# New Lattice Approach for $\beta$-function in the Chirally Broken Phase

Chik Him (Ricky) Wong

Lattice Higgs Collaboration(LatHC):
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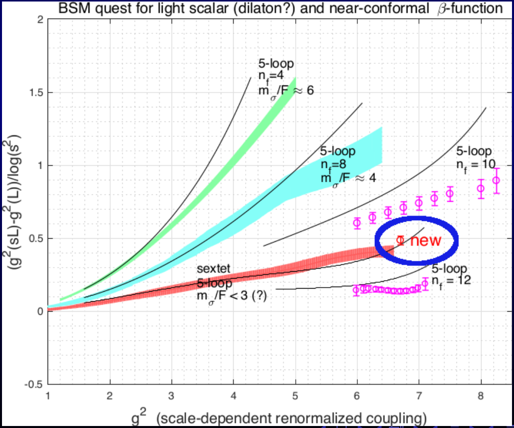
# Outline

- $\beta$-function at strong coupling is crucial in the studies of nearly conformal theories, but simulation becomes hard as we approach the conformal window
- Here we present an alternate way of obtaining the $\beta$-function making use of knowledge in the p-regime for theories outside the conformal window



# Outline

- Review: Lattice Studies of $\beta$-function of nearly conformal gauge theories
- An alternative Lattice approach for $\beta$-function in the $\chi$SB phase
- Application example: $\beta$-function of the Sextet model
- Conclusion

# Review: Lattice Studies of $\beta$-function of nearly conformal gauge theories

- Lattice study of $\beta$-function
  - Gradient Flow [Lüscher JHEP 1008:071,2010]

$$\frac{dA_\mu(t)}{dt} = D_\nu F_{\nu\mu},\ D_\mu = \partial_\mu + [A_\mu, \cdot]$$

  - Perturbation Theory $\overline{MS}$, RG scale: $\mu = 1/\sqrt{8t}$

$$E = \frac{3(N_c^2-1)g^2}{128\pi^2 t^2}(1+\overline{c_1}g^2 + O(g^4)),\ E = \frac{1}{4}(F^a_{\mu\nu})^2$$

  - Non-perturbative definition, (overall normalization depends on boundary conditions etc)

$$g^2(t) \propto \left(\frac{128\pi^2}{3(N_c^2-1)}\right) t^2 \langle E \rangle_{\text{latt}}$$

  - $\beta$-function $\propto -\mu^2 \frac{dg^2}{d\mu^2} = t\frac{dg^2}{dt}$
  - Defined in continuum and infinite volume

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  - Fix $c = \sqrt{8t}/L \Rightarrow \mu = 1/(cL)$
  - Compare $g_c^2$ at $L/a$ with $g_c^2$ at $sL/a$ for some finite ratio $s$

$$\beta(g_c^2, a/L) = \frac{g_c^2(sL/a) - g_c^2(L/a)}{\ln(s^2)}$$

    $g_c^2$ is obtained by either tuning $6/g_0^2$ or interpolating $g_c^2(6/g_0^2)$
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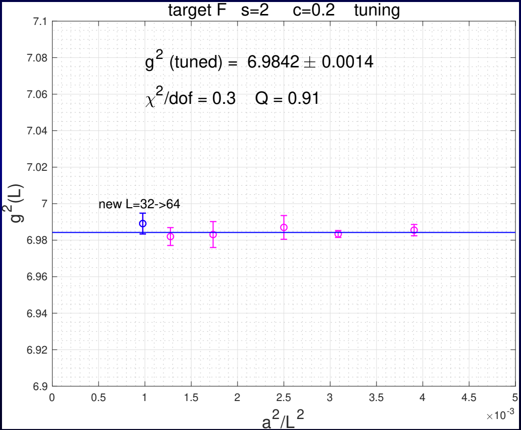


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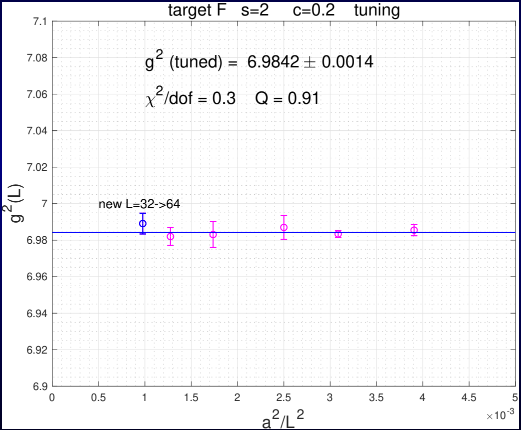


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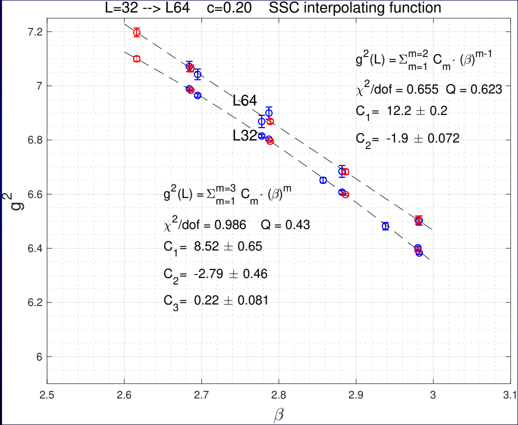


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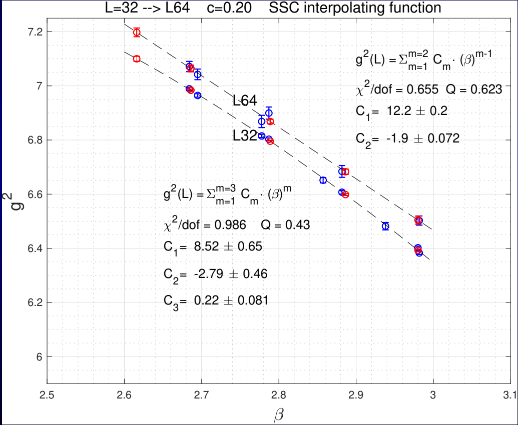


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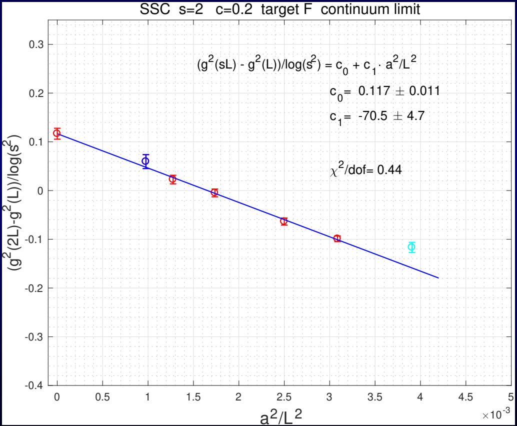

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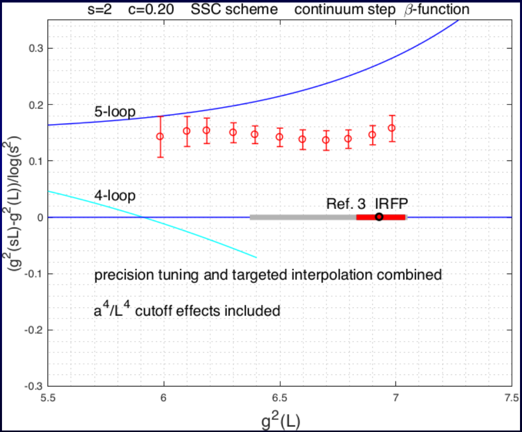


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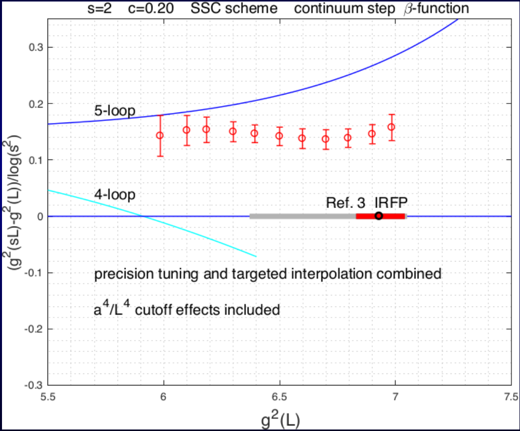


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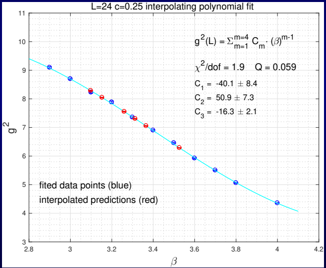


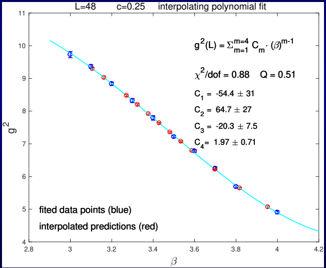

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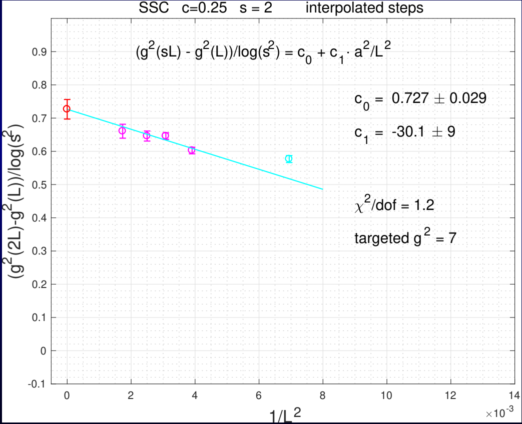

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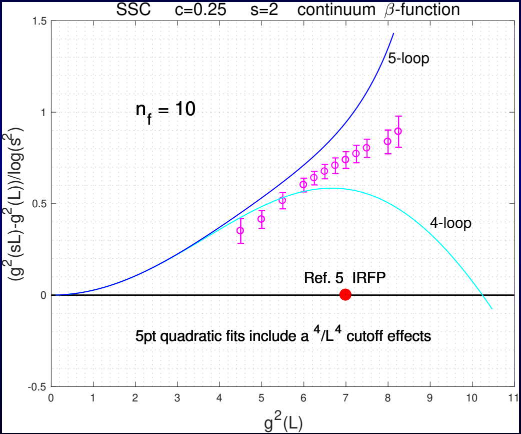


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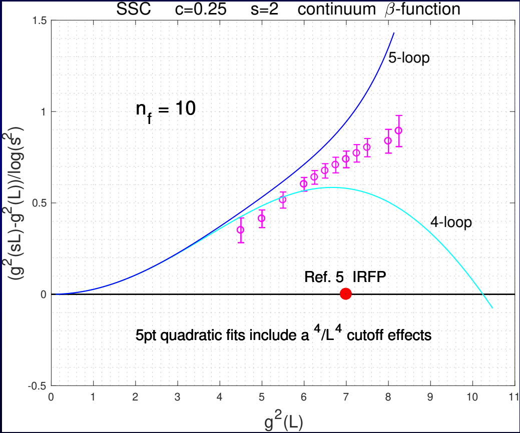


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- Two ways to determine boundary of conformal window
- 1. Rejecting $\chi$SB: Look for IRFP
  - $\beta$-function diminishes as we approach near conformal window
    $\Rightarrow$ higher accuracy and better systematic controls are needed
    $\Rightarrow$ Costly simulations are needed to hopefully resolve controversies
  - Absence of IRFP in limited search range of $g^2$ is not decisive
    $\Rightarrow$ An IRFP is always possible at stronger $g^2$ out of reach
- 2. Rejecting IR conformality: Look for signals of $\chi$SB
  - If the theory is $\chi$SB, chiral symmetry is spontaneously broken beyond certain $g^2_{\text{critical}}$
  - The value of $g^2_{\text{critical}}$ has to be consistent with results from the p-regime simulations
    $\Rightarrow$ p-regime simulations predict how the model should behave beyond $g^2_{\text{critical}}$
  - If we can compute $\beta$-function from p-regime calculation and covers the range of $g^2$ values achievable by step scaling (hopefully includes $g^2_{\text{critical}}$), it is a consistency test for $\chi$SB behavior

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  - $\beta$-function diminishes as we approach near conformal window
    $\Rightarrow$ higher accuracy and better systematic controls are needed
    $\Rightarrow$ Costly simulations are needed to hopefully resolve controversies
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    $\Rightarrow$ An IRFP is always possible at stronger $g^2$ out of reach
- 2. Rejecting IR conformality: Look for signals of $\chi$SB
  - If the theory is $\chi$SB, chiral symmetry is spontaneously broken beyond certain $g^2_{\text{critical}}$
  - The value of $g^2_{\text{critical}}$ has to be consistent with results from the p-regime simulations
    $\Rightarrow$ p-regime simulations predict how the model should behave beyond $g^2_{\text{critical}}$
  - If we can compute $\beta$-function from p-regime calculation and covers the range of $g^2$ values achievable by step scaling (hopefully includes $g^2_{\text{critical}}$), it is a consistency test for $\chi$SB behavior

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# An alternative Lattice approach for $\beta$-function in the $\chi$SB phase

[Fodor et al, LATTICE 2017(2017)]

- Along the flow: Obtain $\beta(g^2) = -\mu^2\, dg^2/d\mu^2 = t\, dg^2(t)/dt$
  Approximated by arbitrarily small $dt = \varepsilon$:

$$\left.\frac{dg^2}{dt}\right|_t = \frac{1}{12\varepsilon}\left(-g^2(t+2\varepsilon) + 8g^2(t+\varepsilon) - 8g^2(t-\varepsilon) + g^2(t-2\varepsilon)\right) + O(\varepsilon^4)$$

- Choose $g^2 = g^2_{\text{target}}$, determine $t_0$ such that $g^2(t_0/a^2, L/a, am) = g^2_{\text{target}}$ at each $L/a$ and $am$
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# Application example: $\beta$-function of the Sextet model

- $SU(3)$ Gauge Theory with $N_f = 2$ fermions in Two-index Symmetric Representation
- Our previous studies show $\chi$SB behaviors, e.g. Hadron spectrum with pseudoscalars as Goldstone bosons of $\chi$SB and non-vanishing Goldstone decay constant in the chiral limit [Fodor et al EPJ Web Conf. 175 08027 (2018)]
- SSC:
  - HMC gauge action: Symanzik, 2 steps of $\rho = 0.15$ stout
  - Flow gauge action: Symanzik
  - Discretization of E: Clover
- Target: $g^2(t_0/a^2, L/a, am) = g^2_{\text{target}} = 6.7$
  - chosen such that it is attainable across all ensembles
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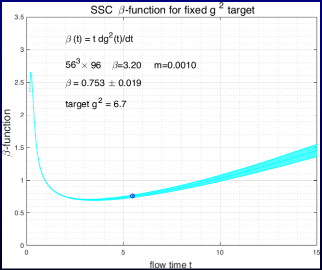
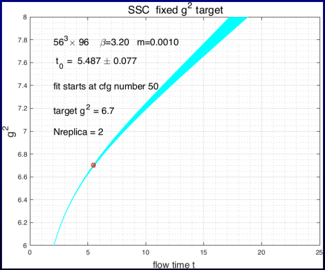
- Example:
  $56^3 \times 96,\ 6/g_0^2 = 3.20,\ am = 0.001 \Leftarrow t_0/a^2 = 5.487 \pm 0.077$
- Approximate derivative
  $\varepsilon = 0.05 \Rightarrow \beta(t_0) = t\left(\frac{dg^2}{dt}\right) = 0.753 \pm 0.019$



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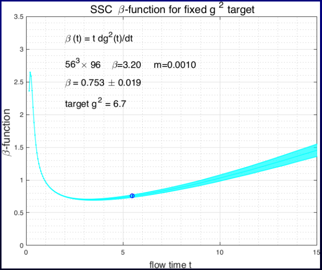
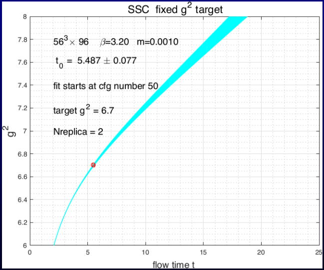
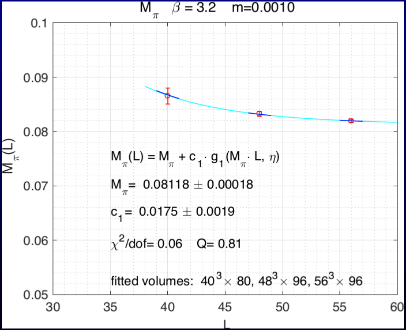


# Application example: $\beta$-function of the Sextet model

- Infinite volume limit using $\chi$PT [Bar and Golterman, Phys. Rev. D 89, 034505 (2014)]
- Point-like source approximation $\sqrt{8t_0}\, M_\pi << 1$
  $\Rightarrow$ Finite volume correction by wrap-around Goldstone bosons $g_1(M_\pi L_t, \eta = L_t/L_s)$
- Infinite volume limit of $M_\pi$: $aM_\pi = 0.08118 \pm 0.00018$



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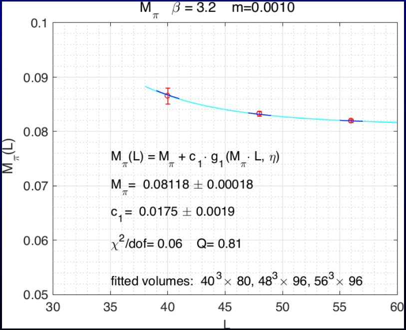

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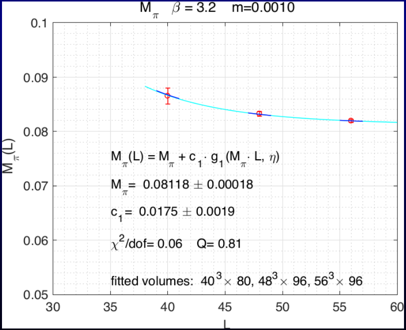

# Application example: $\beta$-function of the Sextet model

- Infinite volume limit using $\chi$PT (ignoring the effects of low lying $0^{++}$ scalar)
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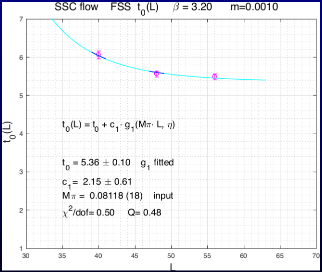


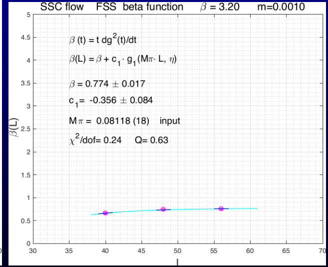

# Application example: $\beta$-function of the Sextet model

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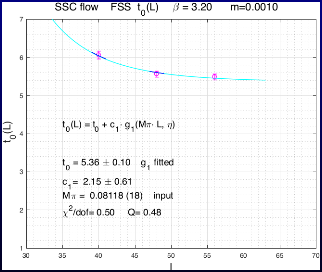


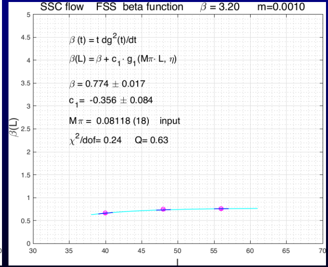

# Application example: $\beta$-function of the Sextet model

- Chiral limit using $\chi$PT [Bar and Golterman, Phys. Rev. D 89, 034505 (2014)] (ignoring the effects of low lying $0^{++}$ scalar)
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$$t_0 = t_0^{(M_\pi=0)}\left(1 + k_1 \frac{M_\pi^2}{(4\pi f)^2} + \ldots\right)$$

$$\beta(t_0) = \beta(t_0^{(M_\pi=0)})\left(1 + l_1 \frac{M_\pi^2}{(4\pi f)^2} + \ldots\right)$$

- In this analysis, only leading order ($M_\pi^2$) is considered
- $M_\pi = 2Bm \Rightarrow$ Linear in $m$ for leading order
- Ansatz:

$$t_0 = t_0^{(m=0)}(1 + c_1 m),$$

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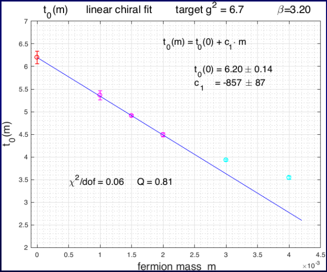


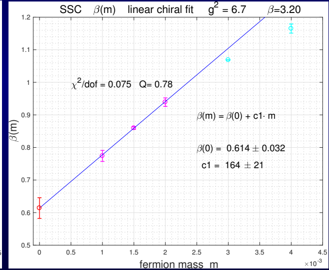

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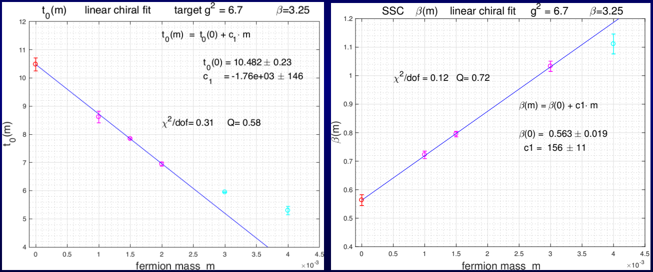

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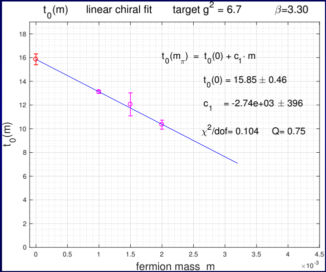


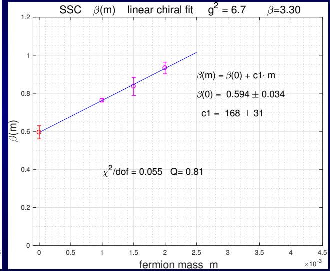


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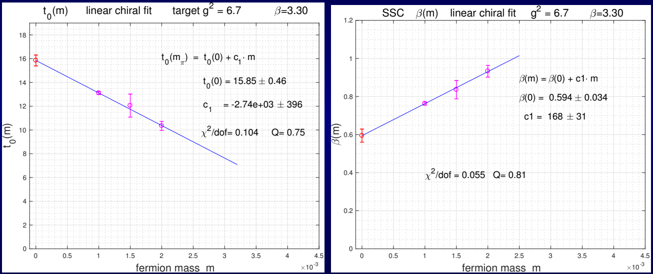


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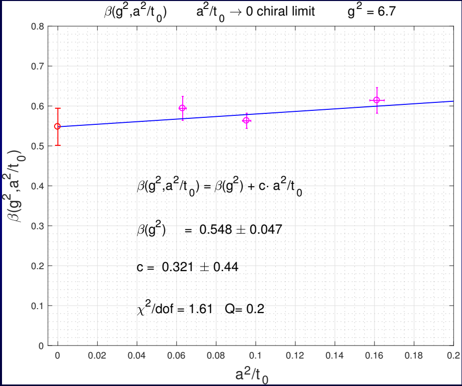
# Application example: $\beta$-function of the Sextet model

- Continuum limit $\beta(g^2_{\text{target}}, a^2/t_0) = \beta(g^2_{\text{target}}) + k\, a^2/t_0$




- Error of $t_0$ is taken into account by $\chi^2 = \sum_k \left[ \frac{(X_k - x_k)^2}{\sigma^2_{x,k}} + \frac{(Y_k - y_k)^2}{\sigma^2_{y,k}} \right]$ $x = a^2/t_0, X = \langle x \rangle;\ y = \beta, Y = \langle y \rangle;\ \sigma$'s : variances [Krystek and Anton, Measurement Science and Technology 18, 3438 (2007)]

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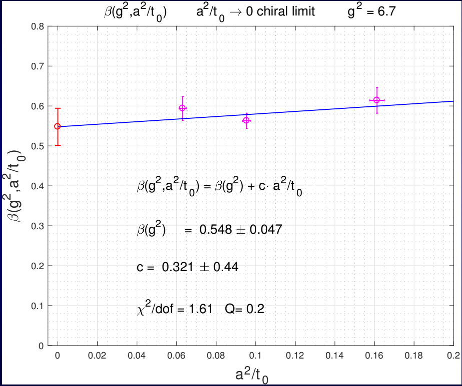


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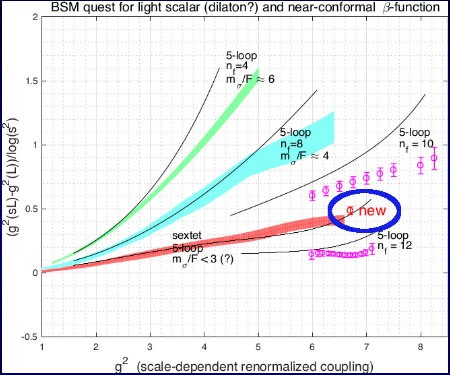


- The new p-regime $\beta$-function is like $c \to 0, s \to 1$ in step scaling, but they are different schemes
  $\Rightarrow$ a bridging between the two has to be done, despite that they seem to be close to each other, possibly due to the insensitivity of $c$ and $s$ values of step scaling when $c$ and $s$ are small enough

# Conclusion

- An alternative approach of computing non-perturbative $\beta$-function of models in $\chi$SB phase is presented
  - It provides a new tool to probe and test $\chi$SB behaviors by bridging p-regime simulations and step scaling $\beta$-function
  - We can now recycle p-regime simulations on $\beta$-function calculation
- Possible improvements:
  - The method was based on $\chi$PT ignoring the existence of light $0^{++}$ scalars near the Conformal Window. The effect of taking this into account is under investigation
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