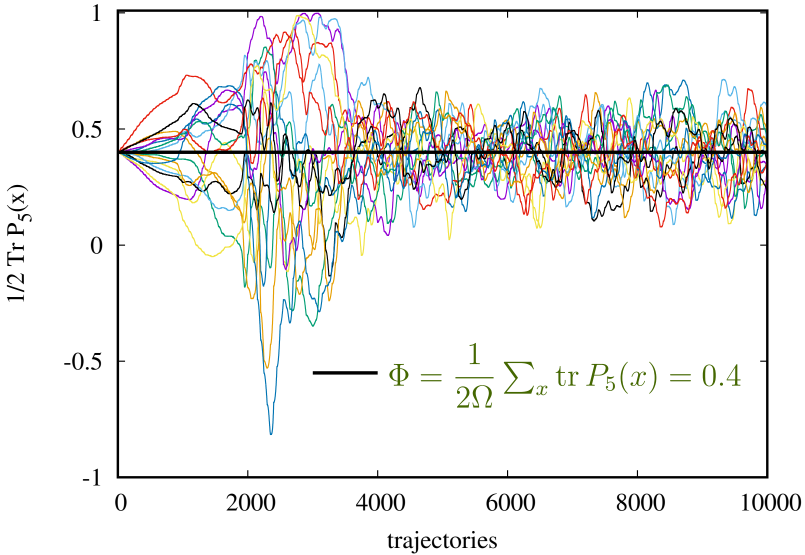
$$
\begin{aligned}
\pi_{n+1/2} &= \pi_n - \frac{h}{2}\left(\frac{\partial S}{\partial U_n} - \frac{\lambda}{8\Omega}\mathrm{tr}\,[...\sigma_i U_n ...]\sigma^i\right) \\
U_{n+1} &= e^{h\pi_{n+1/2}} U_n = [1 + h\pi_n - h^2/2(... + \pi_n^2) + \mathcal{O}(h^3)]U_n \\
0 &= \frac{1}{2\Omega}\sum_x \mathrm{tr}\, P_{n+1} - \Phi = \frac{1}{2\Omega}\sum_x \mathrm{tr} \prod_{n_5=0}^{N_5-1} U_{n+1}(x, n_5) - \Phi \\
\pi_{n+1} &= \pi_{n+1/2} - \frac{h}{2}\left(\frac{\partial S}{\partial U_{n+1}} - \frac{\mu}{8\Omega}\mathrm{tr}\,[...\sigma_i U_{n+1} ...]\sigma^i\right) \\
0 &= \frac{1}{8\Omega}\sum_{x,n_5} \mathrm{tr}\,\{\mathrm{tr}\,[...\sigma_i U_{n+1}(x,n_5)...]\sigma^i \pi_{n+1}(x,n_5)\}
\end{aligned}
$$

the first three equations determine $(\pi_{n+1/2}, U_{n+1}, \lambda)$, whereas the remaining two give (π_{n+1}, μ)

Newton-Störmer-Verlet-leapfrog method on the Torus

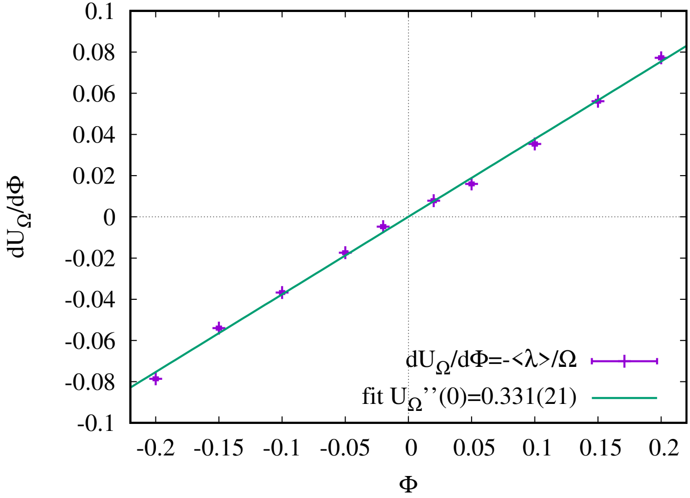
$$
\begin{aligned}
\frac{\lambda}{8\Omega} &= \Big\{ \sum_{x,n_5} \Big(\mathrm{tr}\,[\ldots \Big(\frac{\partial S}{\partial U_n(x,n_5)} - 2\pi_n(x,n_5)/h \Big) U_n(x,n_5)\ldots] \\
&\quad -2 \sum_{m_5>n_5}^{N_5-1} \mathrm{tr}\,[\ldots \pi_n(x,n_5) U_n(x,n_5) \ldots \pi_n(x,m_5) U_n(x,m_5) \ldots] \\
&\quad -\mathrm{tr}\,[\ldots \pi_n^2(x,n_5) U_n(x,n_5) \ldots] \Big) \Big\} / \\
&\quad \sum_{x,n_5} \mathrm{tr}\,\{\ldots \mathrm{tr}\,[\ldots \sigma_i U_n(x,n_5)\ldots] \sigma^i U_n(x,n_5) \ldots\} + \mathcal{O}(h^3)!!! \\
\frac{\mu}{8\Omega} &= \sum_{x,n_5} \big(\mathrm{tr}\,[\ldots \sigma_i U_{n+1}(x,n_5)\ldots] \mathrm{tr}\,[\sigma^i \partial S / \partial U_{n+1}(x,n_5)] \\
&\quad -2\mathrm{tr}\,[\ldots \sigma_i U_{n+1}(x,n_5)\ldots] \mathrm{tr}\,[\sigma^i \pi_{n+1/2}(x,n_5)]/h \big) / \\
&\quad \sum \mathrm{tr}\,\{ (\mathrm{tr}\,[\ldots \sigma_i U_{n+1}(x,n_5)\ldots] \sigma^i)^2 \}
\end{aligned}
$$

Newton-Störmer-Verlet-leapfrog method on the Torus



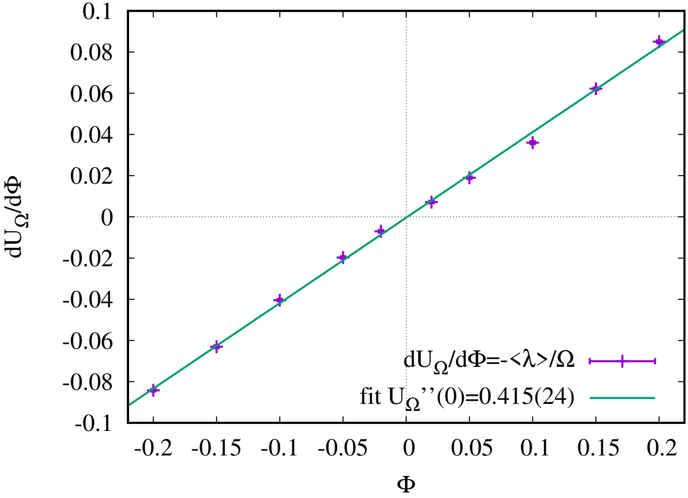
5D Torus Effective Higgs Potential

$$\beta_4 = 1.0, \beta_5 = 2.8, \Omega = 8^4, N_5 = 4, \langle \mathrm{tr}\, P_5 \rangle = 0, a_4 m_H \approx 0.5(1)$$



5D Torus Effective Higgs Potential

$$\beta_4 = 0.8, \beta_5 = 3.8, \Omega = 8^4, N_5 = 6, \langle \mathrm{tr}\, P_5 \rangle = 0, a_4 m_H \approx 0.7(1)$$



Conclusions & Outlook

Constraint HMC Algorithms for gauge-Higgs models

- implemented the constraint HMC for 4D Abelian gauge-Higgs model and computed the effective Higgs potential in the spontaneously broken phase
- developed a new symplectic constraint HMC algorithm for 5D torus and found no SSB in accordance with the absence of stick symmetry

Open Questions and Outlook

- interpretation of torus results, $P_5 = e^{N_5 H}$
- constraint HMC algorithm for 5D orbifold $\Phi = \langle \mathrm{tr}\, \mathcal{Z} \rangle$
- other applications: (constraint) effective Polyakov loop action for finite temperature QCD