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Walking RG Flows and Complex CFTs

with Slava Rychkov and Bernardo Zan

1808.04380, 1807.11512

Walking RG flows

– Two well-known physical examples

  – Weak 1st order phase transitions (CM)

  – Walking technicolor (BSM)

- We propose that, it is the same RG phenomena, which, in some cases, can be described with the help of complex CFTs

Walking RG flows for BSM

– WTC is a way to get around some of the constraints on composite Higgs models.

– Consider a strongly coupled sector which has approximate scale invariance in some range of energies $\Lambda_{IR} < E < \Lambda_{UV}$, and assume Higgs is a part of this sector.

– Then, in this range of energies, Higgs operator has some Δ_H

– We couple it to the rest of the Standard Model:

$$S_{Full} = S_{SM \backslash H} + S_{Strong} + g_i \Lambda_{UV}^{1-\Delta_H} H \Psi_i \bar{\Psi}_i$$

Walking RG flows for BSM

$$S_{Full} = S_{SM\setminus H} + S_{Strong} + g_i \Lambda_{UV}^{1-\Delta_H} H \Psi_i \bar{\Psi}_i$$

– H gets a vev at the scale $\Lambda_{IR} \equiv E_{EW}$,

– So Yukawas are given by

$$y_i = g_i \left(\frac{\Lambda_{IR}}{\Lambda_{UV}} \right)^{\Delta_H - 1}$$

– There are two contradicting requirements:

– $y_t \sim 1 \to \Lambda_{UV} \sim \Lambda_{IR}$

– FCNC should be suppressed $\dfrac{\Psi^i \Psi^j \bar{\Psi}^k \bar{\Psi}^l}{\Lambda_{UV}^2} \to \Lambda_{UV} \gg \Lambda_{IR}$

Walking RG flows for phenomenology

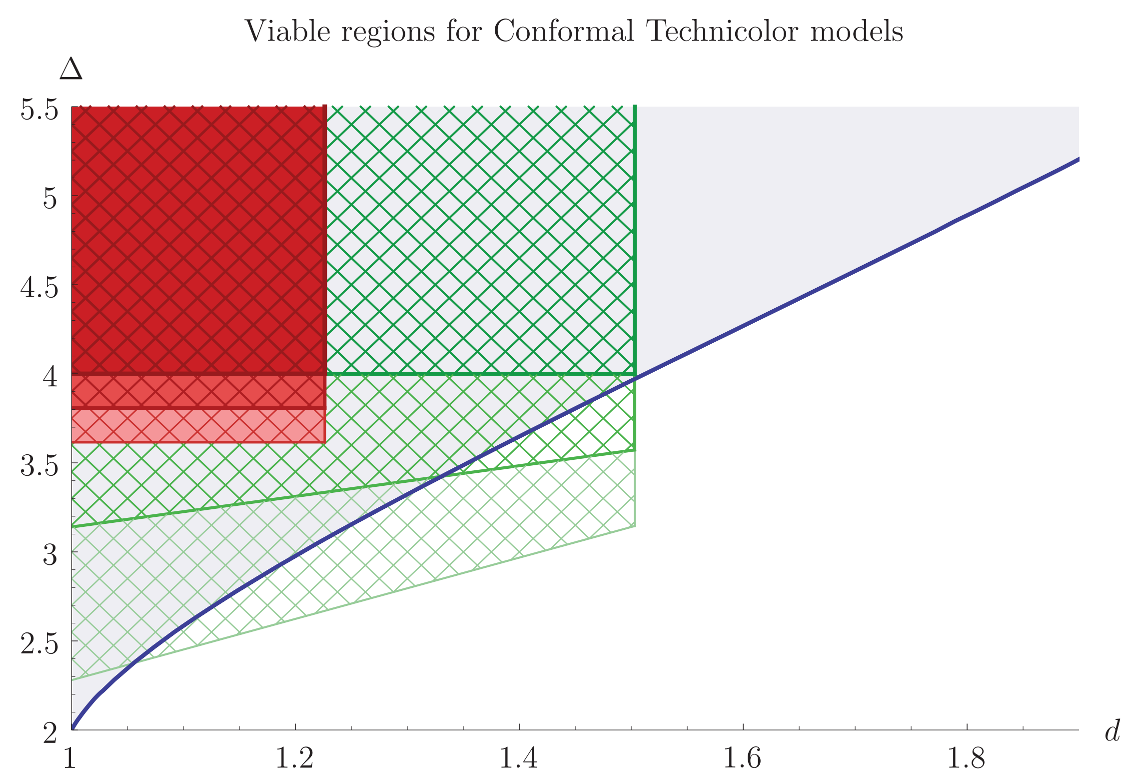
$$y_i = g_i \left(\frac{\Lambda_{IR}}{\Lambda_{UV}}\right)^{\Delta_H - 1} \qquad \frac{\Psi^i \Psi^j \bar{\Psi}^k \bar{\Psi}^l}{\Lambda_{UV}^2} \to \Lambda_{UV} \gg \Lambda_{IR}$$

– Way out is to have $\Delta_H \sim 1$

– But this leads to another potential problem: in unitary CFTs,

$$\Delta_H \sim 1 \to \Delta_{[HH^\dagger]} < 4$$

so we are back to fine-tuning problem due to a relevant singlet operator.



Poland et al, 2012

Slow RG flows

Two ways of producing slow RG flows

“Mild Tuning”

(Deformed CFT)

“Walking”

(Vicinity of a CFT merger)

Mild Tuning

– Conformal Technicolor (Luty-Okui ’04)

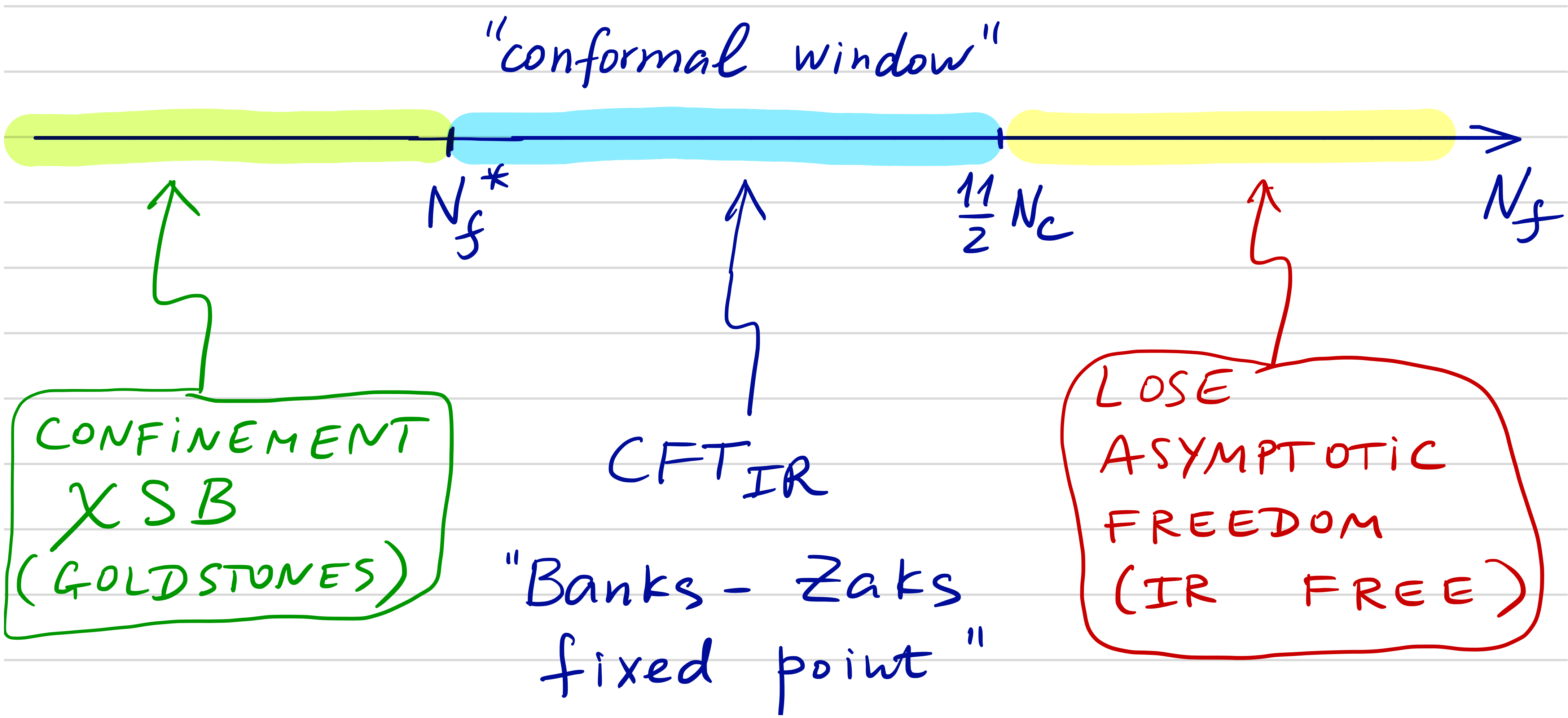
$$S_{Strong} = CFT_{UV} + c\Lambda_{UV}^{4-\Delta}\mathcal{O}$$

– Scale invariance is broken at $\Lambda_{IR} = \Lambda_{UV} c^{\frac{1}{4-\Delta}}$

- $c \ll 1$ (symmetry)
- $\Delta \sim 4$ (“Mild” tuning)
- $\Delta = 4$ Asymptotic freedom is a particular case of Mild tuning

Walking (Ex: Conformal window)

– Consider QCD with Nf flavors:

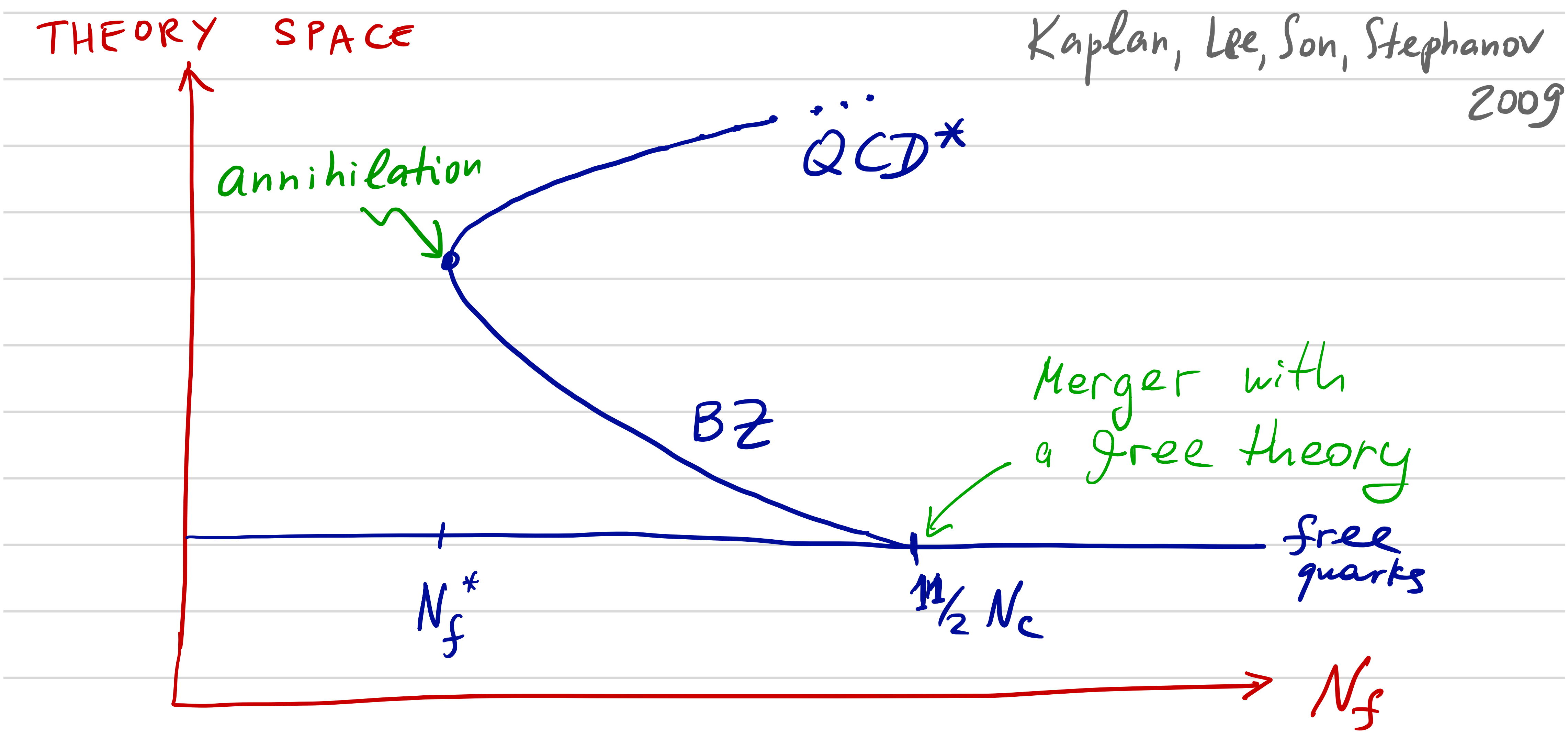


–What happens at the ends of the conformal window?

Walking (Conformal window)

– What happens at the ends of the conformal window?

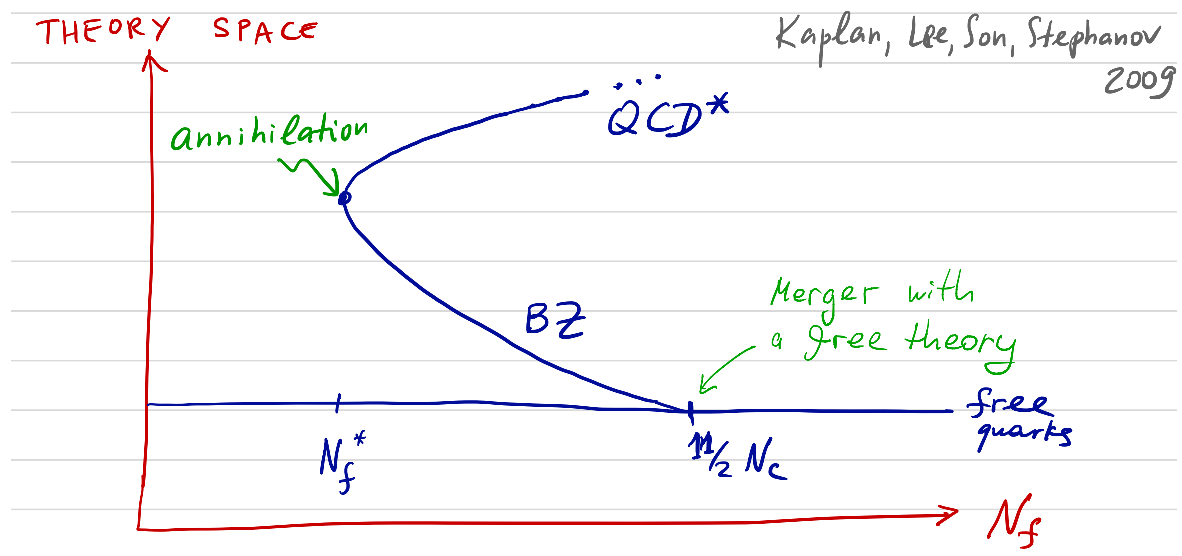
– A fixed point cannot just disappear



- We can discuss more exotic scenarios later

Walking (Ex: Conformal window)

– Let us discuss the vicinity of the annihilation point:



– Consider a “toy model”

$$\beta_\lambda = (N_f - N_f^*) - \lambda^2$$

– For $N_f > N_f^*$ there are two fixed points, but what happens for $N_f \lesssim N_f^*$ - for which walking has been observed on the lattice?

Walking (Toy model)

$$\beta_\lambda = (N_f - N_f^*) - \lambda^2 \qquad N_f \lesssim N_f^*$$

– There are no real CFTs, but there are two complex solutions!

$$\lambda = \pm i\sqrt{N_f^* - N_f}$$

– Note that (the real) RG flow is walking and there is an exponentially large hierarchy of scales:

$$\frac{\Lambda_{UV}}{\Lambda_I R} = \exp\left(-\int_{\lambda_{UV}}^{\lambda_{IR}} \frac{d\lambda}{\beta_\lambda}\right) \sim \exp\left(\frac{\pi}{\sqrt{N_f^* - N_f}}\right)$$

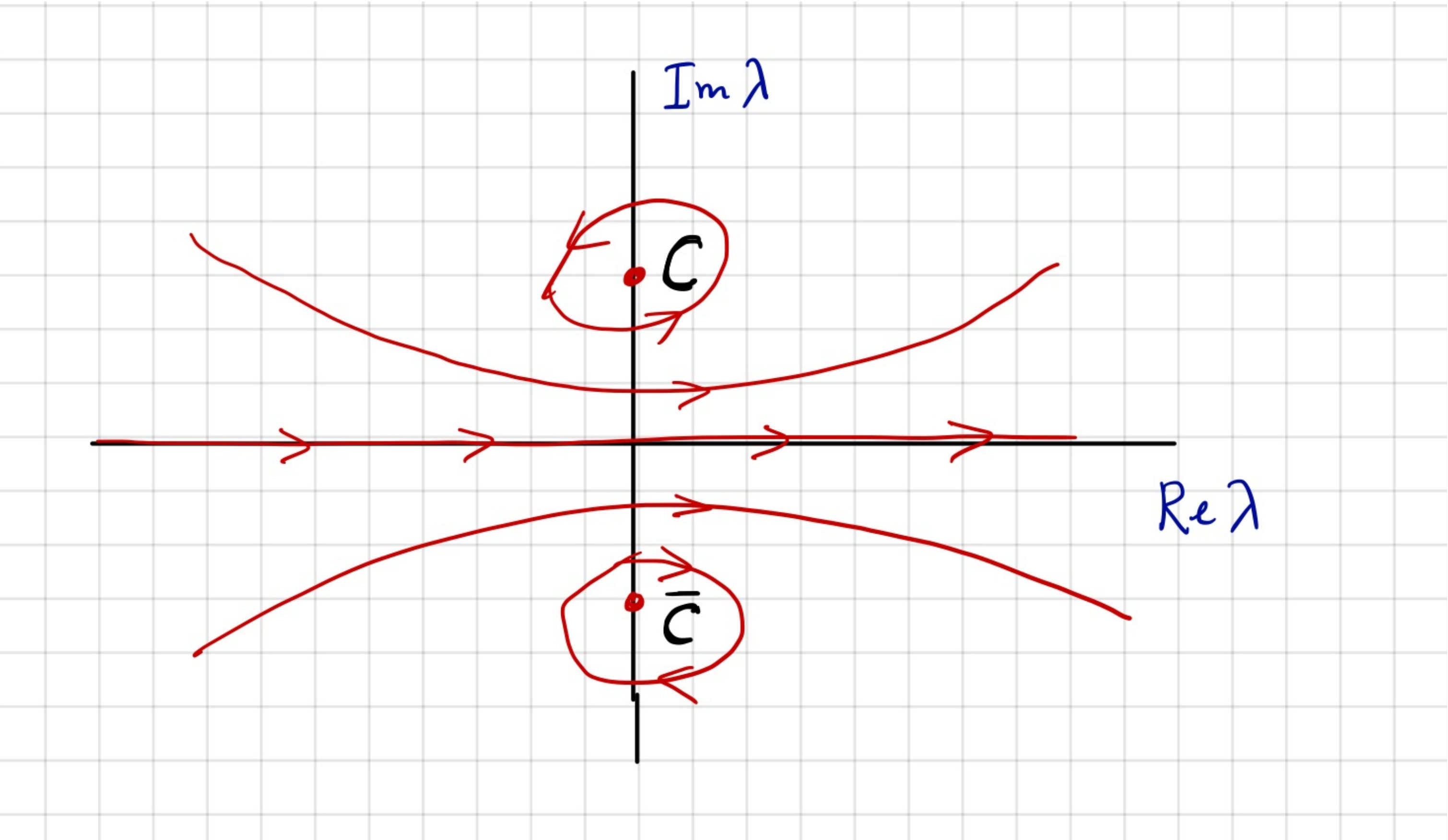
– Walking occurs in the vicinity of a generic CFT merger point

Walking RG flows

$$\beta_\lambda = (N_f - N_f^*) - \lambda^2 \qquad \lambda = \pm i\sqrt{N_f^* - N_f}$$

– Our main point is that these complex CFTs are still very useful to describe the real unitary RG flow, as long as imaginary parts are small.

– We can do Conformal Perturbation theory around $\mathcal{C}$ or $\bar{\mathcal{C}}$



Walking RG flows

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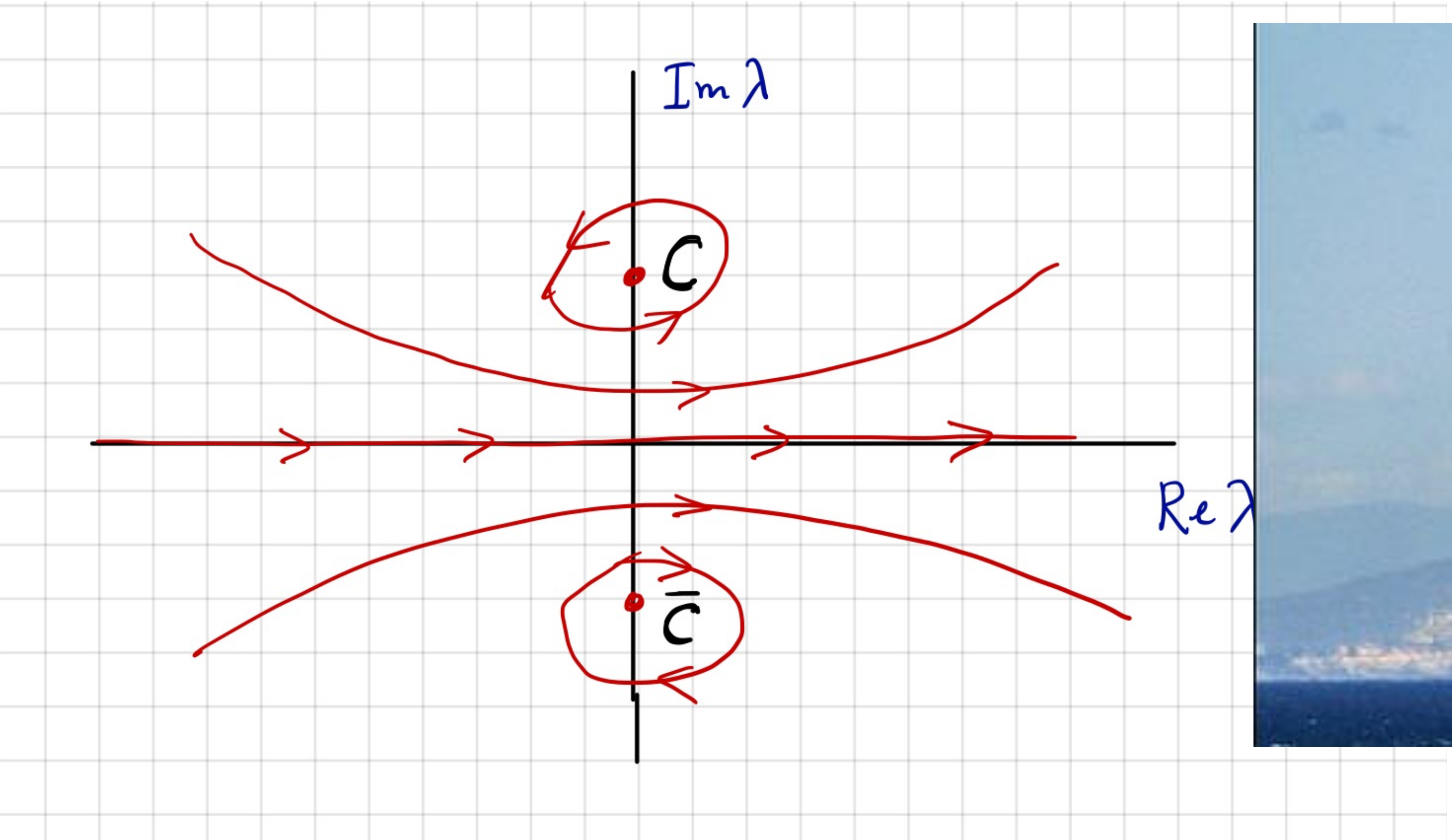
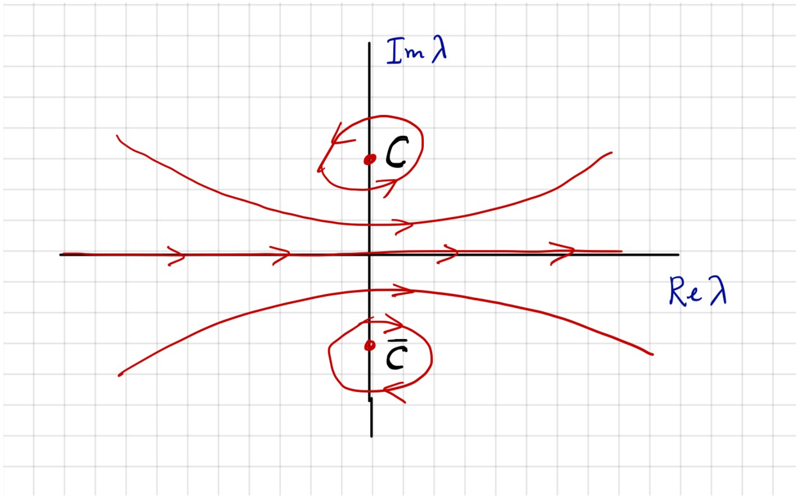


CPT around a complex CFT

– Imagine a complex CFT with an operator of dimension

$$\Delta = d + i\epsilon + O(\epsilon^2)$$



– Deform CFT with this operator:

$$S_{CFT} + g\int d^d x O$$

– Then

$$\beta(g) = i\epsilon g + \pi C_{OOO} g^2$$

beta-function is real for

$$g = iu^* + v = \frac{-i\epsilon}{2\pi} C_{OOO} + v$$

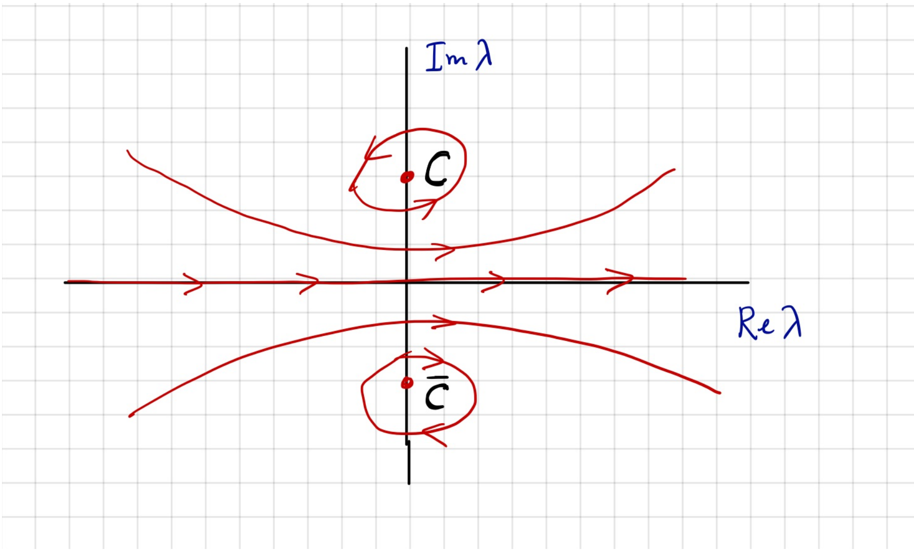
and has the form

$$\beta(iu^* + v) = -A\epsilon^2 - Bv^2$$

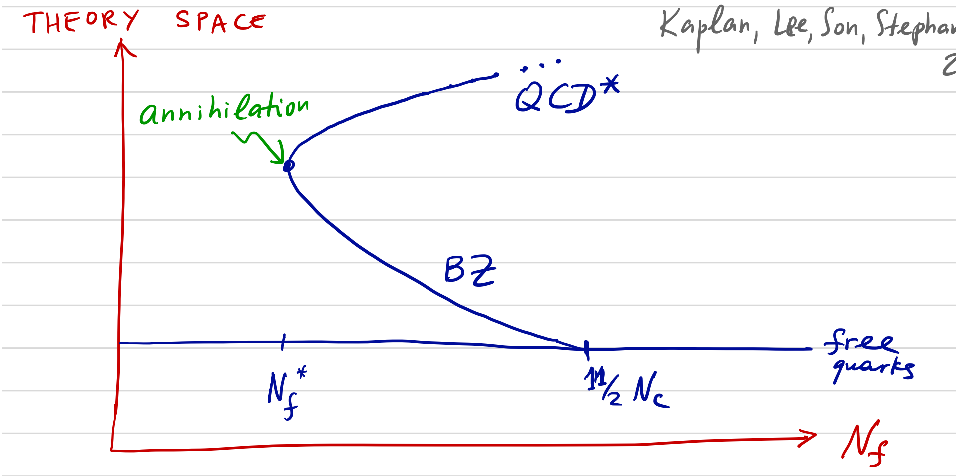
CPT around a complex CFT

$$\beta(iu^* + v) = -A\epsilon^2 - Bv^2$$

$$\beta_\lambda = (N_f - N_f^*) - \lambda^2$$



– The "Toy model" describes a generic behavior near the merger point of two fixed points with the same symmetries, hence walking behavior is generic in this case (no need to turn on the operator).



A calculable example: Potts model (in 2D)

– Generalization of Ising model in which spin field can take Q values

$$H[\{s_i\}] = -\beta \sum_{\langle ij \rangle} \delta_{s_i, s_j}$$

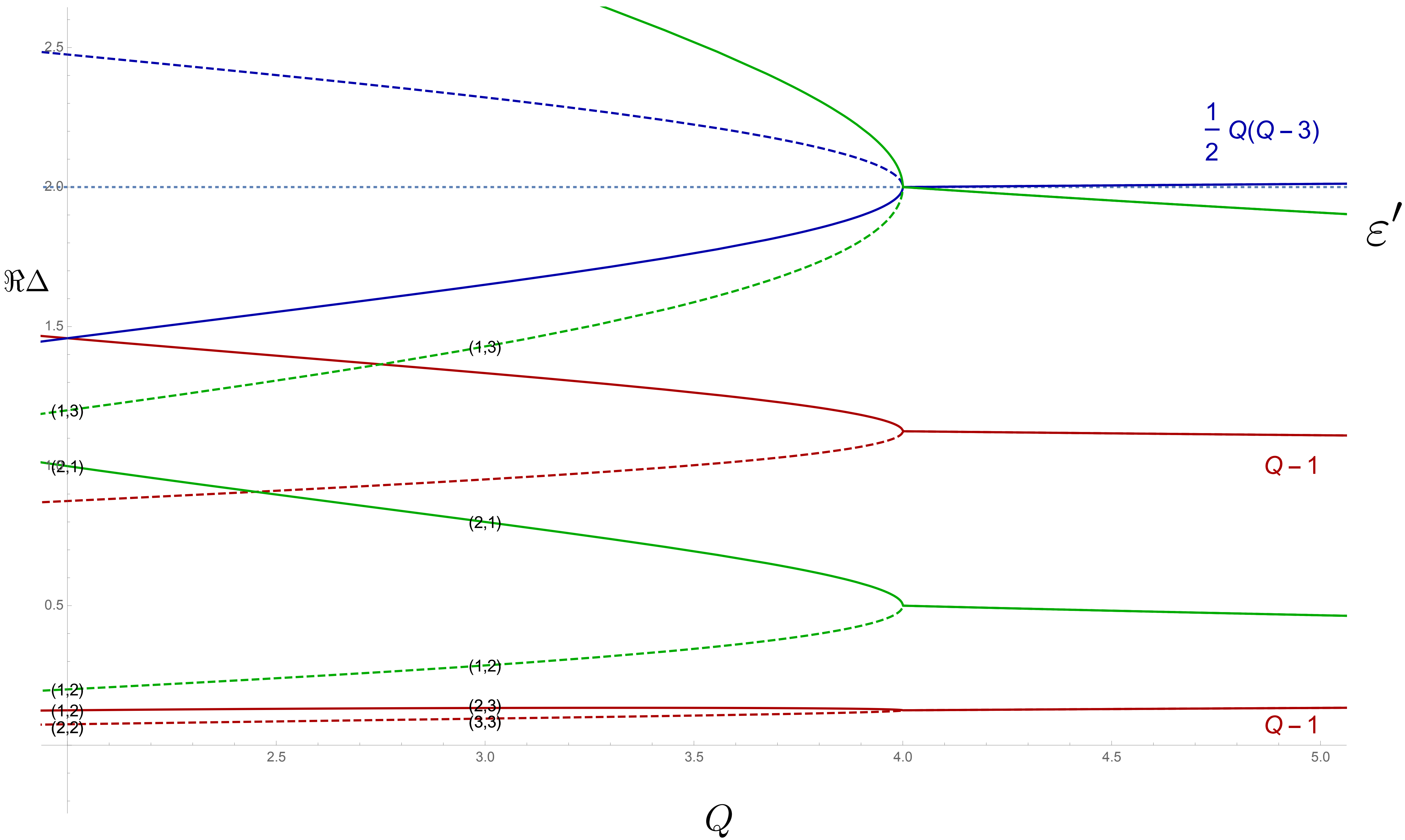
– For Q<4 there are two CFTs (critical and tricritical) that merge at Q=4 $\longleftrightarrow$ conformal window.

– For Q>4 there is no critical point, but a very weak 1st order phase transition:

Q	5	6	7	8	9	10
ξ	2512.2	158.9	48.1	23.9	14.9	10.6

A calculable example: Potts model (in 2D)

– We analytically continue CFT data to Q>4 to get a complex CFT:



A calculable example: Potts model (in 2D)

– We analytically continue CFT data to Q>4 to get a complex CFT

– There is a close to marginal operator: ε'

$$\gamma = -\frac{2i\epsilon}{\pi} - \frac{\epsilon^2}{\pi^2}, \quad C_{\varepsilon'\varepsilon'\varepsilon'} = \frac{4}{\sqrt{3}} - \frac{2i\sqrt{3}}{\pi}\epsilon, \qquad \epsilon \equiv \sqrt{Q-4}$$

– CPT with imaginary coupling:

$$\beta_u^{2-loop} = \frac{\sqrt{3}\epsilon^2}{4\pi^3} + \frac{4\pi u^2}{\sqrt{3}} + \frac{u\epsilon^2}{2\pi^2} + \frac{8\pi^2 u^3}{3}$$

$$\xi_{Potts} = \exp\left(\int_{u_{IR}}^{u_{UV}} \frac{du}{\beta_u(iv_* + u)}\right) = \exp\left(\frac{\pi^2}{\epsilon}\right) \quad \sim \left(\frac{\Lambda_{UV}}{\Lambda_{IR}}\right)_{WTC}$$

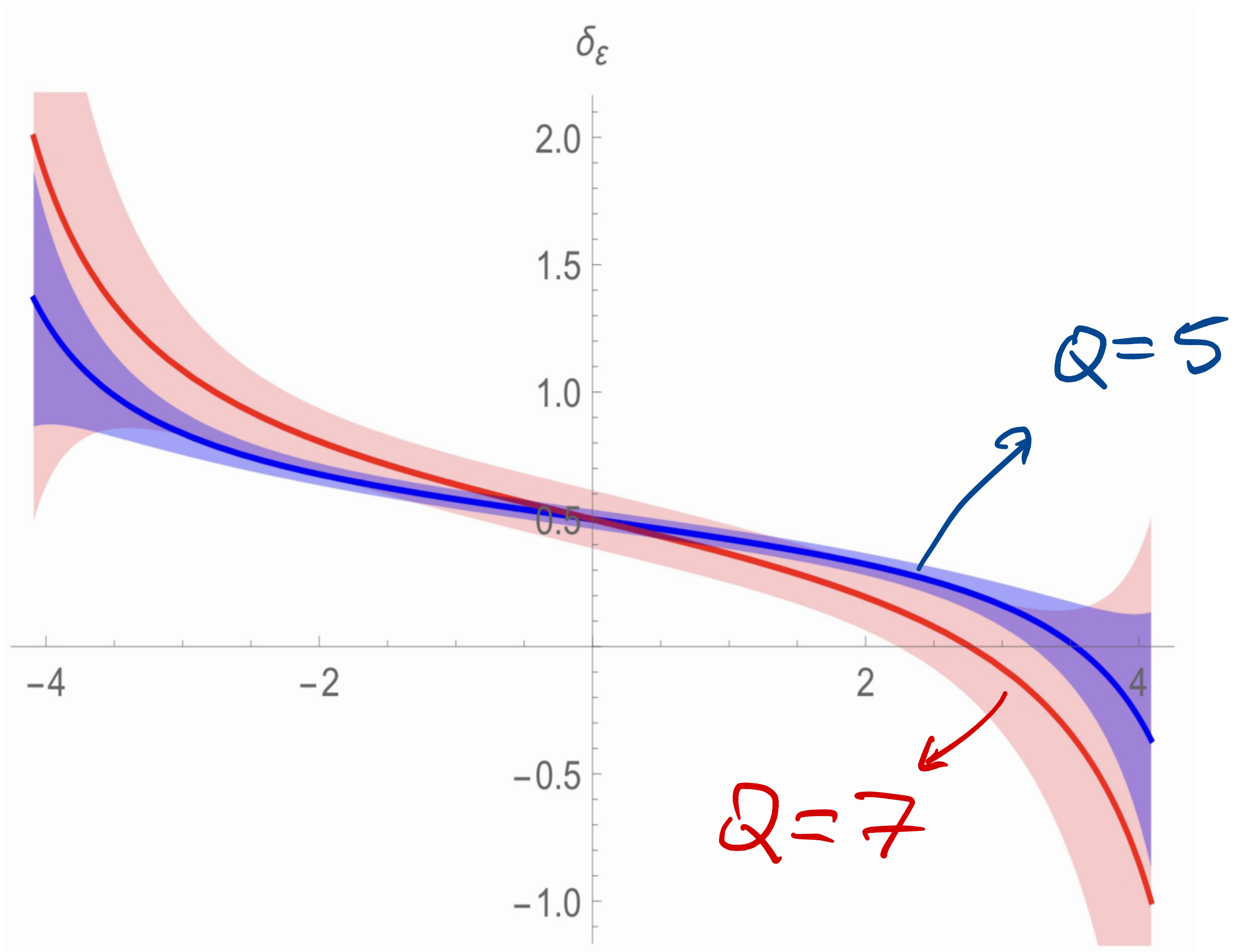
Q	5	6	7	8	9	10
ξ	2512.2	158.9	48.1	23.9	14.9	10.6

A calculable example: Potts model (in 2D)

Drifting scaling dimensions:

$$G_\varepsilon(r) = \langle \varepsilon(0)\, \varepsilon(r) \rangle$$

$$\delta_\varepsilon(r) = -\frac{1}{2}\frac{1}{G_\varepsilon(r)}\frac{\partial G_\varepsilon(r)}{\partial \ln r} = \mathrm{Re}\,\Delta_\varepsilon^c - \frac{\sqrt{3}}{2\pi} C_{\varepsilon\varepsilon\varepsilon'} \tan\frac{\varepsilon \ln r/r_0}{\pi}$$



A calculable example: Potts model (in 2D)

Further Lattice checks, Ma and He 1811.11189

Entanglement entropy of a subregion (in the unitary theory) scales as

$$S(r) \sim \frac{1}{3} \mathrm{Re}\, c \cdot \ln r$$

Other CM examples: Deconfined criticality

RG flow that appears in Néel-VBS transition can be described by bosonic QED_3 with $N=2$ flavors.

Monte Carlo suggests

- Sym. enhancement $SO(3)\times U(1) \to SO(5)$
- Very large correlation length

Bootstrap

- Excludes an $SO(5)$ CFT with measured critical exponents

Resolution:

- Complex $SO(5)$ CFT and walking.

Conclusions:

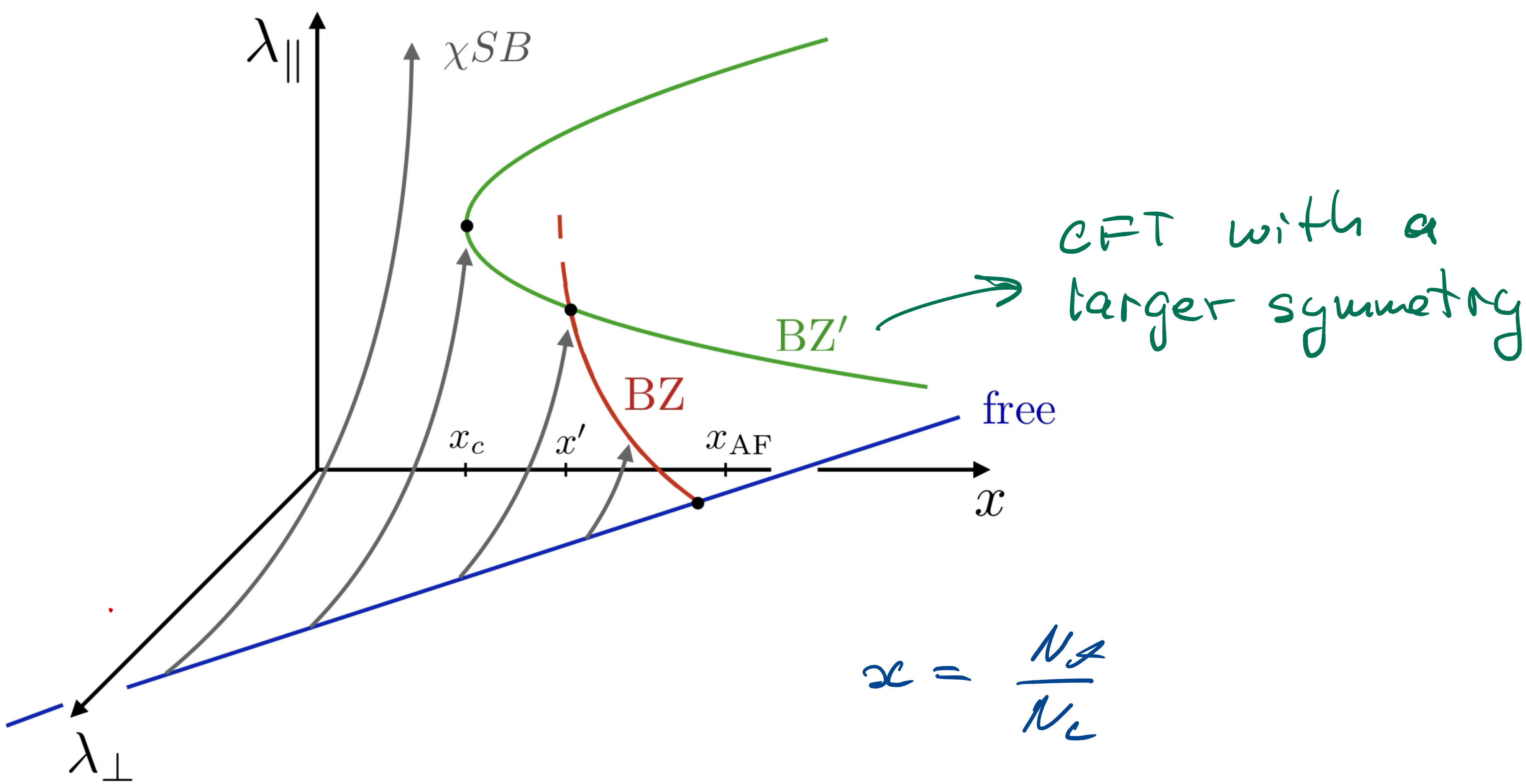
- Walking occurs in the vicinity of a merger of CFts
- Complex CFTs can control real, unitary RG flows
- CPT is under control (iff $\Delta = d + i\epsilon$)
- This knowledge can (and should) be used to study models of Walking Technicolor
- In particular, bootstrap constraints get partially alleviated
- Applications in Condensed Matter systems with weak 1st order phase transitions.
- What about the *light dilaton*?

light dilaton?

- (pseudo-) Goldstone boson of *spontaneously* broken conformal symmetry.
- Was conjectured to be present in WTC models and used in various ways for Hierarchy problem.
- Walking = small *explicit* breaking of conformal symmetry
- **Not** a sufficient condition to have a *light dilaton*
- Instead, one needs a moduli space in a CFT for spontaneous breaking of scale invariance.
- Marginal operator $\neq$ moduli space
- Potts model doesn't have a dilaton in the walking regime.

Alternatives for the end of the conformal window

If CFTs with different symmetries cross they do not merge. They cross and exchange stability



$$x = \frac{N_f}{N_c}$$

Alternatives for the end of the conformal window

We only know that χSB happens for small enough x "experimentally. It could be that BZ CFT exists for arbitrary small x, but RG flows that we know miss it: