



Lattice for Beyond the Standard Model Physics - 4/6/2018 - University of Colorado, Boulder, CO

High-precision tests of the gauge/gravity duality and new applications

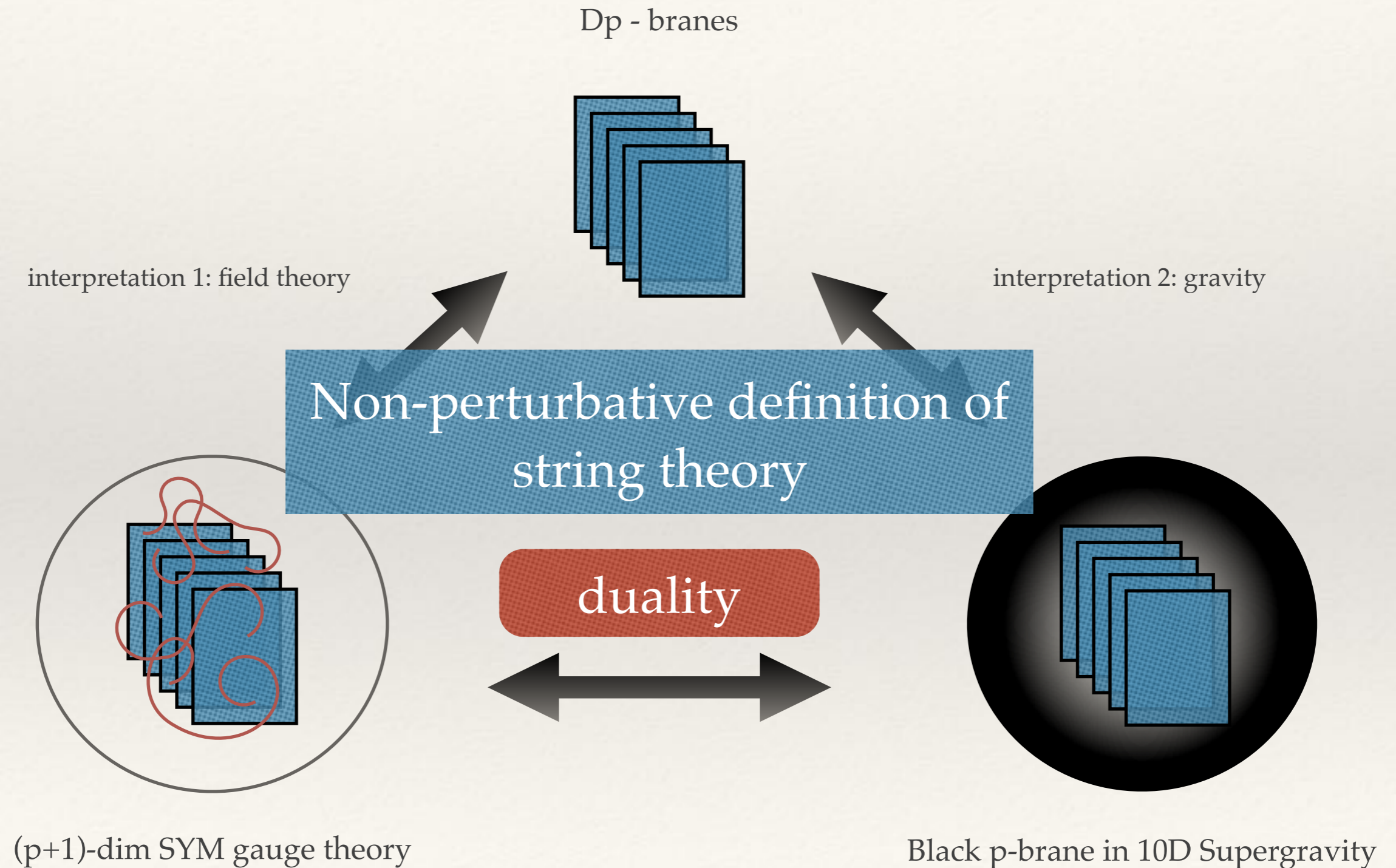
Enrico Rinaldi



RIKEN BNL Research Center

* *with the Monte-Carlo String+M-theory Collaboration (MCSSMC)*

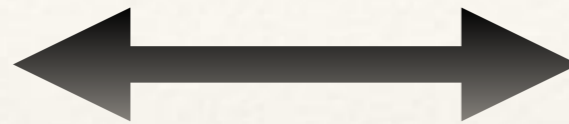
Gauge/Gravity duality



Gauge/Gravity duality

Gauge

supersymmetric SU(N) gauge theory



Gravity

Superstring / M-Theory

Use numerical methods

Test the gravity predictions

strong coupling

HARDER



EASIER

Einstein gravity

Make predictions for gravity!

finite coupling

EASIER



HARDER

string effects

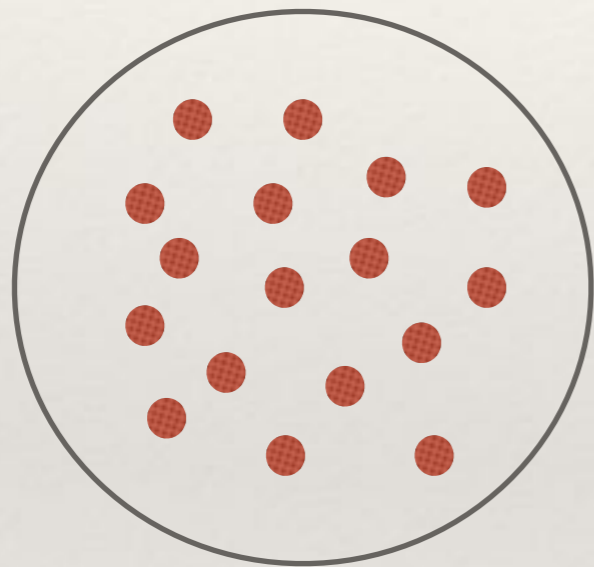
interaction strength

quantum string corrections

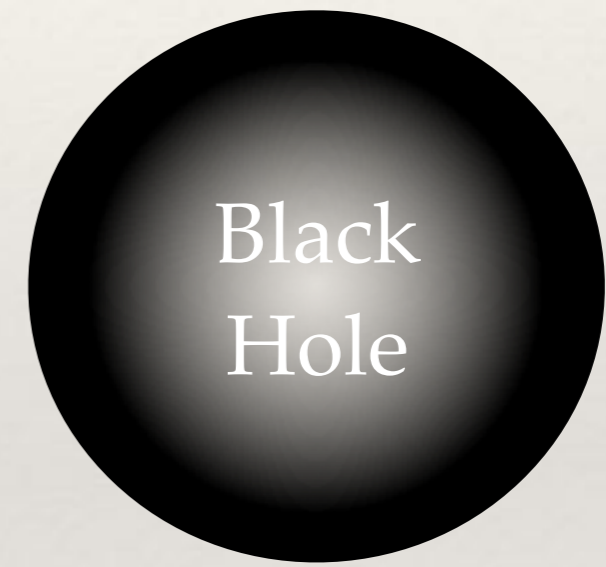
Testing the gauge/gravity duality

internal energy

black hole mass



dictionary



(0+1)-dim maximally supersymmetric gauge theory

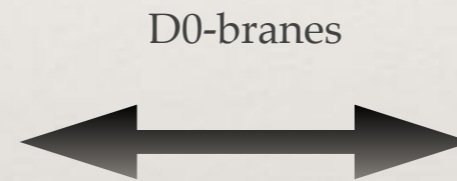
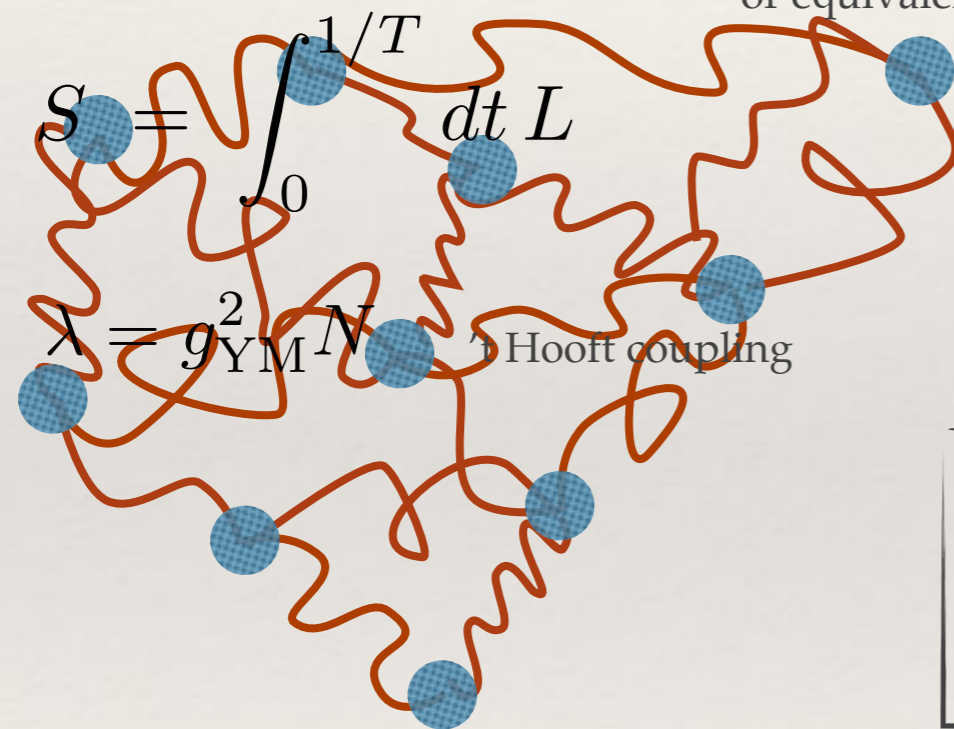
Type IIA superstring theory
in the near horizon limit of
a black 0-brane geometry

Compute observables that are dual to each other on both sides and compare in an easy case (low-d)

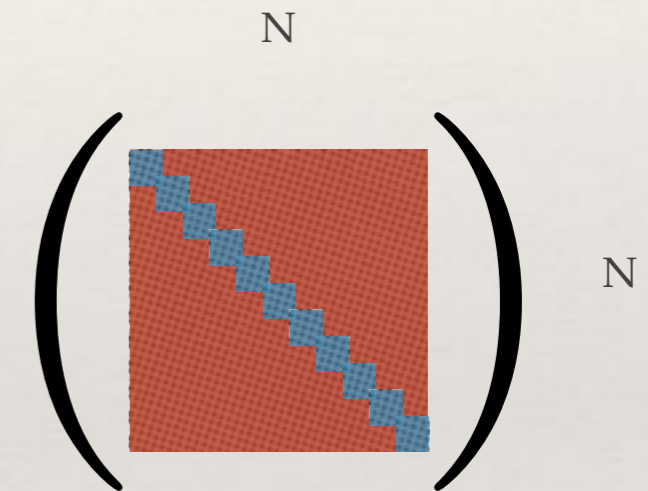
Supersymmetric quantum mechanics

$$L = \frac{1}{2g_{YM}^2} \text{Tr} \left\{ (D_t X_M)^2 + [X_M, X_{M'}]^2 + i\bar{\psi}^\alpha D_t \psi^\beta + \bar{\psi}^\alpha \gamma_{\alpha\beta}^M [X_M, \psi^\beta] \right\}$$

obtained from $\mathcal{N}=1$ U(N) SYM in (9+1)d via dimensional reduction to (0+1)d
or equivalently from $\mathcal{N}=4$ U(N) SYM in (3+1)d



coordinates
couplings



conjectured to be
equivalent to M-
theory

$X_M, M = 1, \dots, 9$ ($N \times N$) \rightarrow hermitian scalars

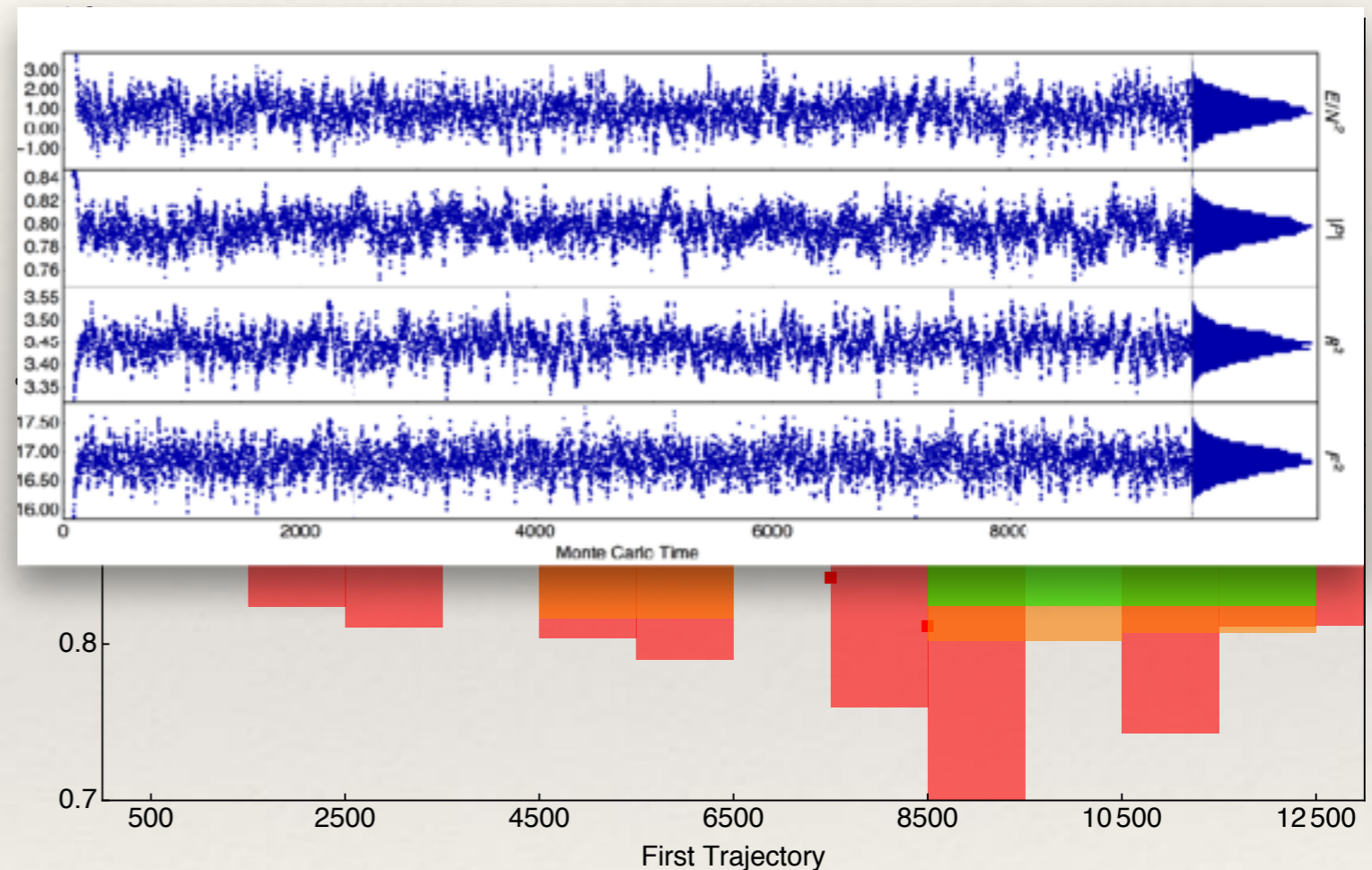
$\psi^\alpha, \alpha = 1, \dots, 16$ ($N \times N$) \rightarrow adjoint fermions

$D_t \cdot = \partial_t \cdot - i[A_t, \cdot]$ \rightarrow gauge covariant derivative

Monte Carlo simulations

- ❖ Observables: E/N^2 , $|P|$,
 $R^2 = \text{Tr}[X^2]/N$,
 $F^2 = \text{Tr}([X_i, X_j]^2)/N$
- ❖ Large statistics for all parameters (N, L, T) is needed
- ❖ Dedicated autocorrelation analysis is paramount due to long fluctuations
- ❖ All data is published in tables and can be used for future benchmarks!

$T=0.5$ $N=24$ $L=32$ O(a) improved action

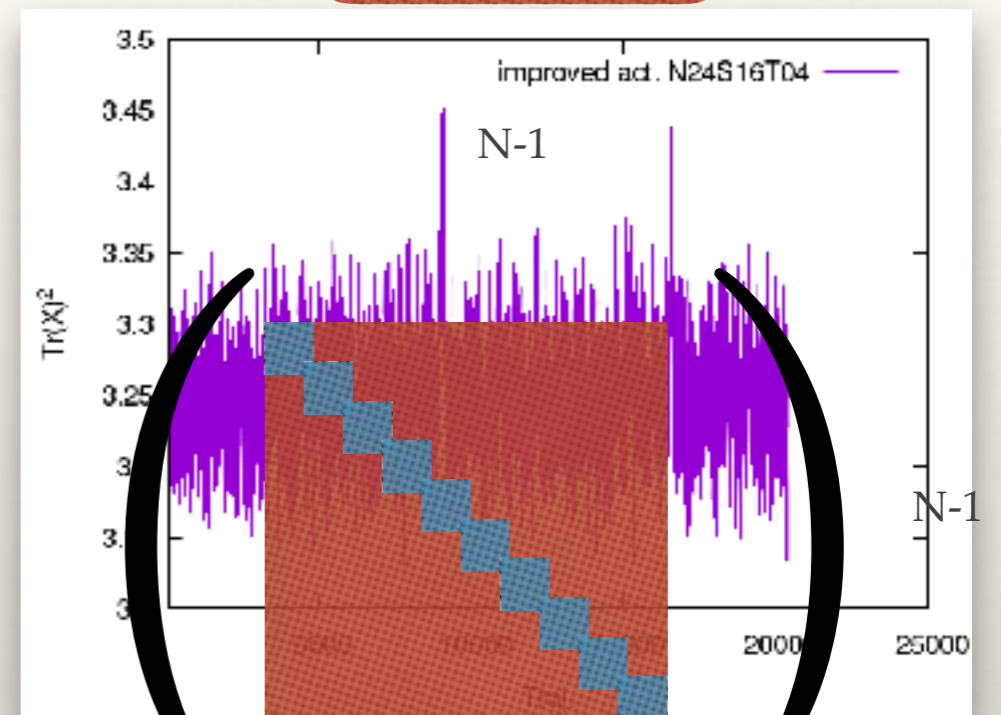
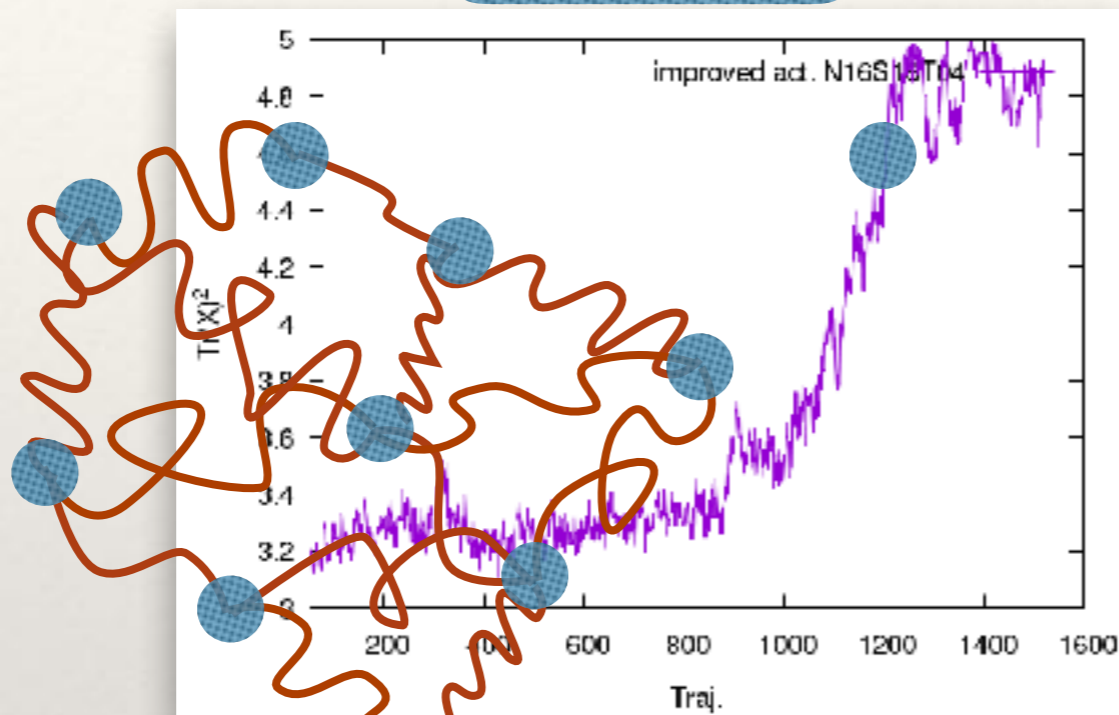


Challenge: flat direction

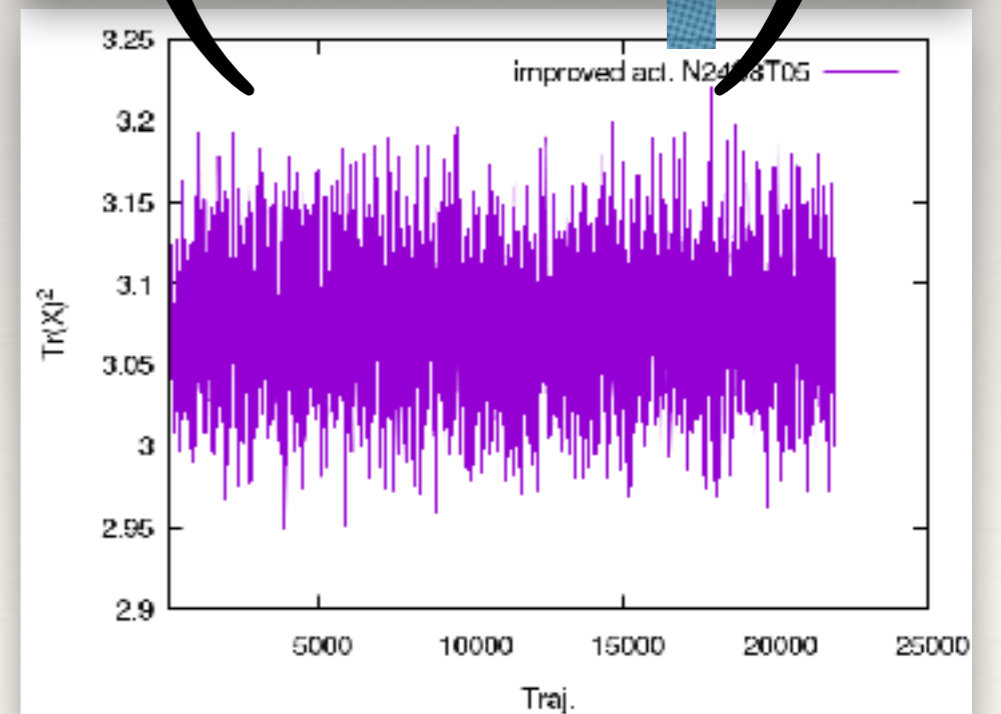
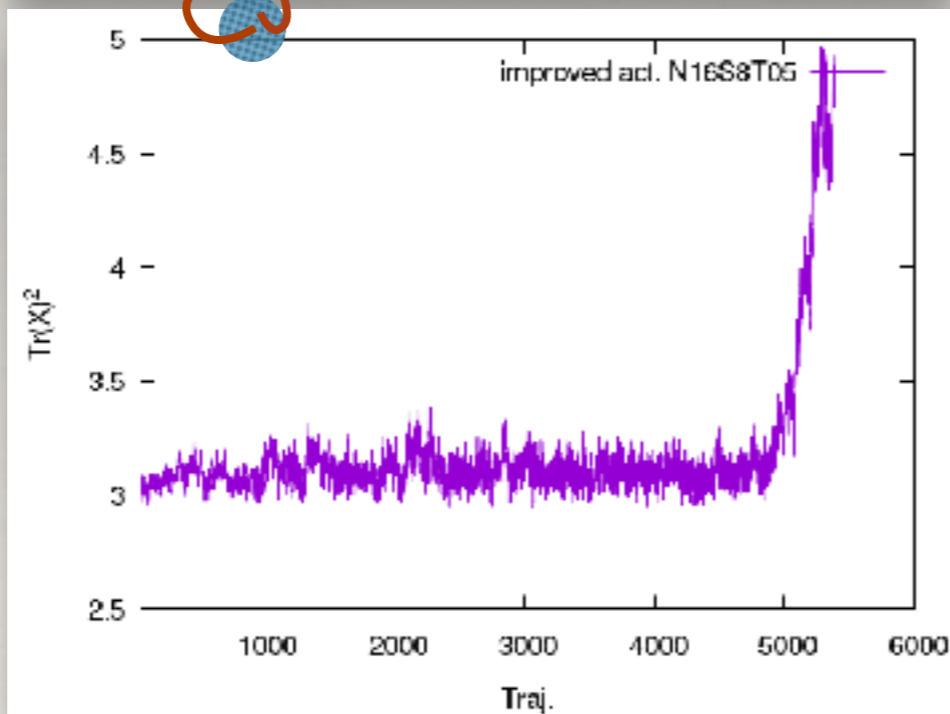
N=16

N=24

T=0.4



T=0.5



Previous results

- ❖ Different cutoff regulator
- ❖ Different discretizations
- ❖ **Finite N**
- ❖ **Finite cutoff**
- ❖ Qualitative agreement
- ❖ Not enough precision for **quantitative predictions**

[Agnastopoulos et al. arxiv:0707:4454]

[Catteral, Wiseman arxiv:0803.4273]

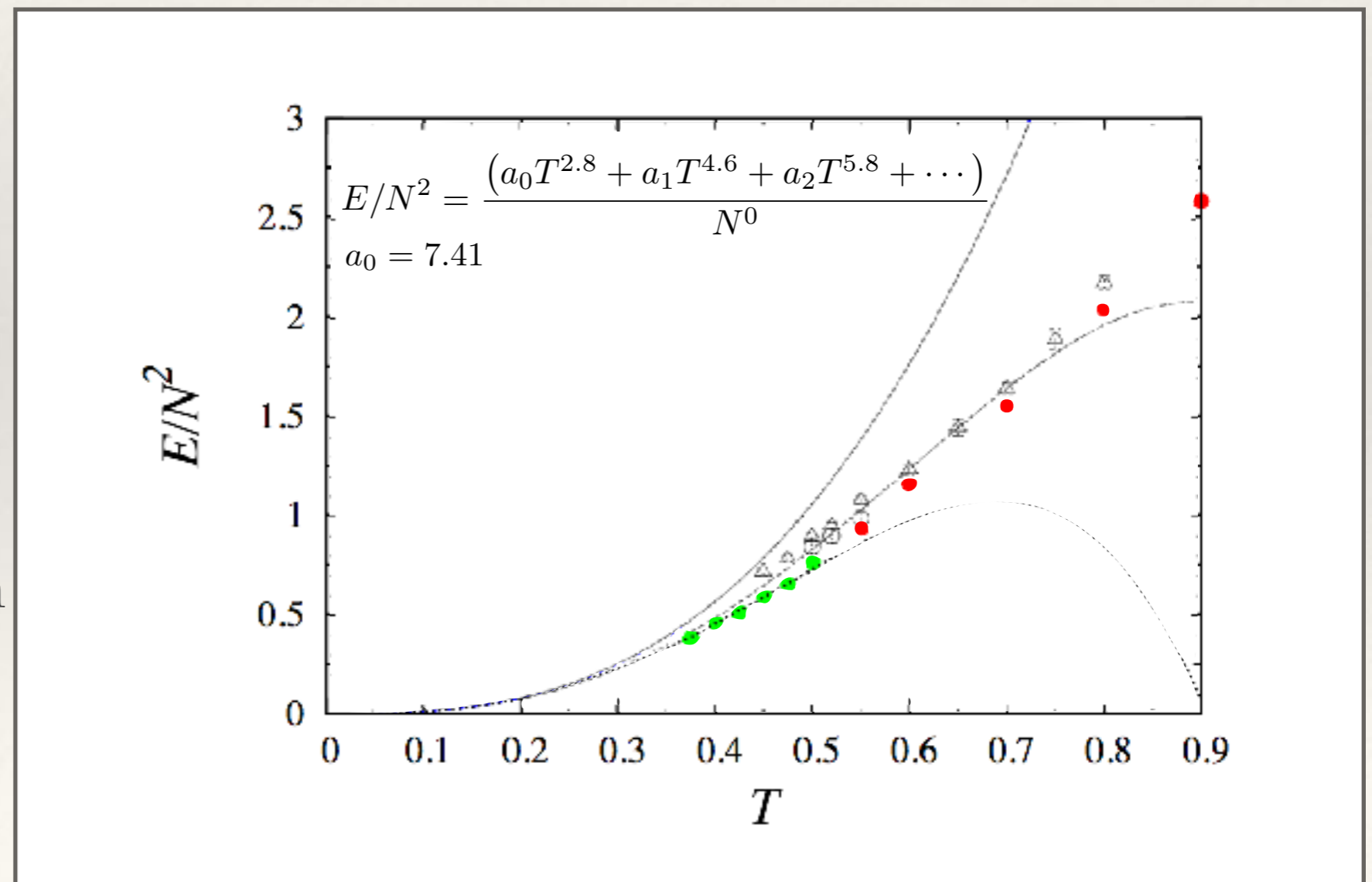
[Hanada et al. arxiv:0811.3102]

[Hanada et al. arxiv:1311.5603]

[Kadoh, Kamata arxiv:1503.08499]

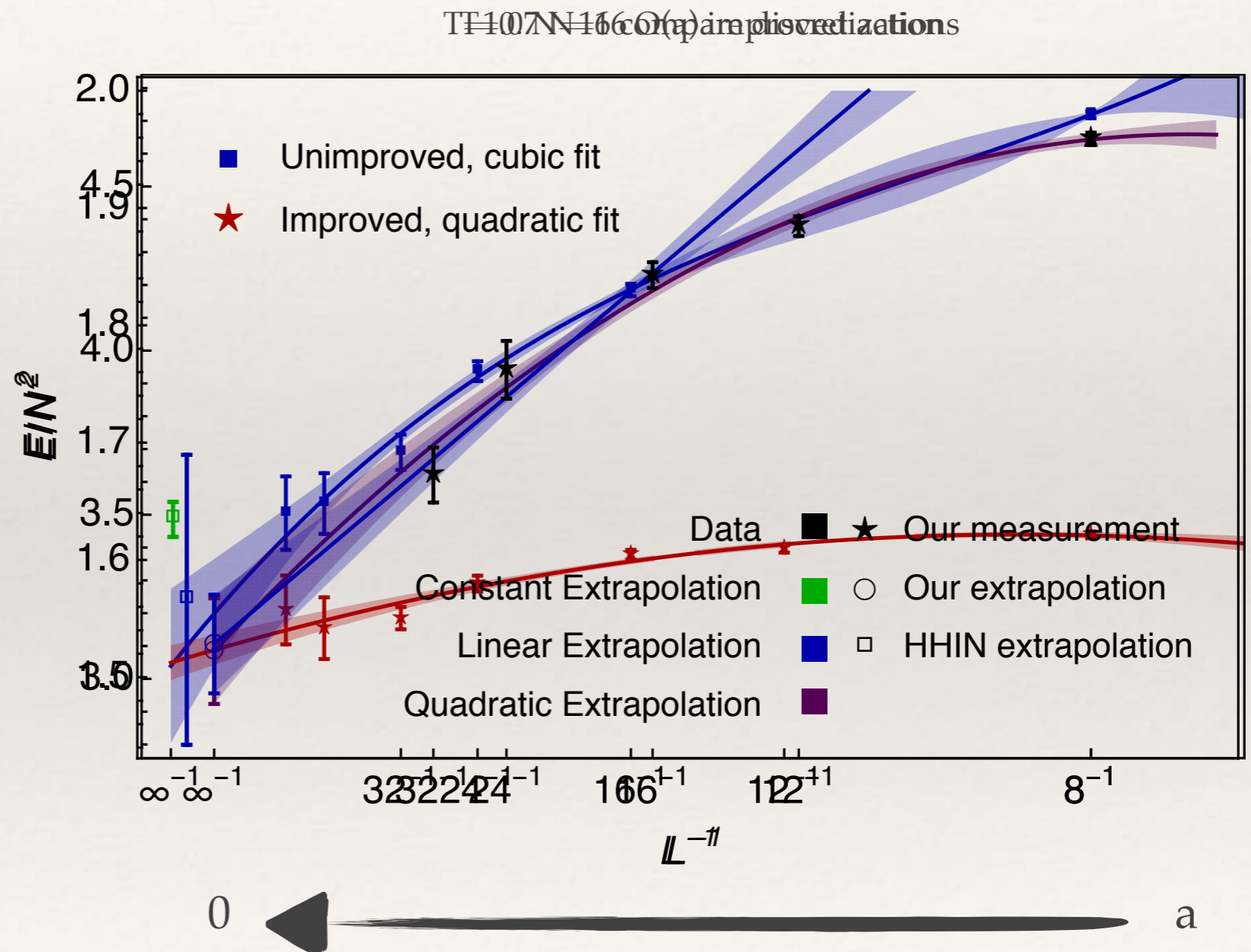
[Filev, O'Connor arxiv:1506.01366]

...



Remove regularization effects

- ❖ Numerical simulations are done at fixed cutoff e.g. a
- ❖ **Need to remove regulator effects**
- ❖ Different discretization forms give **consistent results**
- ❖ Compare with previous results in the literature
- ❖ Physics is independent of the regulator once cutoff effects are accurately removed

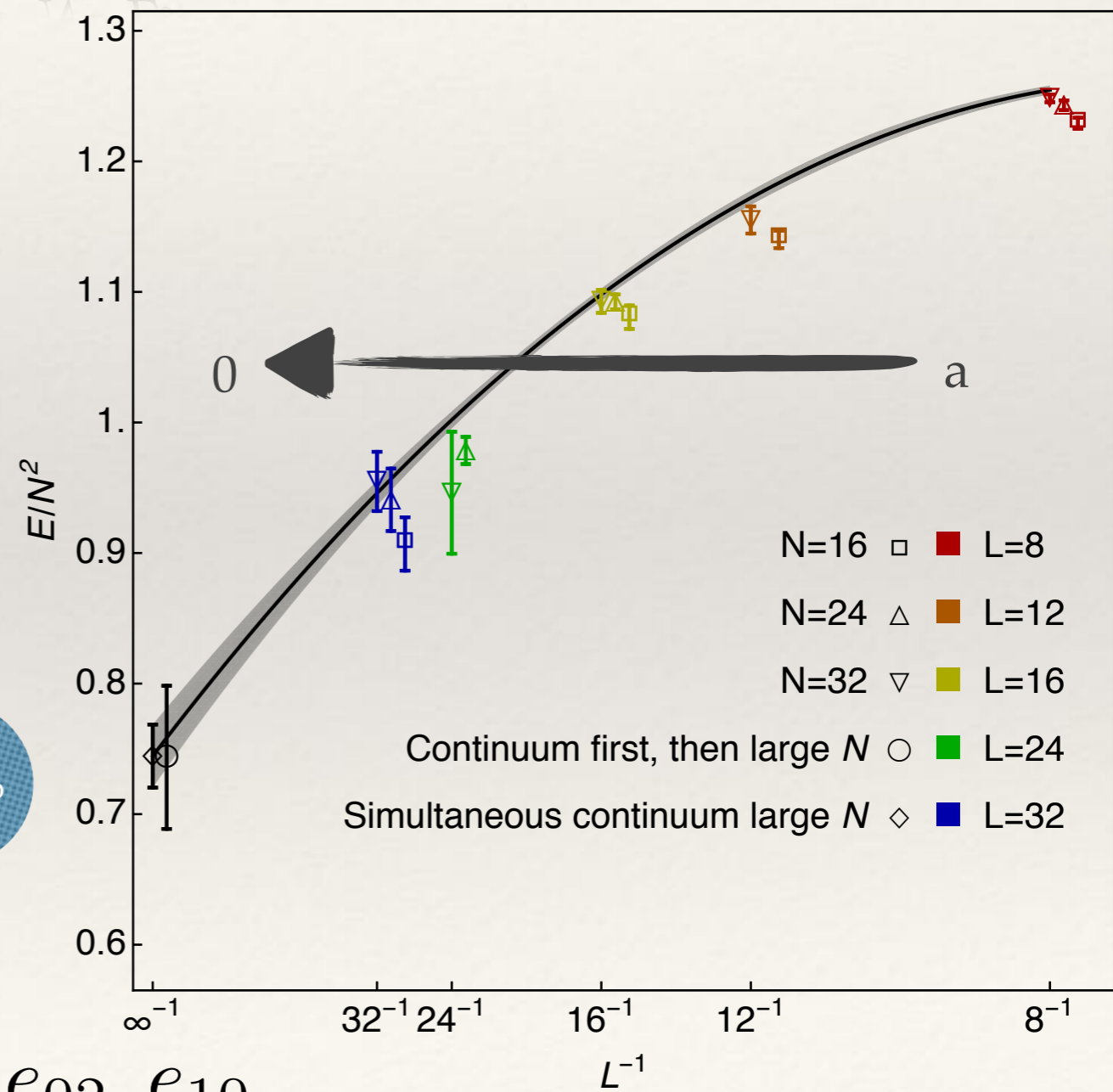
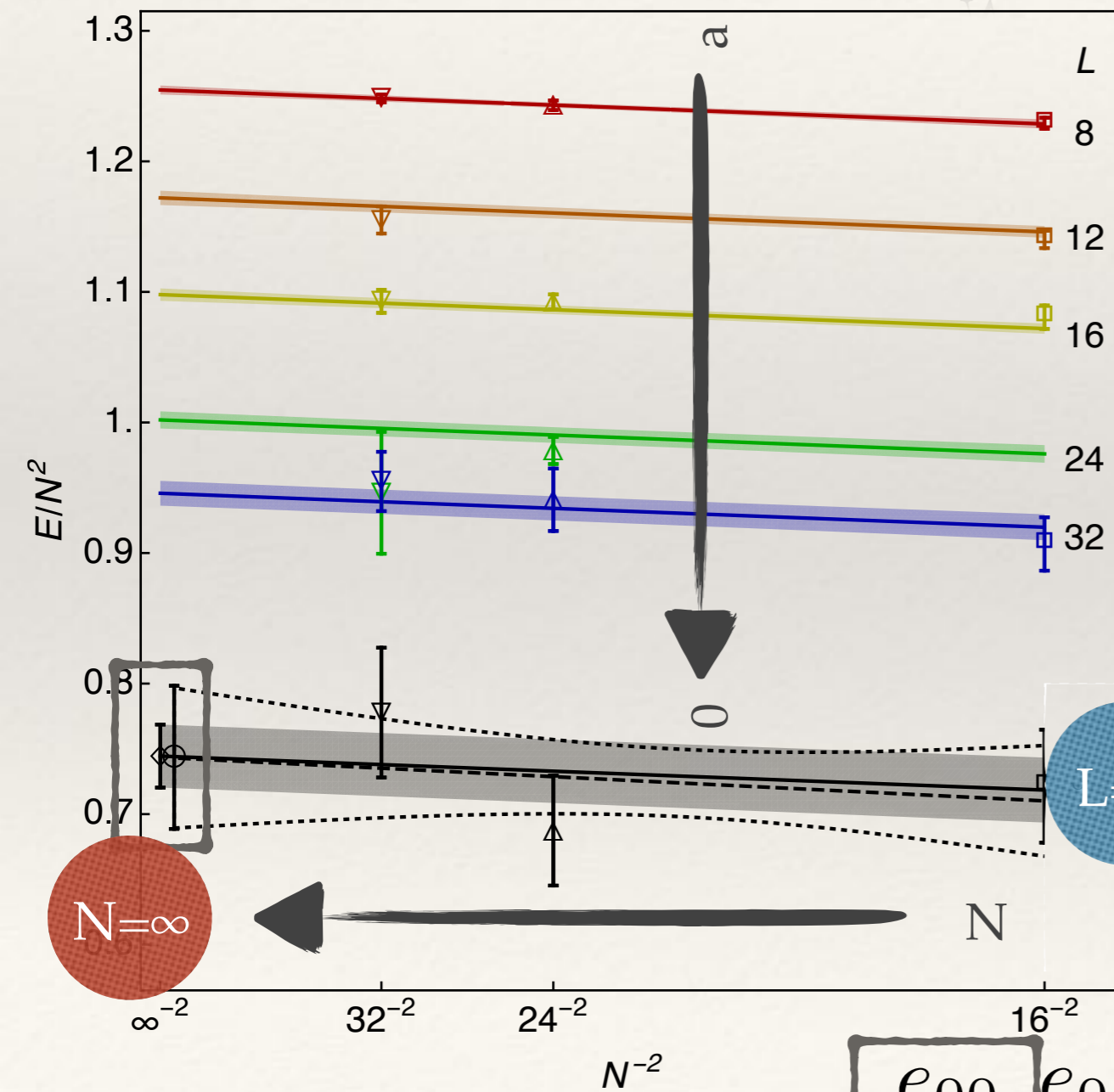


Towards the large-N limit

$$\frac{E}{N^2} = \sum_{\substack{ij \geq 0 \\ sj \geq 0}} \frac{e_{ij}}{N^{2i} L^j}$$

T=0.5 O(a) improved action

T=0.5 O(a) improved action



Extract predictions on the gauge side

❖ LO

$$E_0 = a_0 T^{2.8}$$

❖ NLO

$$E_0 = a_0 T^{2.8} + a_1 T^{4.6}$$

PREDICT

❖ NNLO

$$E_0 = a_0 T^{2.8} + a_1 T^{4.6} + a_2 T^{5.8}$$

❖ NNNLO

$$E_0 = a_0 T^{2.8} + a_1 T^{4.6} + a_2 T^{5.8} + a_3 T^{6.4}$$

❖ NLOP1(fixed LO)

$$E_0 = 7.41 T^{2.8} + a_1 T^{p_1}$$

❖ NNLOP1(fixed LO)

$$E_0 = 7.41 T^{2.8} + a_1 T^{p_1} + a_2 T^{p_1+1.2}$$

❖ NNLOP0

$$E_0 = a_0 T^{p_0} + a_1 T^{p_0+1.8} + a_2 T^{p_0+3}$$

TEST

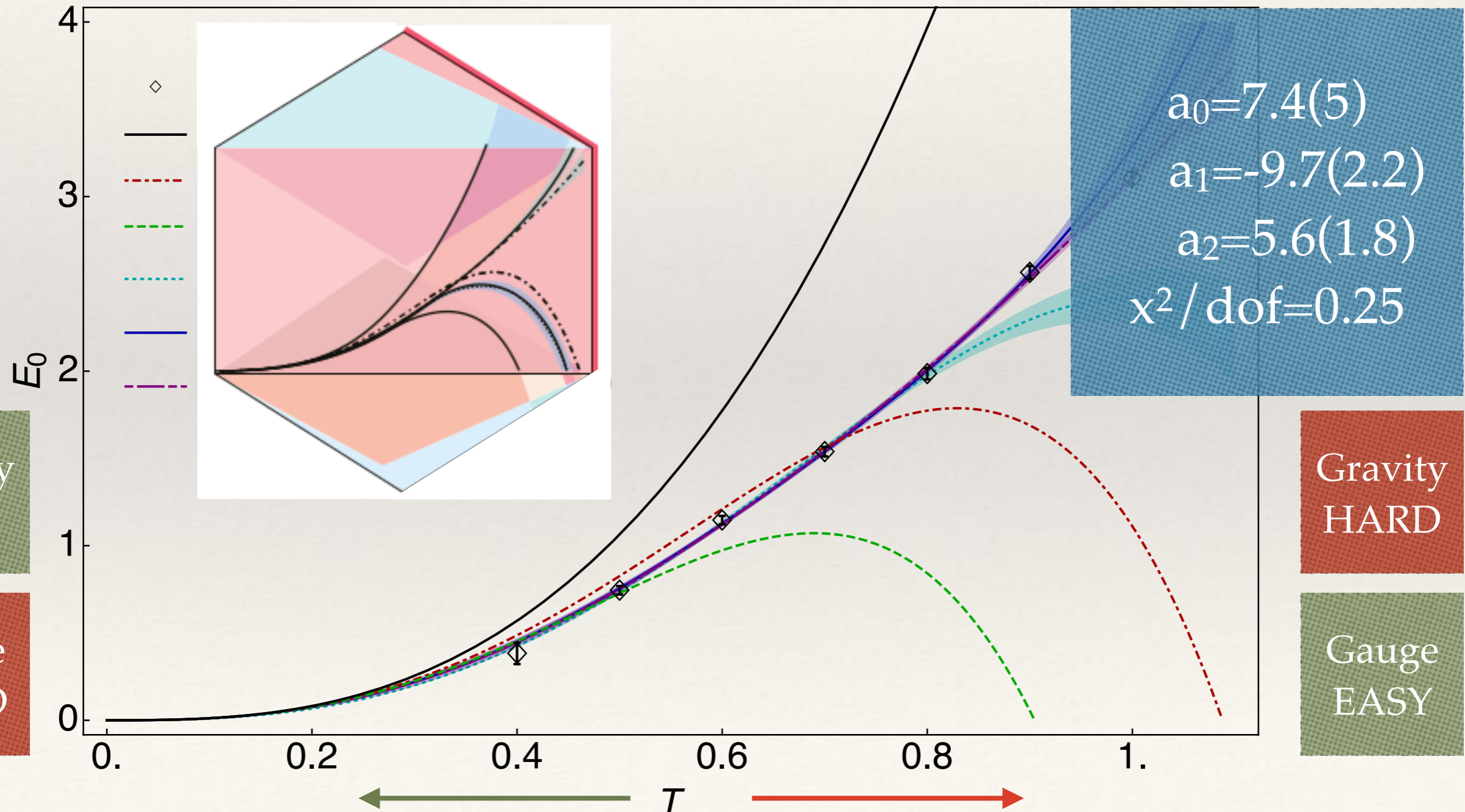
TEST

Large-N and continuum limit results

$$\frac{E}{N^2} = a_0 T^{2.8} + a_1 T^{4.6} + a_2 T^{5.8}$$

$a_0 = 7.41$ Supergravity

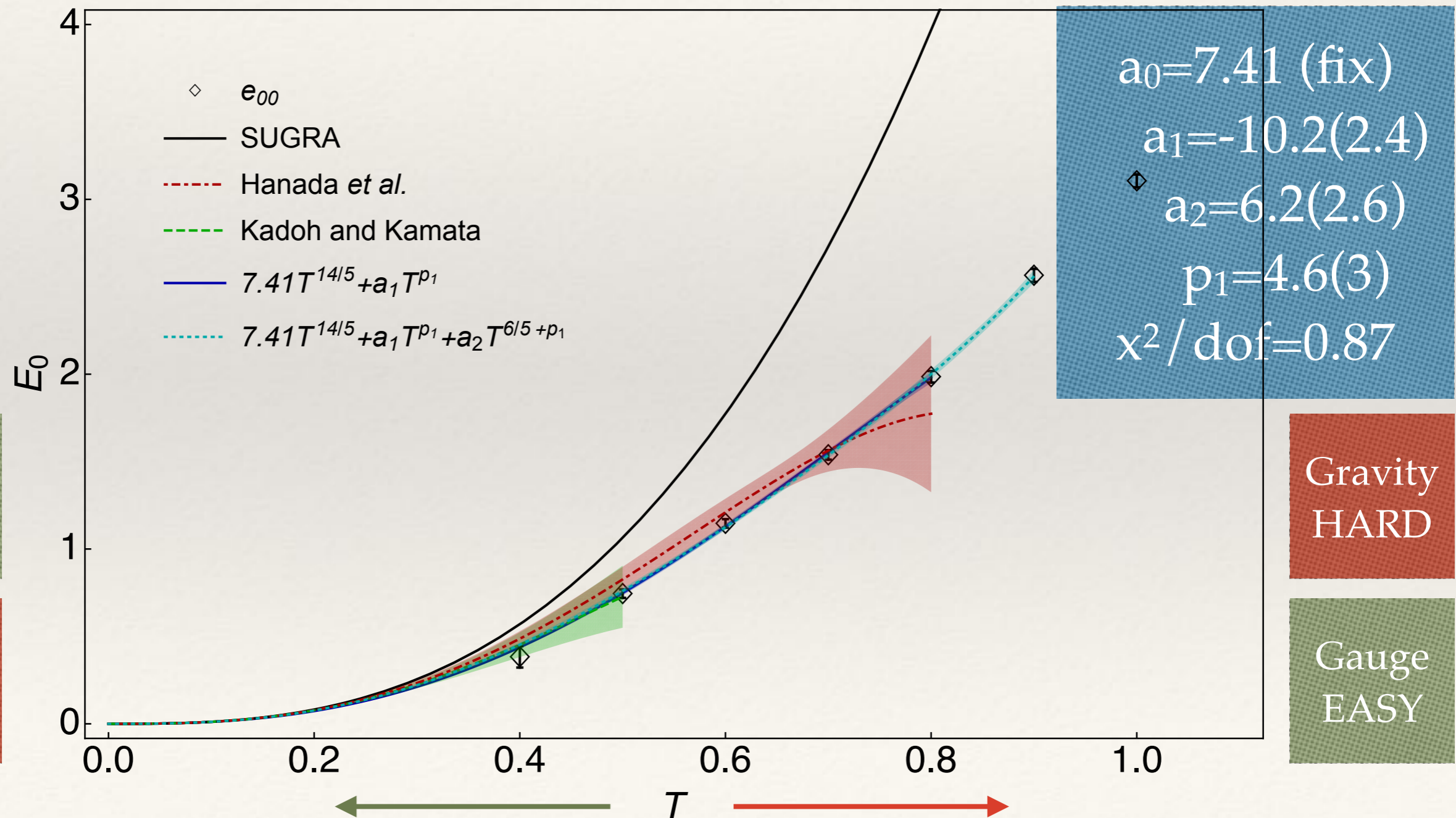
$a_1 = ?$ $a_2 = ?$ String effects



Large-N and continuum limit results

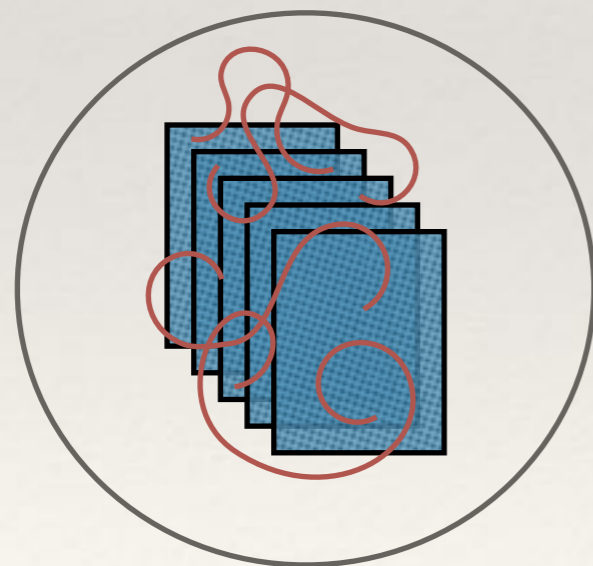
$$\frac{E}{N^2} = a_0 T^{2.8} + a_1 T^{p_1} + a_2 T^{p_1+6/5}$$

$a_0 = 7.41$ Supergravity
 $a_1 = ? \quad p_1 = 4.6$ String effects



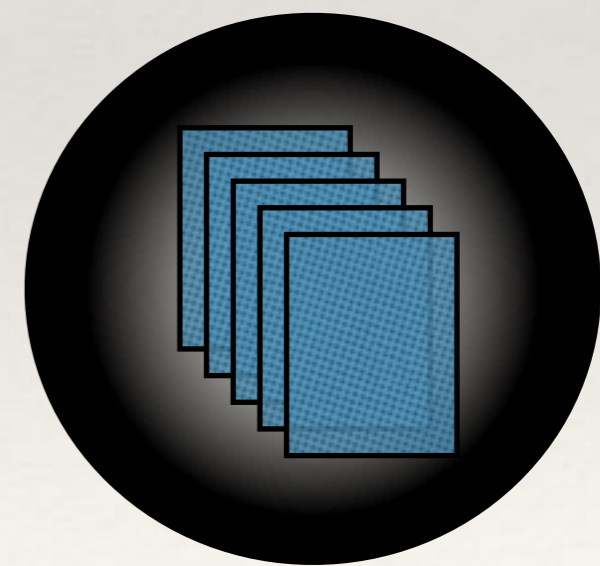
Geometry from gauge theories

Goal: determine how geometry emerges from the dual gauge theory



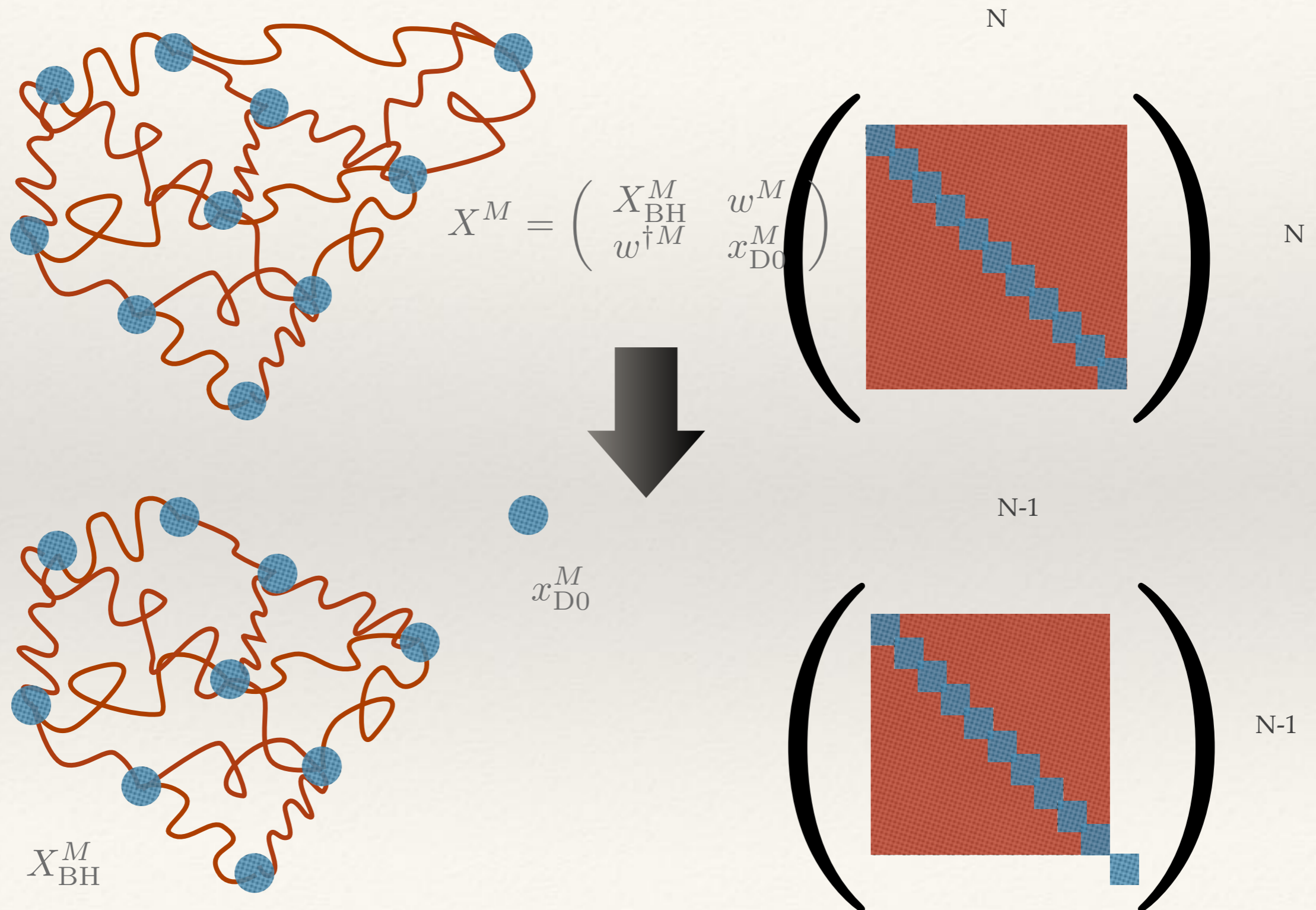
(p+1)-dim SYM gauge theory

- ★ interior
- ★ exterior
- ★ horizon
- ★ evaporation



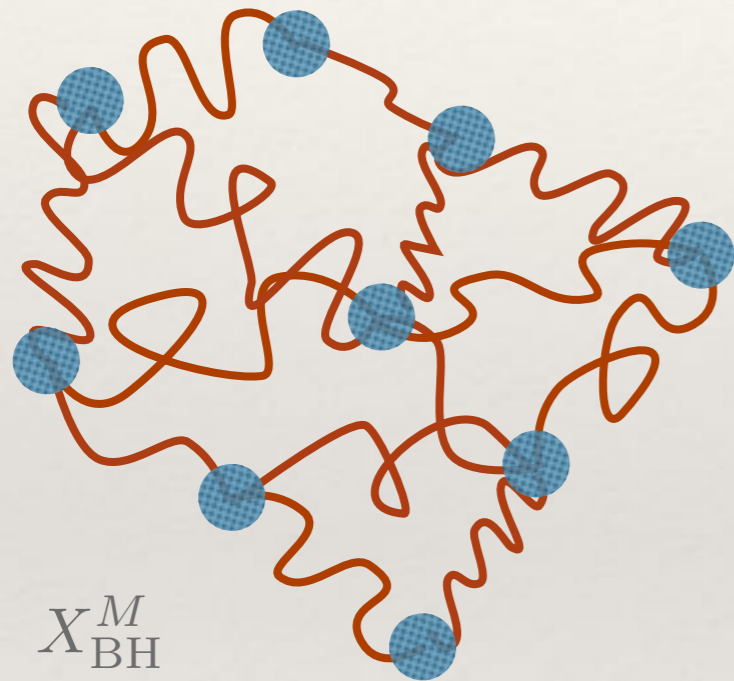
Black p-brane in 10D Supergravity

Numerical approach



“Probing” the force

$$X^M = \begin{pmatrix} X_{\text{BH}}^M & w^M \\ w^{\dagger M} & x_{\text{D0}}^M \end{pmatrix}$$



Probe

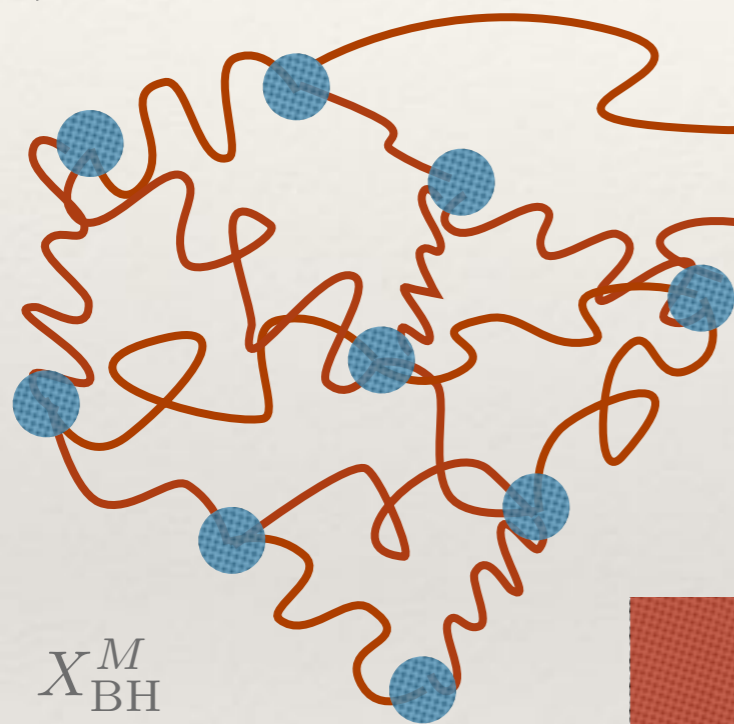


distance
e.g. fix $M=1$

$$r \equiv \sqrt{\sum_{M=1}^9 \left(\frac{\text{Tr} X_{\text{BH}}^M}{N-1} - x_{\text{D0}}^M \right)^2}$$

“Measure” the force

$$r \equiv \sqrt{\sum_{M=1}^9 \left(\frac{\text{Tr} X_{\text{BH}}^M}{N-1} - x_{\text{D}0}^M \right)^2}$$



$$F(N, r_0; c) = -2 c (r - r_0)$$

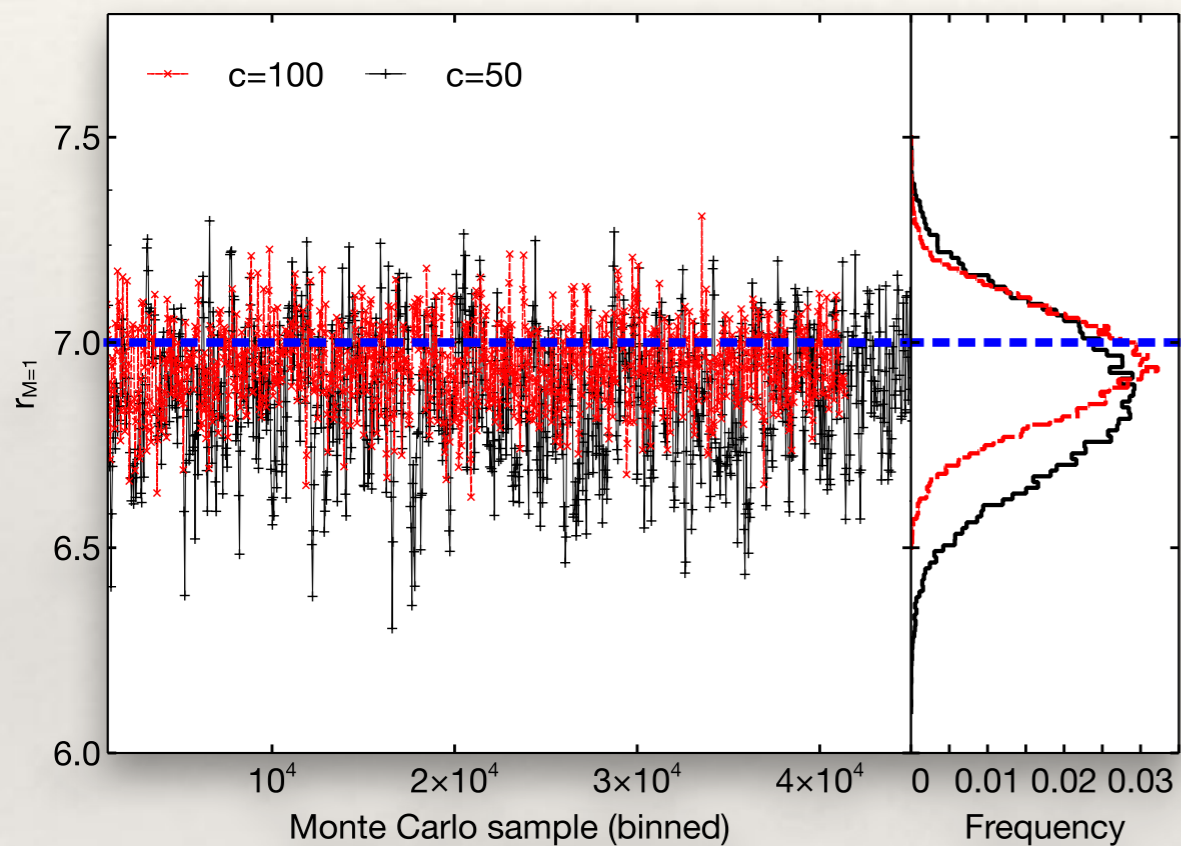
“potential”



$$\Delta L = -c \left\{ \left(\frac{\text{Tr} X_{\text{BH}}^1}{N-1} - x_{\text{D}0}^1 - r_0 \right)^2 + \sum_{M=2}^9 \left(\frac{\text{Tr} X_{\text{BH}}^M}{N-1} - x_{\text{D}0}^M \right)^2 \right\} - c' |w_1|^2$$

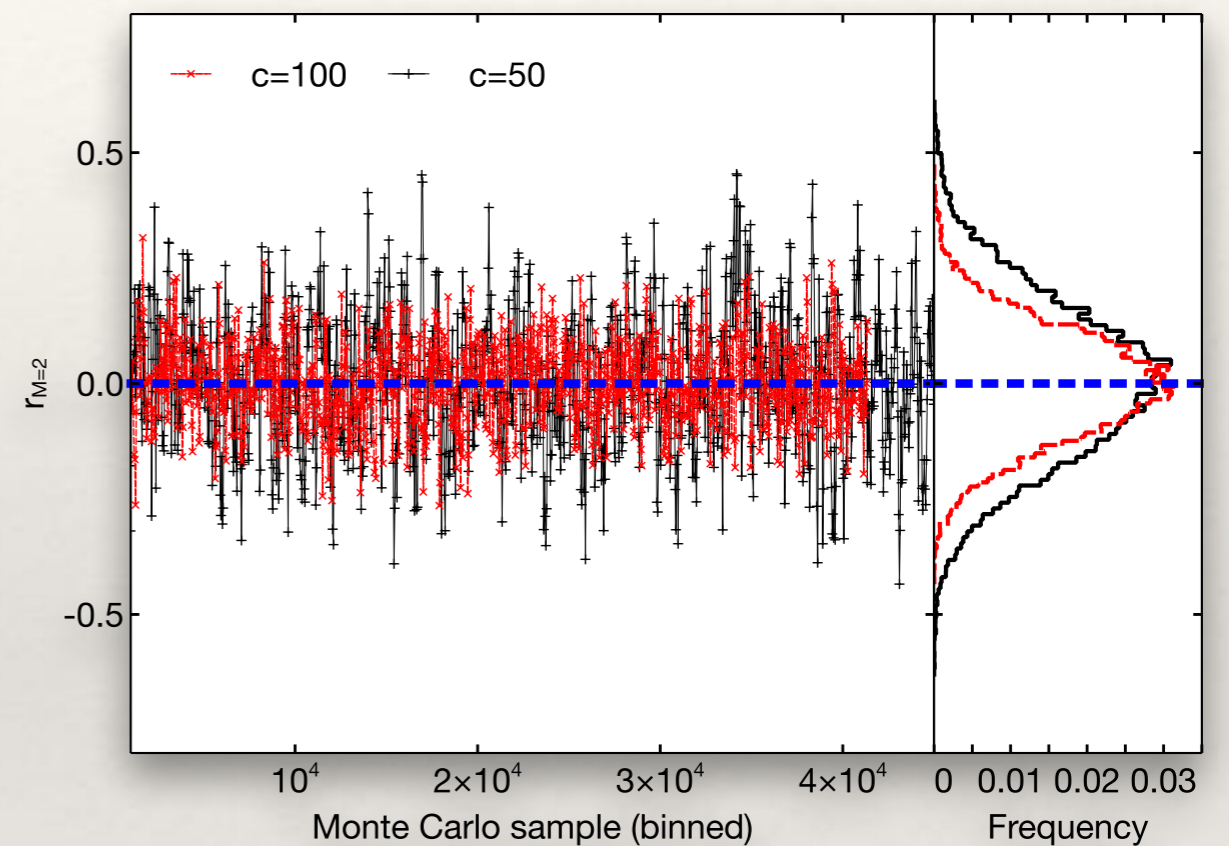
Numerical demonstration

distance
in $M=1$ direction



$$r_0 = 7.0$$

distance
in $M=2$ direction

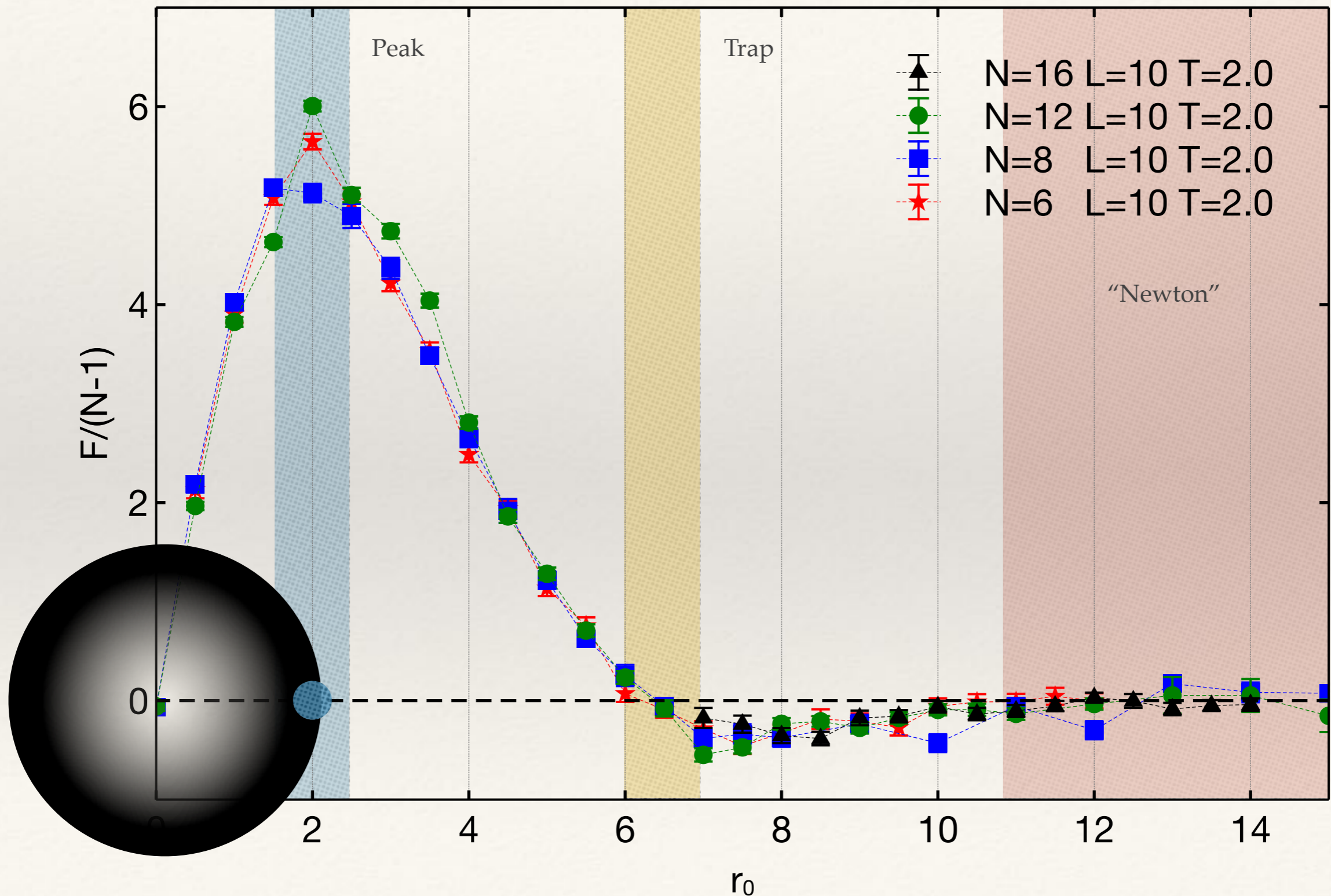


$$r_0 = 0.0$$

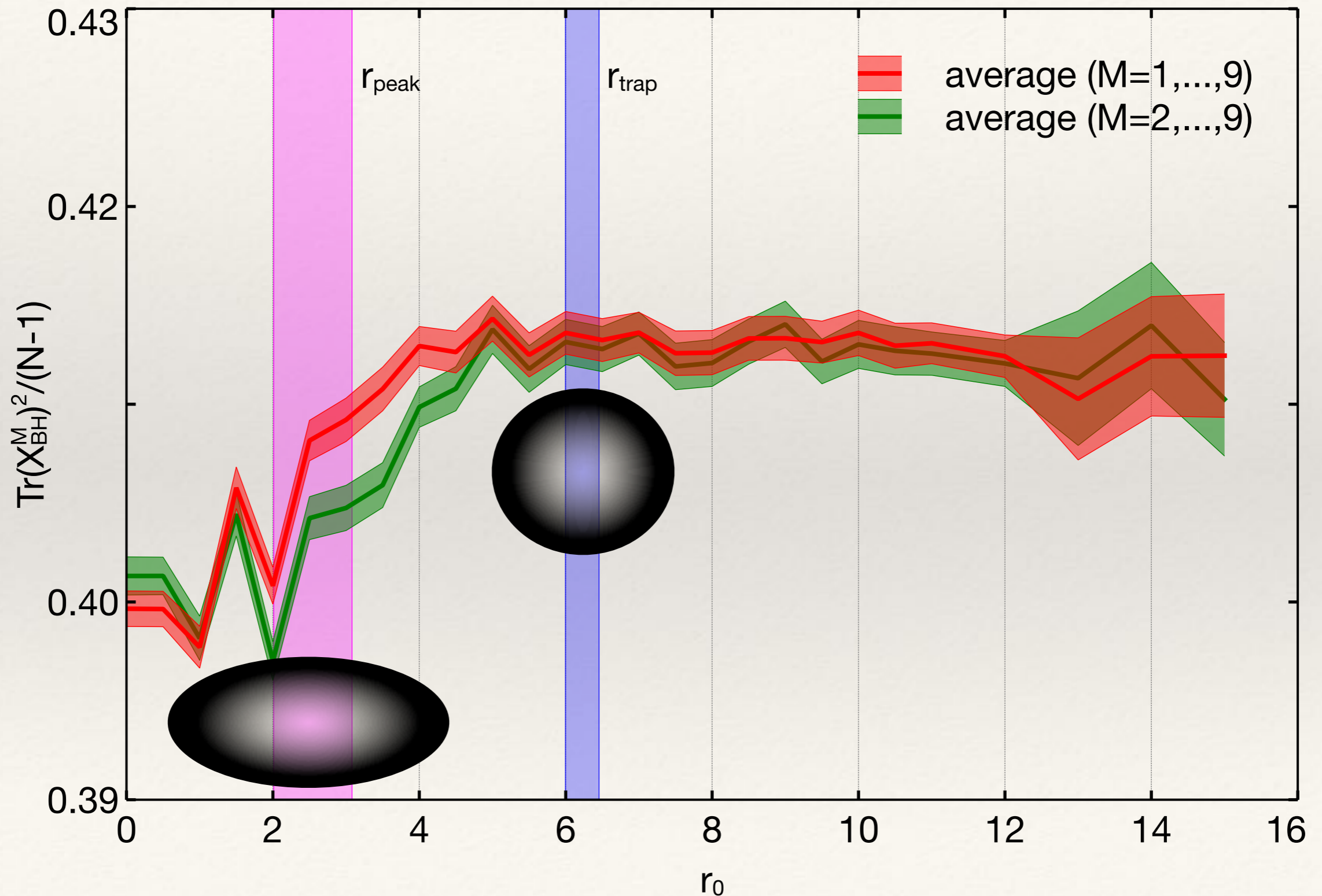
$$c = 100$$

$$c = 50$$

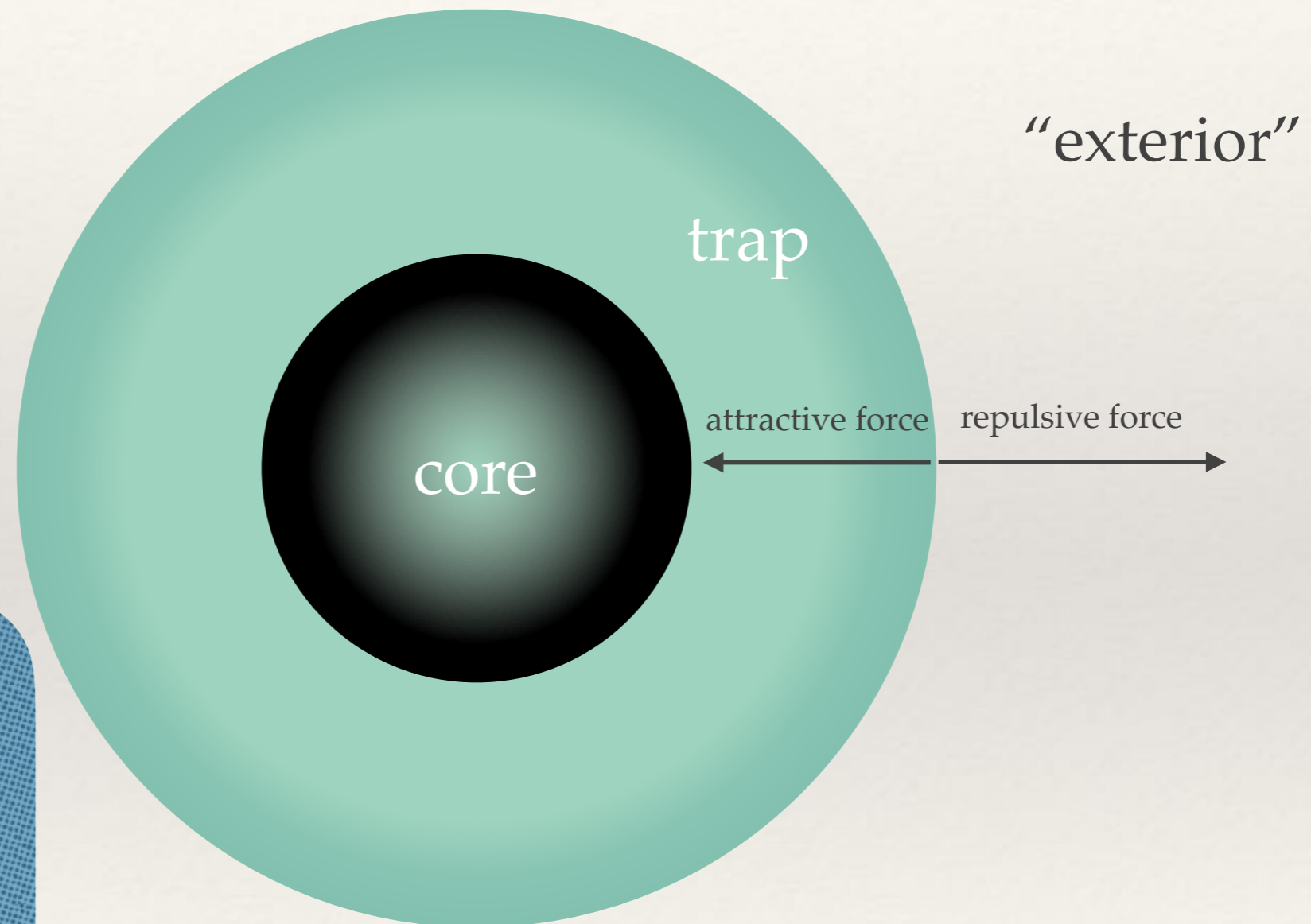
Lattice Monte Carlo results



Core radius



Possible interpretations



Caveats:

- not accounting the potential
- no continuum limit
- no large-N limit

~~trap~~ = horizon

[see also: Filev, O’Connor arXiv: 1512.02536, 1612.09281]

Summary and future work

- ❖ Testing the gauge / gravity duality is the first step towards using it to define a quantum theory of gravity
- ❖ Numerical tests are now mature for low-dimensional systems: control over continuum and planar limit (only for $p=0$)
- ❖ Lower temperatures are needed to directly extract the leading SUGRA behavior from data (unless assumptions are made)
- ❖ New numerical simulations can help extracting bulk geometrical properties of gravity theories
- ❖ New conjectures can be tested non-perturbatively in relatively short time [see MCSMC 1802.02985 and Maldacena, Milekhin 1802.00428]

thanks to my collaborators and funding sources

Masanori Hanada



Evan Berkowitz

Pavlos Vranas



RIKEN BNL Research Center

Goro Ishiki

Shinji Shimasaki

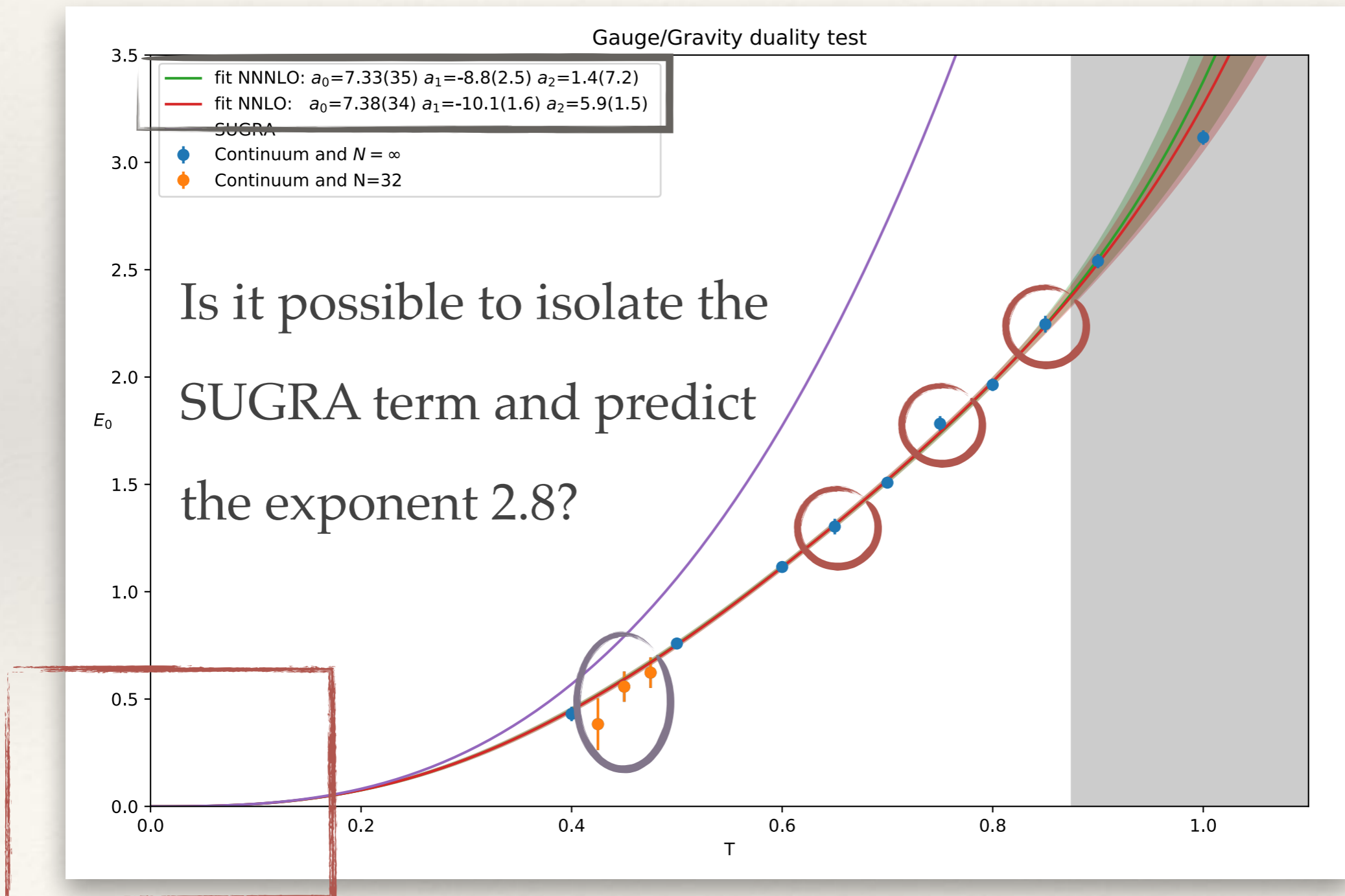


Jonathan Maltz

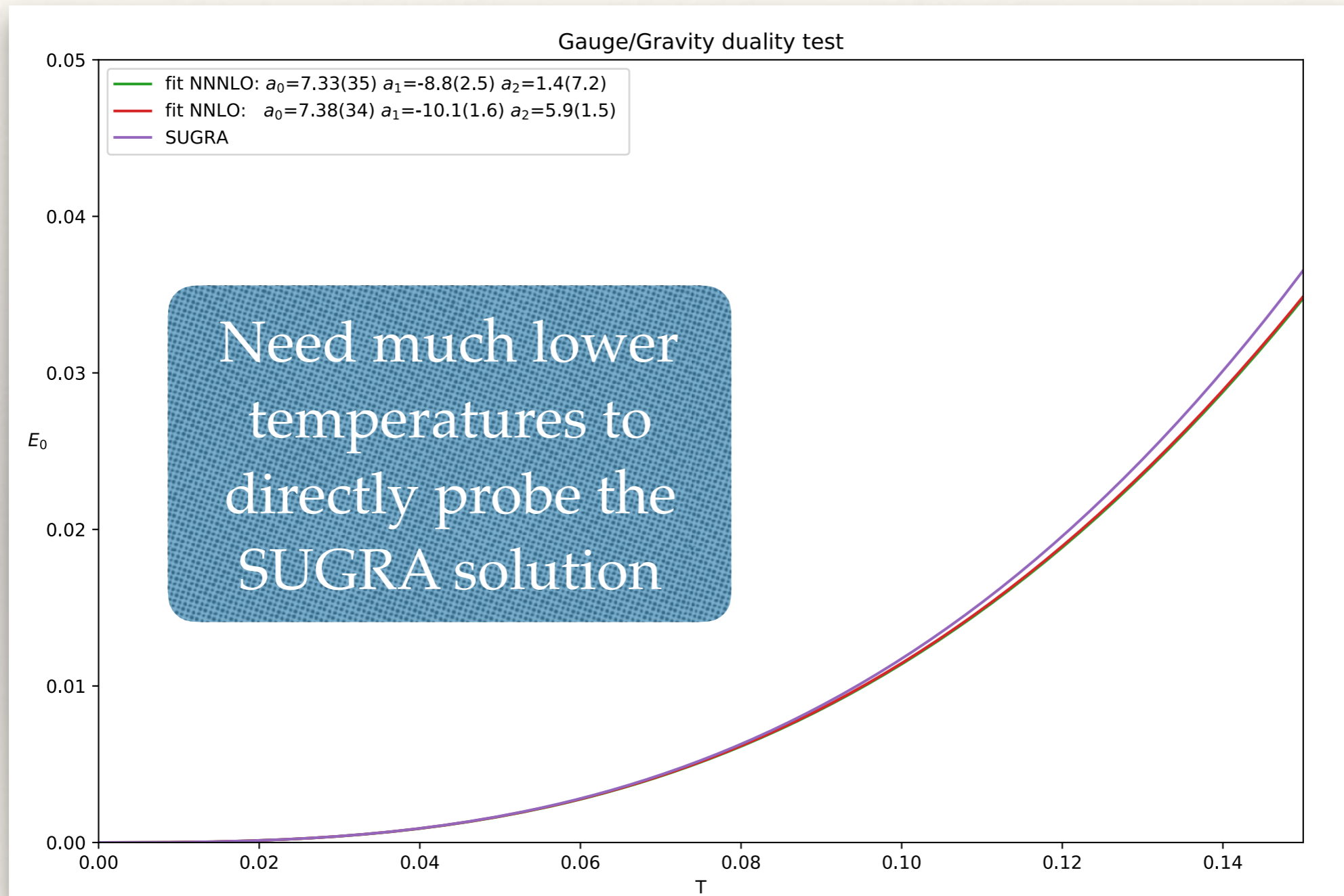
Extra slides

Preliminary new results

$$\frac{E}{N^2} = a_0 T^{2.8} + a_1 T^{4.6} + a_2 T^{5.8}$$



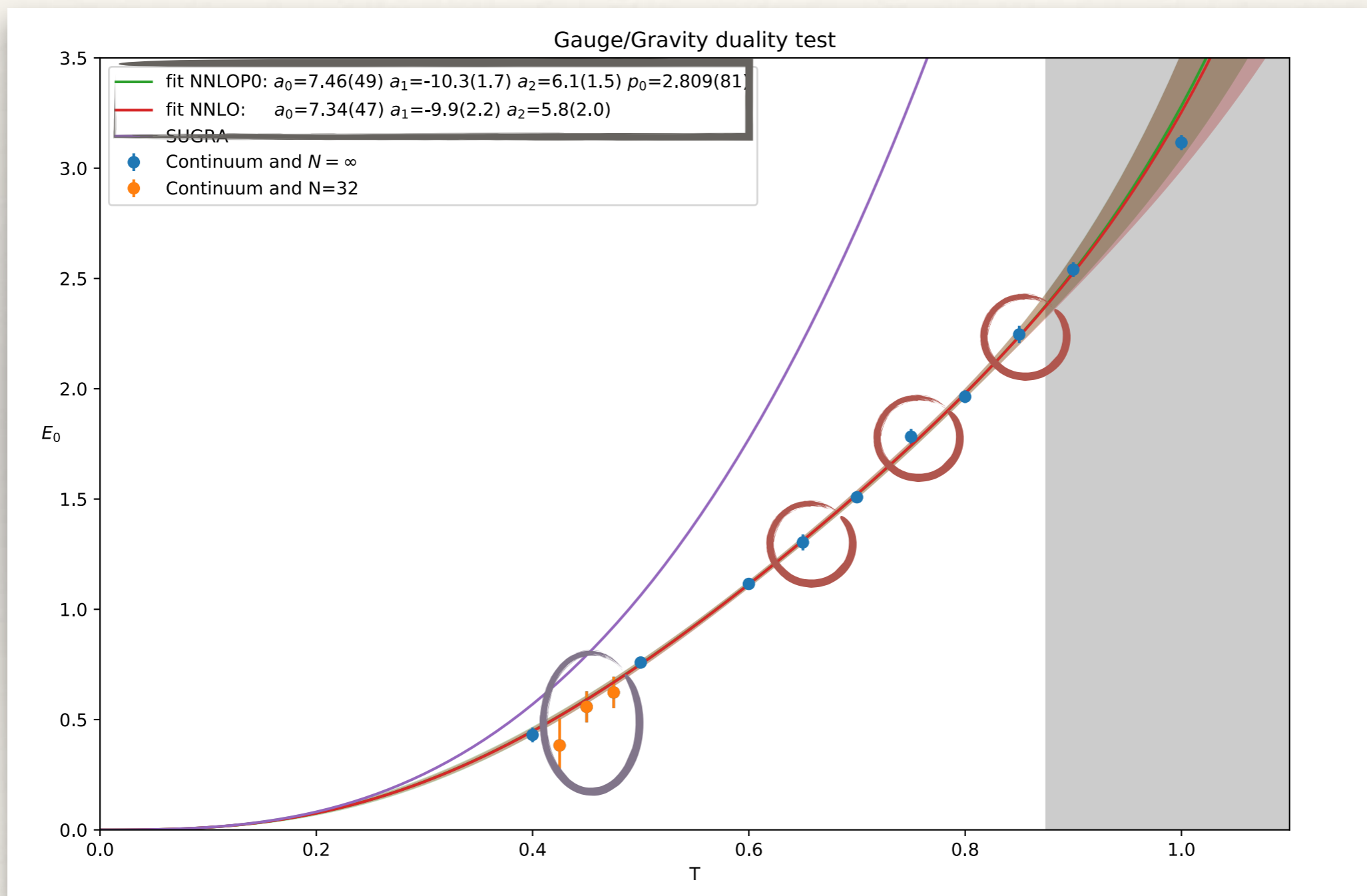
Preliminary new results



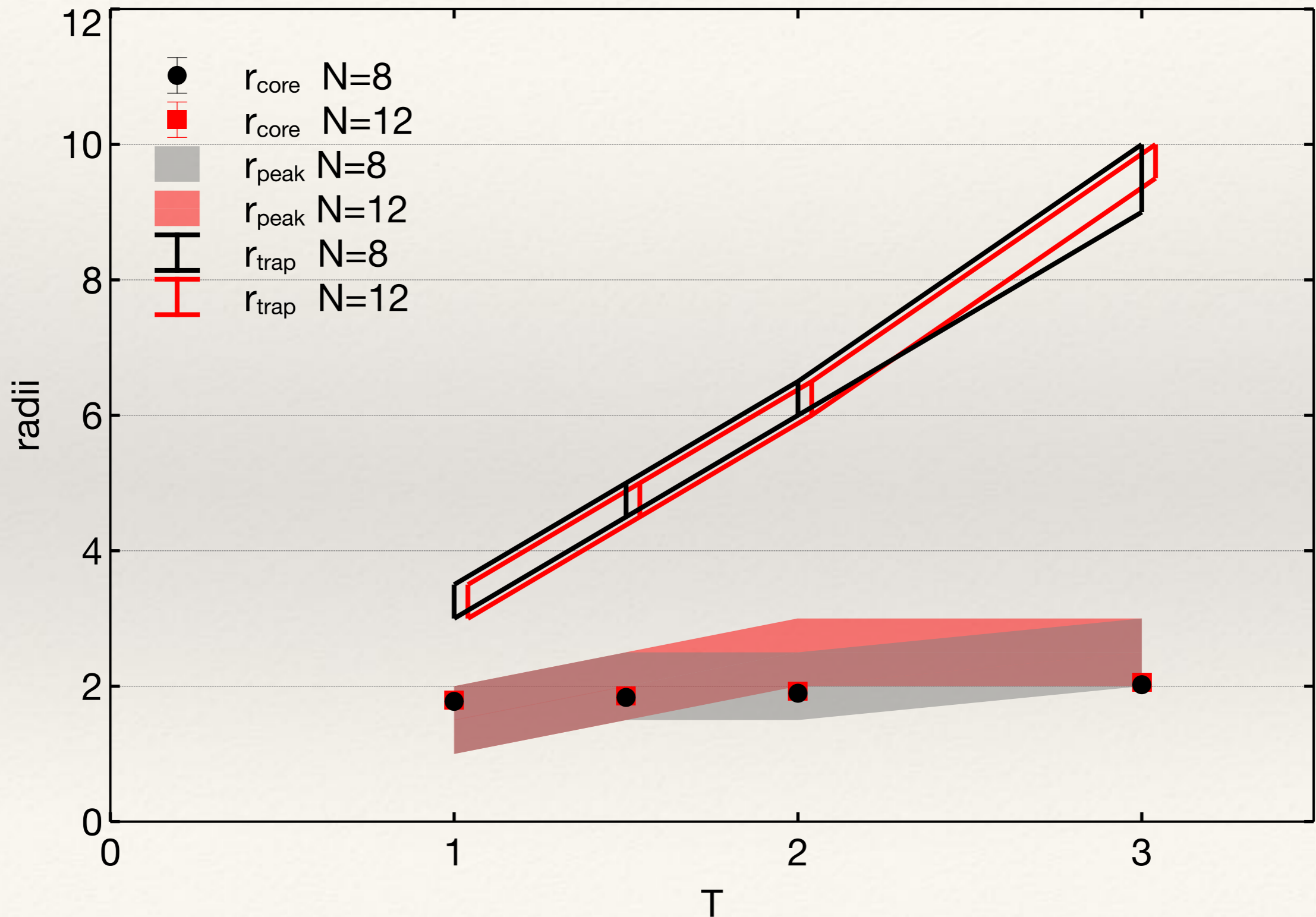
Preliminary new results

$$\frac{E}{N^2} = a_0 T^{p_0} + a_1 T^{p_0+1.8} + a_2 T^{p_0+3}$$

- Bayesian priors are used
- Assumptions on the low-energy gravity theory are used



Temperature dependence



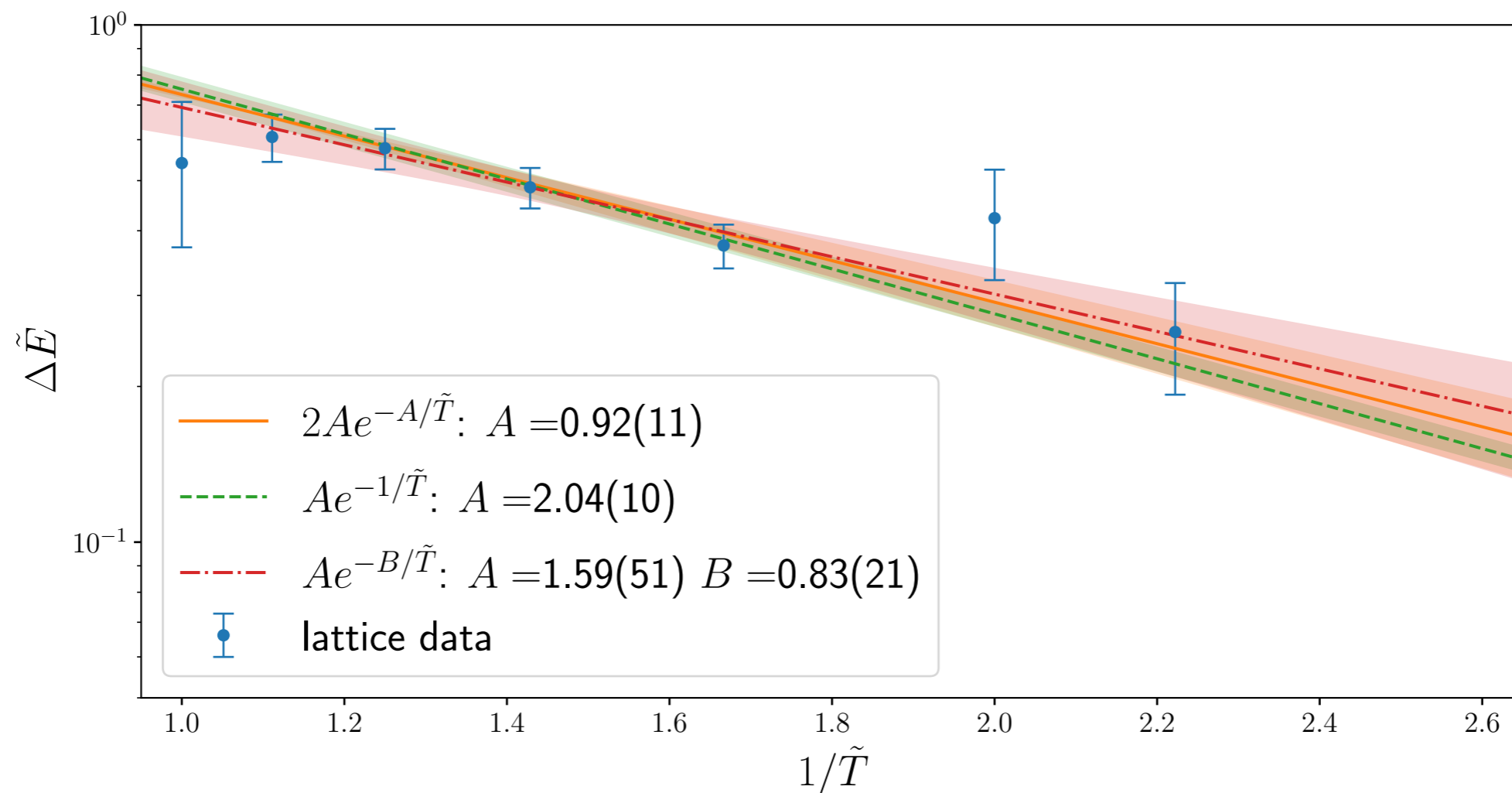
“to gauge or not to gauge”

“un” gauge / gravity
duality

- ★ D0-brane matrix model (dual)
- ★ $SU(N)$ symmetry is broken (quantum mechanics)
- ★ Remove gauge singlet constraint (Gauss Law)
- ★ Introduce non-singlet states
- ★ Effects on non-singlets are suppressed at low energy
- ★ Where the gravity solution is valid, gauge singlets dominates

Easier to simulate on a
quantum computer!

Energy difference

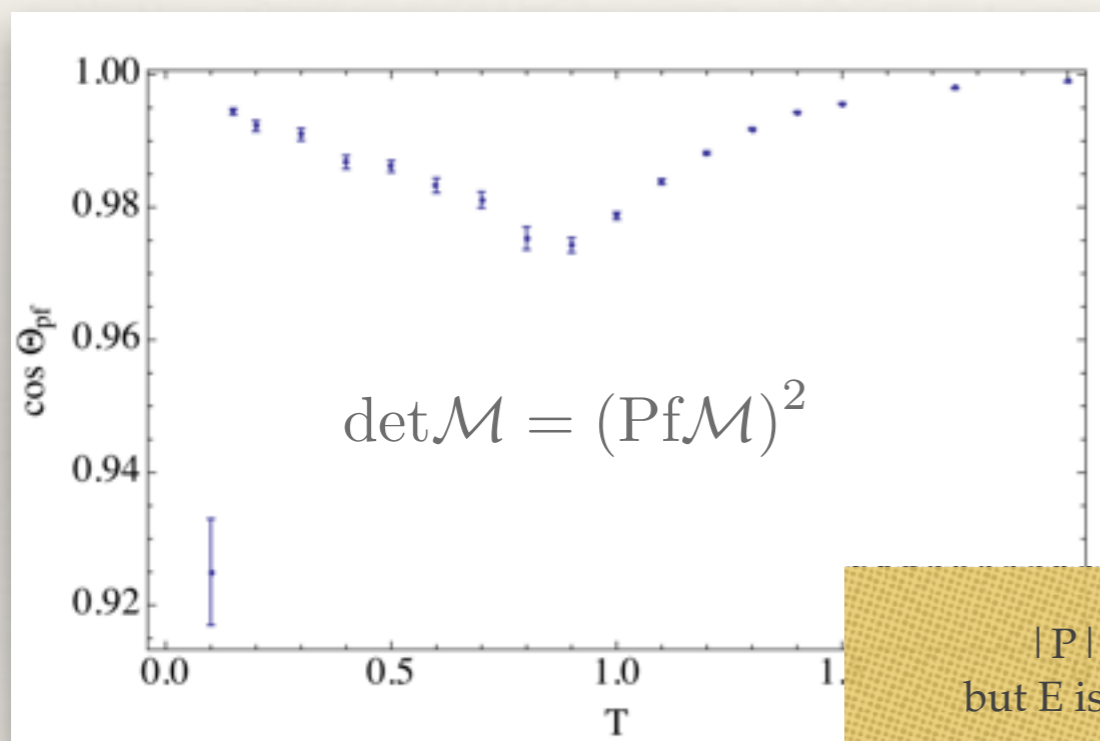


$$\Delta E = E_{\text{ungauged}} - E_{\text{gauged}} = d_{\text{adj}} C_{\text{adj}} N^2 e^{-C_{\text{adj}}/T} + \dots$$

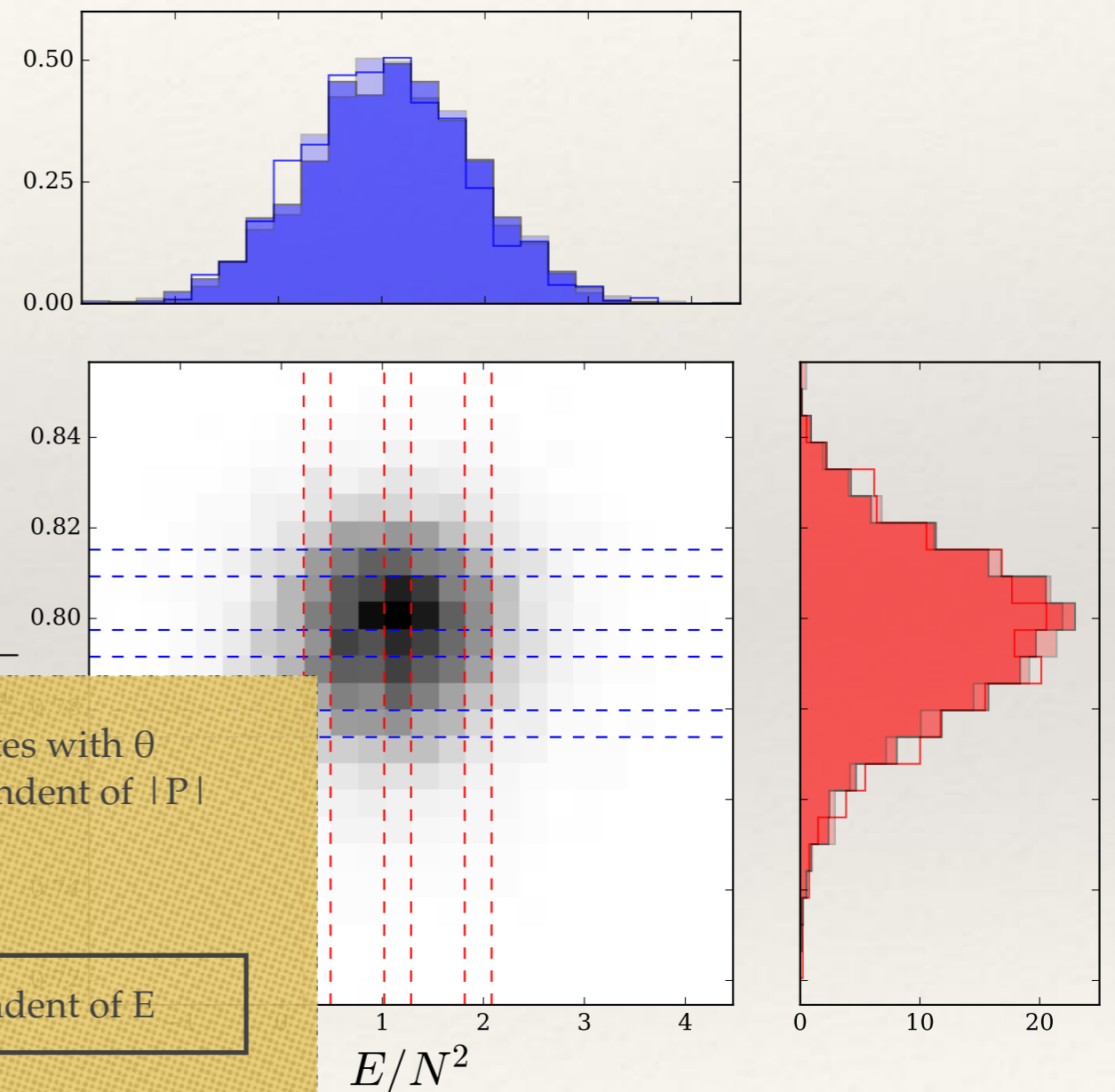
Challenge 2: Pfaffian

phase reweighting: $\langle \mathcal{O} \rangle_F = \frac{\langle \mathcal{O} \cdot e^{i\theta} \rangle_{PQ}}{\langle e^{i\theta} \rangle_{PQ}}$

N=3 L=4 (+ small regulator mass)



T=0.5 N=16 L=16 O(a) improved action



|P| correlates with θ
but E is independent of |P|

↓

θ is independent of E

No correlations between |P| and E

$$\langle \mathcal{O} \rangle_{PQ} = \int dx x \rho(x)$$

$$\langle \mathcal{O} \cdot e^{i\theta} \rangle_{PQ} = \int dx x \rho(x) w_x$$

$$\langle e^{i\theta} \rangle_{PQ} = \int dx \rho(x) w_x$$

Analytical expectations at finite T

$$E/N^2 = \frac{(a_0 T^{2.8} + a_1 T^{4.6} + a_2 T^{5.8} + \dots)}{N^0} + \frac{(b_0 T^{0.4} + b_1 T^{2.2} + \dots)}{N^2} + \mathcal{O}\left(\frac{1}{N^4}\right)$$

small T, infinite N, strong coupling

$$a_0 = 7.41$$

finite T, infinite N, finite coupling

$$\ell_s^6 \dots$$

Include effects of finite string length

small T, finite N, strong coupling

$$b_0 = -5.77$$

Include effects of finite string coupling

small T, finite N, strong coupling

$$\sim \ell_s^{12} g_s^2$$

quantum string corrections

