Conformal dimensions in large charge sectors using "qubit" formulations

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Use a large conserved charge "Q" sectors to identify a small parameter.

Then, use Effective Field Theory ideas and "radial" quantization to solve for the conformal dimensions as a perturbative expansion.

In O(N) models conformal dimensions emerge as an expansion of the form

$$D_Q = c_{3/2}Q^{3/2} + c_{1/2}Q^{1/2} + c_0 + \mathcal{O}(1/Q^{1/2})$$

 $c_{3/2}$, $c_{1/2}$ are low energy constants that are unknown.

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Challenge: Computing D_Q using Monte Carlo methods suffers from severe signal to noise ratio problems with conventional methods for large Q.

Simplest Example: O(2) model at the 3d Wilson-Fisher fixed point

$$\left\langle e^{iQ\theta_x} e^{-iQ\theta_y} \right\rangle \sim \left(\frac{1}{|x-y|}\right)^{D_Q}$$

For large Q, we have to average quantities of unit magnitude to obtain small numbers!

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Non-trivial Example: O(4) model at the 3d Wilson-Fisher fixed point.

 $SO(4) \sim SU(2) \times SU(2)$



Representations: (q_L, q_R)

Hence we now need to compute

$$\left\langle O_{x}^{q_{L},q_{R}} \left(O^{\dagger} \right)_{y}^{q_{L},q_{R}} \right\rangle \sim \left(rac{1}{|x-y|}
ight)^{D_{q_{L},q_{R}}}$$

New ideas for studying CFTs using Monte Carlo Methods!

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Banerjee, SC, Orlando PRL 120, (2016) 061603

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Traditional



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Traditional

Worldline

$$Z = \int [d\theta] e^{\beta \sum_{x,\alpha} \cos(\theta_x - \theta_{x+\alpha})}$$





Banerjee, SC, Orlando PRL 120, (2016) 061603

Traditional

Worldline



The worldline approach allows us to efficiently create and annihilate charges at various space-time separations using worm algorithms.

$$Z_{Q} = \sum_{[q]} \left[\prod_{x,\alpha} I_{q_{x,\alpha}}(\beta/2) \right] \left[\prod_{x \neq x_{i}, x_{f}} \delta \left(\sum_{\alpha} (q_{x,\alpha} - q_{x-\alpha,\alpha}) \right) \right]$$
$$\delta \left(\sum_{\alpha} (q_{x_{i},\alpha} - q_{x_{i}-\alpha,\alpha} - Q) \delta \left(\sum_{\alpha} (q_{x_{f},\alpha} - q_{x_{f}-\alpha,\alpha} + Q) \right) \right]$$

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eq x_i, x_f} \deltaigg(\sum_lpha (q_{x,lpha} - q_{x-lpha,lpha}) igg)
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L x L box

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Scaling: $Z_Q \sim 1/L^{D_Q}$

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L x L box

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Worm algorithms can compute

 $Z_Q/Z_{Q-1}\sim 1/L^{\Delta_Q}$

$$\Delta_Q = D_Q - D_{Q-1}$$





Previous work only up to Q=4 Hasenbusch, Vicari, PRB 84 (2011) 125136

Q: How well does the Q-expansion work?

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Fit Data: $D_Q = 1.195(10) Q^{3/2} + 0.075(10) Q^{1/2} - 0.094$ analytic calculation
Q: What about the O(4) Wilson-Fisher fixed point, especially since it has two charges (q_L,q_R) that characterizes "charged sectors."

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Qubit formulation of the O(4) Wilson-Fisher fixed point!

Canonical commutation relation of QFTs requires an infinite dimensional Hilbert space per lattice site.

$$[\phi(x),\pi(y)]=i\delta_{x,y}$$

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Fermions are already qubits, but with anti-commutation relations.



"renormalizable couplings"



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It is important to identify the Quantum Critical Points that lead to the QFT of interest.

Identifying QCPs usually requires tools beyond perturbation theory!



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This talk: They helped us to explore the large Q-expansion in the O(4) model!

T. Bhattacharya, SC, R. Gupta, H.Singh and R. Somma

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Euclidean action of the traditional theory

$$S = \frac{1}{2g} \int d^d x \ d\tau \ \partial_\mu \vec{\phi} \cdot \partial_\mu \vec{\phi}$$

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Features of the QFT fixed point

d=1, asymptotically free fixed point

- d=2, Wilson-Fisher fixed point
- d=3, Gaussian free fixed point

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Q: Can we reproduce these features using a Qubit Hamiltonian? A: Yes! Here we focus on d=2 Wilson-Fisher point!



 $|s, \mathbf{r}\rangle$ $|m, \mathbf{r}\rangle, m = 0, +1, -1$ singlet triplet















Hopping term



Hopping term



Pair Creation/Annihilation term


$$Z = \sum_{k} \int [dt_k \dots dt_1] \operatorname{Tr} \left(e^{-(\beta - t_k)H_1} (-H_2) e^{-(t_k - t_{k-1})H_1} \cdots (-H_2) e^{-(t_1)H_1} \right)$$

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Relativistic Limit

$$\varepsilon = 1$$

 $W_t = W_s$



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Hamiltonian limit $\varepsilon \to 0$

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 $\begin{array}{l} \varepsilon = 1 \\ \text{Relativistic Limit} \\ W_t = W_s \end{array}$

Hamiltonian limit $\varepsilon \rightarrow 0$

Can study using classical QMC (directed loop/worm algorithms)



Order Parameter Suceptibility

$$\chi = \frac{1}{ZL^{d}} \sum_{\mathbf{r},\mathbf{r}'} \int_{0}^{\beta} dt \operatorname{Tr}\left(e^{-(\beta-t)H} a_{\mathbf{r},\mathbf{m}} e^{-tH} a_{\mathbf{r}',\mathbf{m}}^{\dagger}\right)$$



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Winding Number Susceptibility



 $\nu = 0.7113(11), \eta = 0.0378(6)$ Pelisetto and Vicari Phys. Repts. (2002)

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We see the Gaussian fixed point in d=3+1. We also see asymptotic freedom in d=1+1 but with caveats!

Banerjee, SC, Orlando, Reffert, 1902.09542

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Needs five states per lattice site.



Every monomer has weight U

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arge results at the O(4) Wilson-Fisher fixed point

Q	D(Q,Q)		Q	D(Q,Q)	
	(this work)	(from [26])		(this work)	(from [26])
1/2	0.515(3)	0.5180(3)	1	1.185(4)	1.1855(5)
3/2	1.989(5)	1.9768(10)	2	2.915(6)	2.875(5)
5/2	3.945(6)	-	3	5.069(7)	-
7/2	6.284(8)	-	4	7.575(9)	-
9/2	8.949(10)	-	5	10.386(11)	-

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 $D(Q, Q) = 1.068(4) Q^{3/2} + 0.083(3) Q^{1/2} - 0.094$

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A. First perform Monte Carlo calculations to identify the quantum critical point where the correct QFT emerges.

B. Then study the theory close to the quantum critical point on the quantum computer.