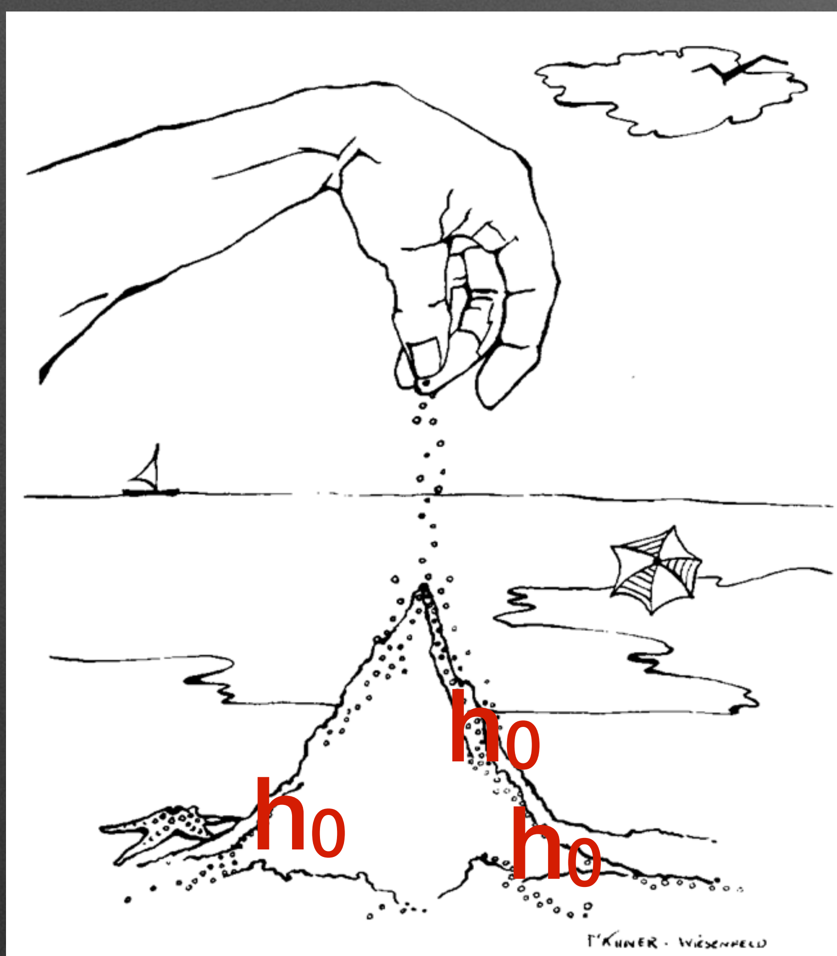
Self-Organized Higgs Criticality



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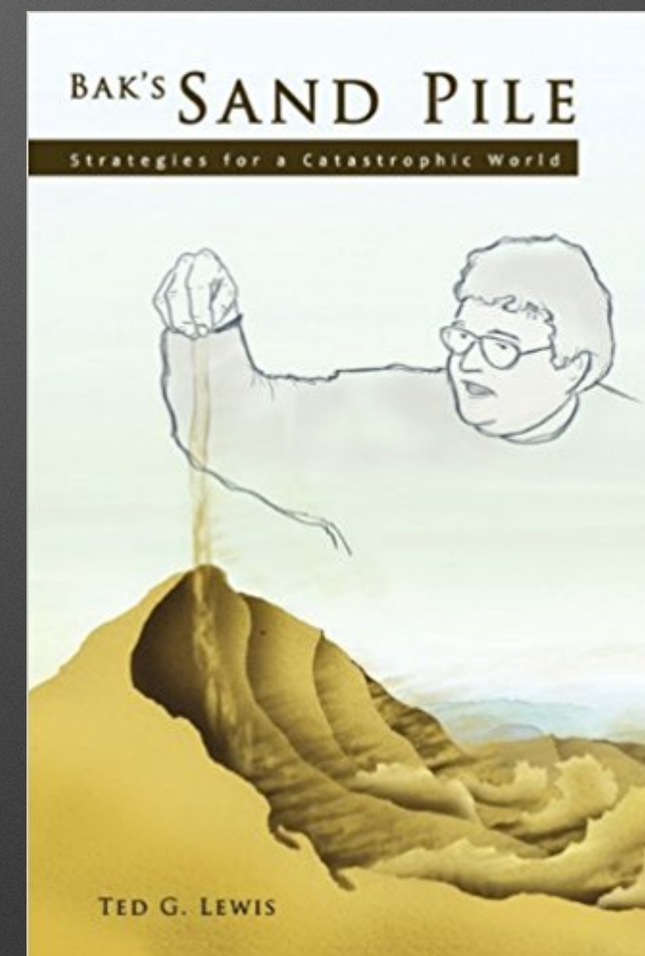
Syracuse University



Lattice for BSM Physics 2018

Eröncel, JH, Rigo 1804.00004 [hep-ph]

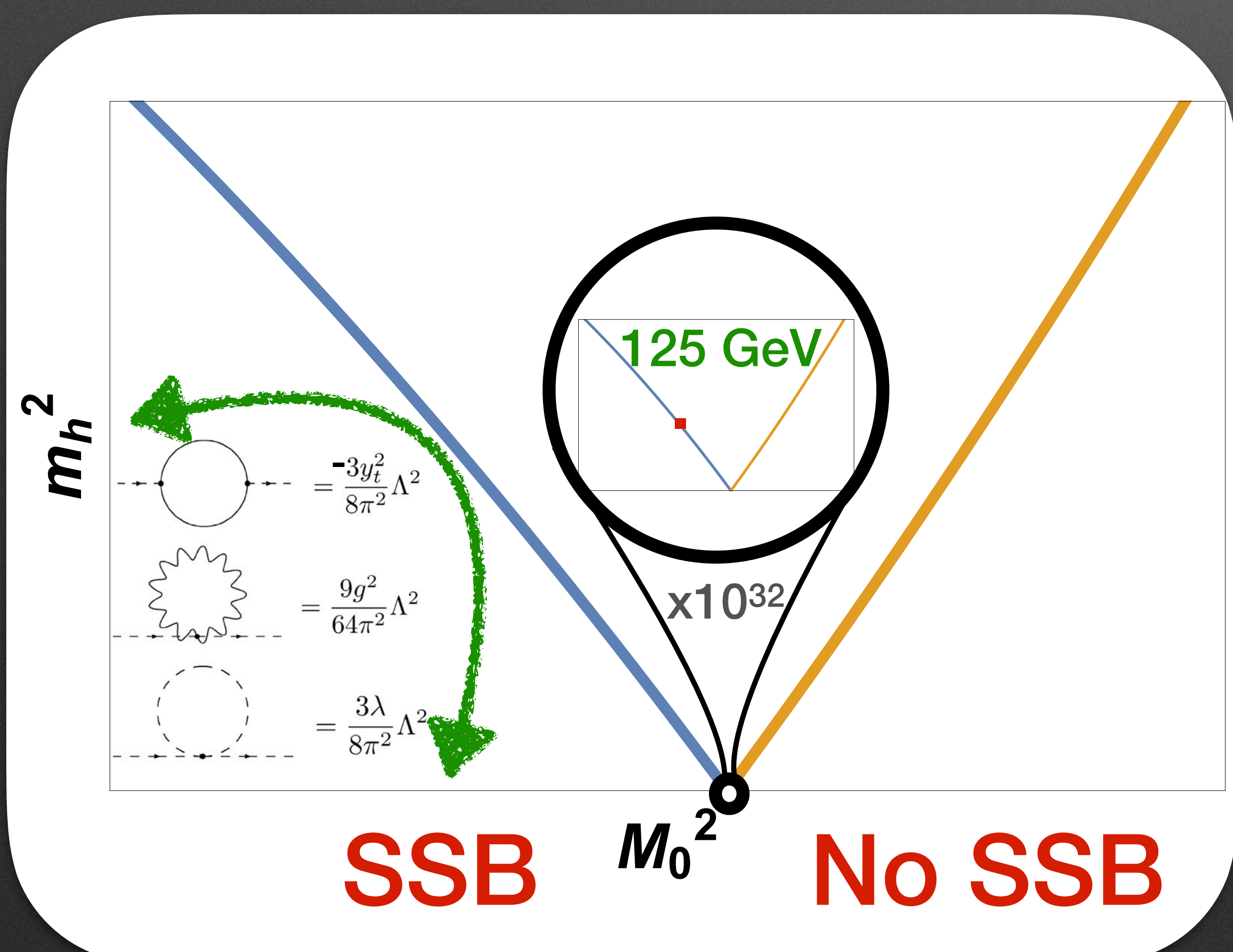
U.C. Boulder, April 5

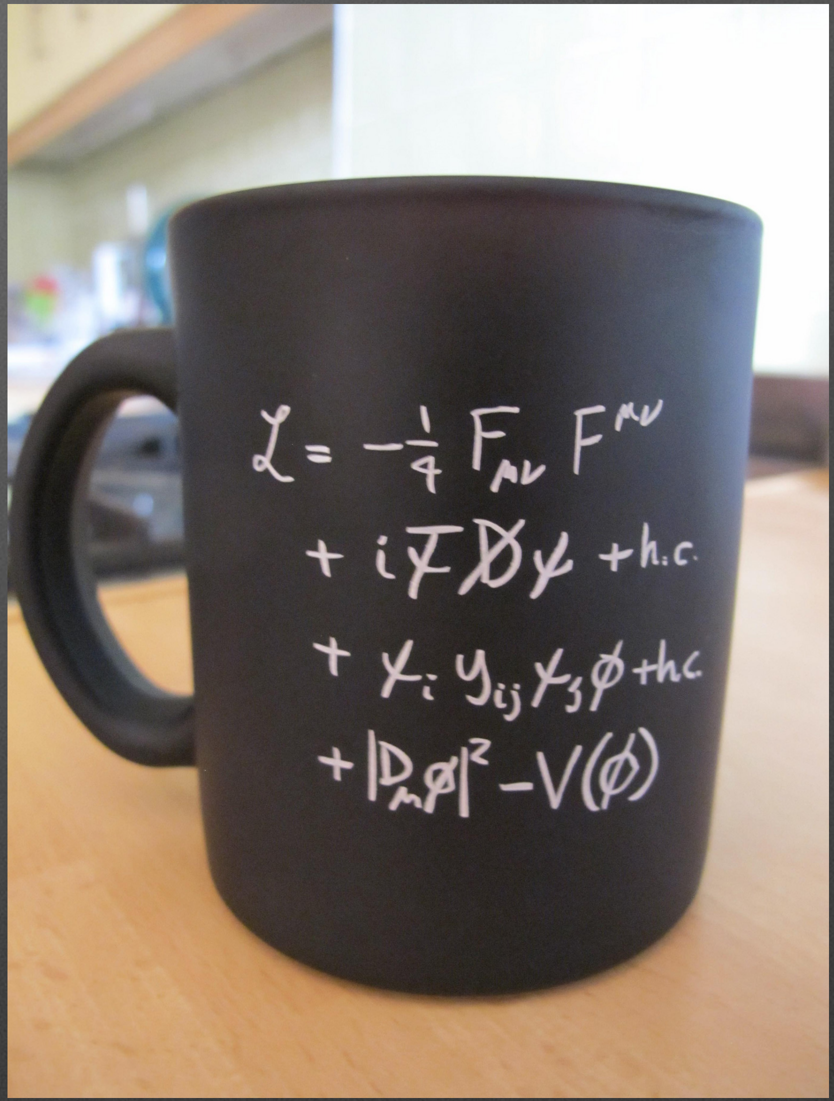
The Standard Model Higgs is Fine Tuned

We write this in the microscopic theory:

$$V(\phi) = M_0^2|\phi|^2 + \frac{\lambda_0}{4}\left(|\phi|^2 - v^2\right)^2$$

Defined at some high (physical) mass scale like Planck or GUT





Our SM is outrageously close to a **critical point** that appears thus far to be unprotected by symmetry

Self-organized criticality (SOC)?

Per Bak, Chao Tang, and Kurt Wiesenfeld (1988)

- In many pockets of the world, systems are naturally driven to (and through) critical points - Self Tuned Phase Transitions
  - Sandpiles - you keep on slowly adding sand, but system creates avalanches of grains to maintain same critical slope
  - Earthquake fault lines - techtonic drift builds force slowly, culminating in eventual slippage to new equilibrium point
  - Internal market pressures can create bubbles prior to financial crashes/“re-adjustment”

Commonality:

Slow forced (temporal) driving of system to a precipice of catastrophy - all length/time scales become important, perturbations exhibit scaling (critical exponents)

Giudice 2008 “Naturally Speaking” - Can this be part of why Higgs is light?

Complex Scaling Dimensions

- It has been guessed at that (at least some) systems with SOC exhibit log-periodicity at threshold of catastrophy (e.g. work of Didier Sornette et. al.)
- The log-periodic power law is the signature of discrete scale invariance
- Discrete scale invariance is not an “allowed” RG flow
- The Breitenlohner-Freedman bound is the holographic dual to this RG instability - the AdS tachyon
- Scalar solutions with bulk mass m^2 scale like z^{Δ}

$$\Delta(\Delta - 4) = m^2$$ Complex for $m^2 < -4$

- Time driving = spatial gradients (Relativity) = Breaking of scale inv. (AdS/CFT)

BKT and Conformality Lost

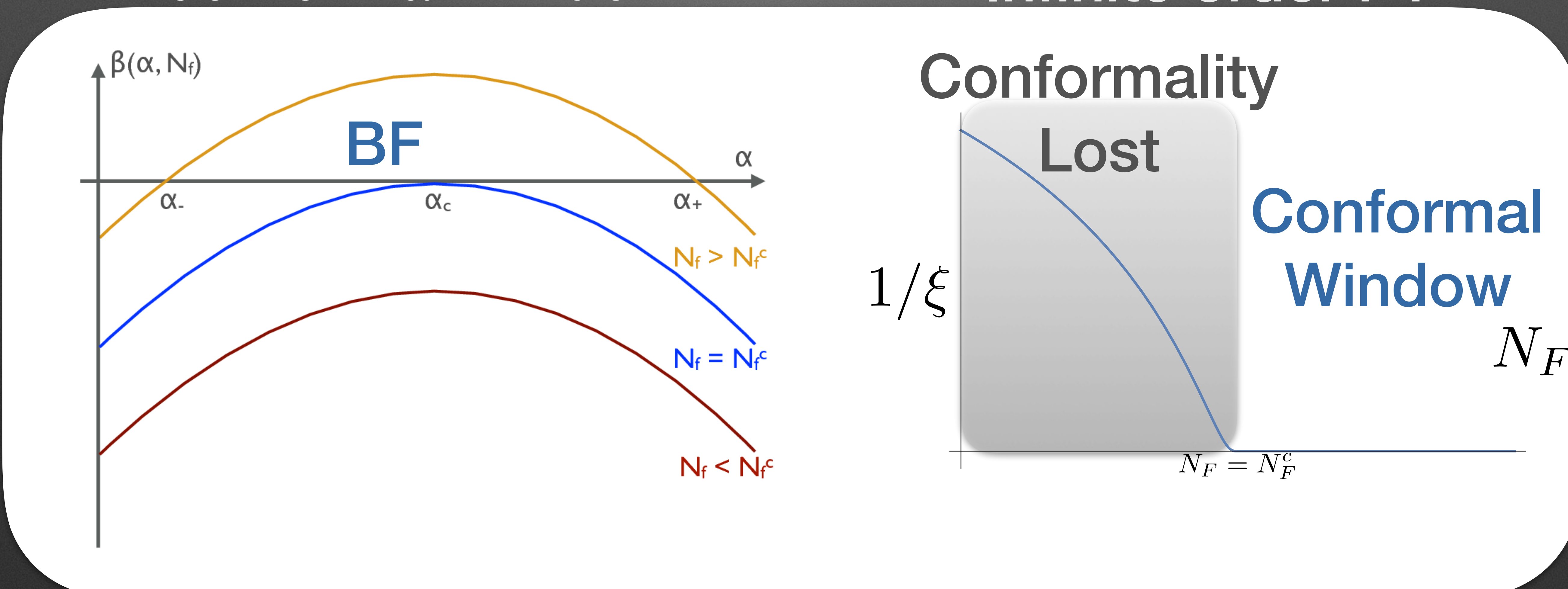
Kaplan, Lee, Son, Stephanov 2009

E.g. QCD in/out of conformal window

BKT

Extended critical region

Infinite order PT

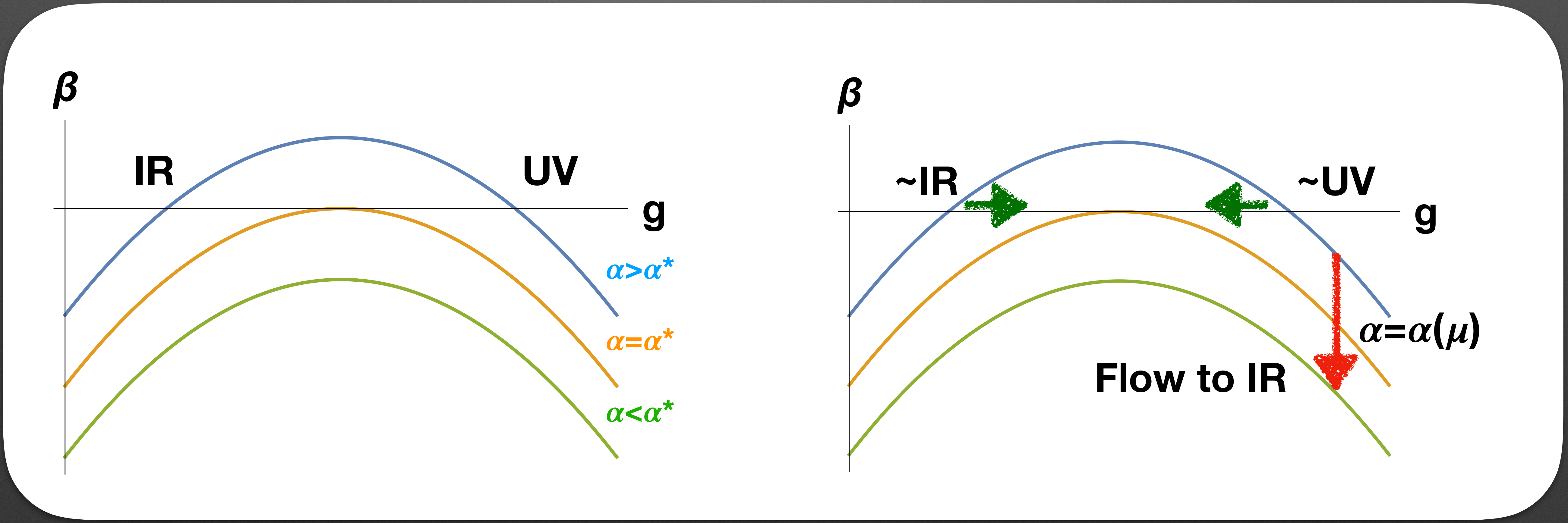


BKT type scaling below critical N_f

Slowly Driving off the Edge

QCD in/off conformal window

Hypothetical Model with healthy UV FP



BKT scaling past critical N_f

Radiatively driven off CFT Fixed point in C-plane

How does theory rectify instability?

Spontaneously broken ~CFT/TC type Confinement

Holographic Conformality Lost

Two Theories are One:

Scalar Solutions in AdS_5:

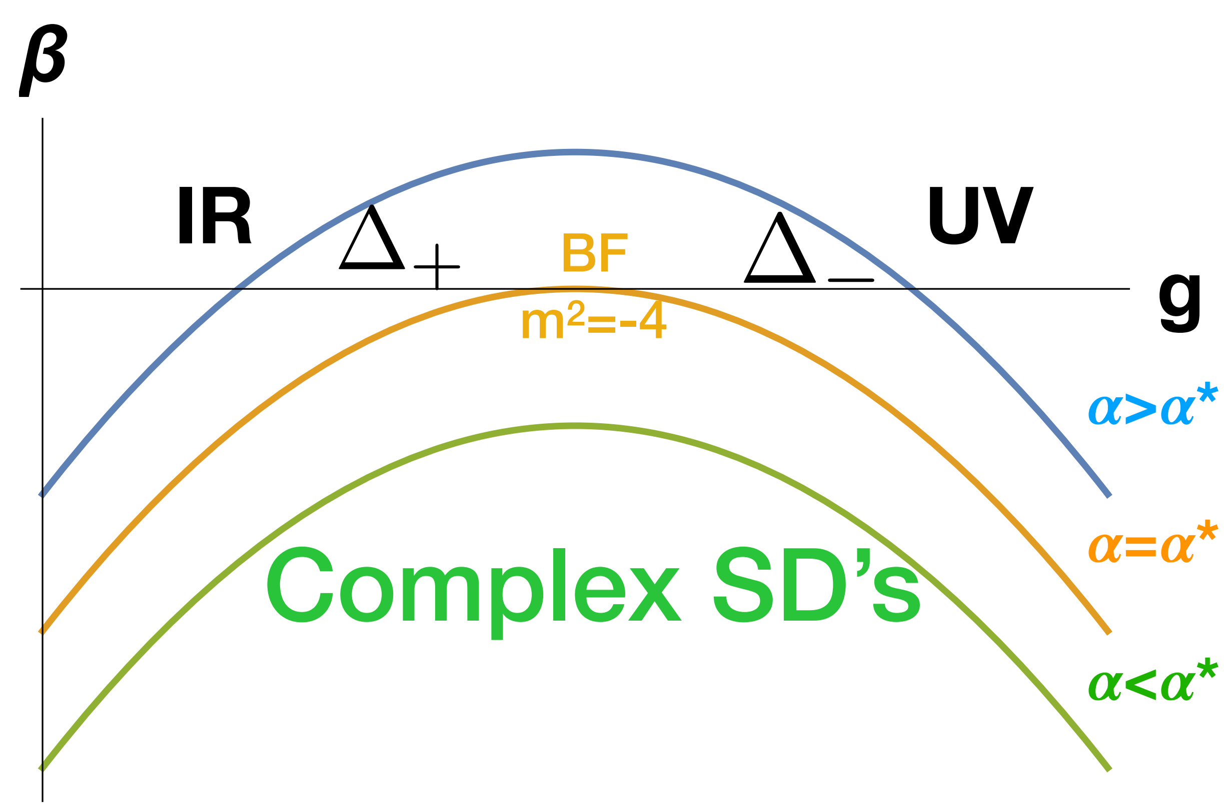
$$\phi \propto z^{\overbrace{2\pm\sqrt{4+m_\phi^2}}^{\Delta_\pm}}$$

scaling dimensions at fixed points

Conjecture of KLSS:

two solutions are same microscopic theory at different FP

fine tune AdS boundary theory to hug UV FP



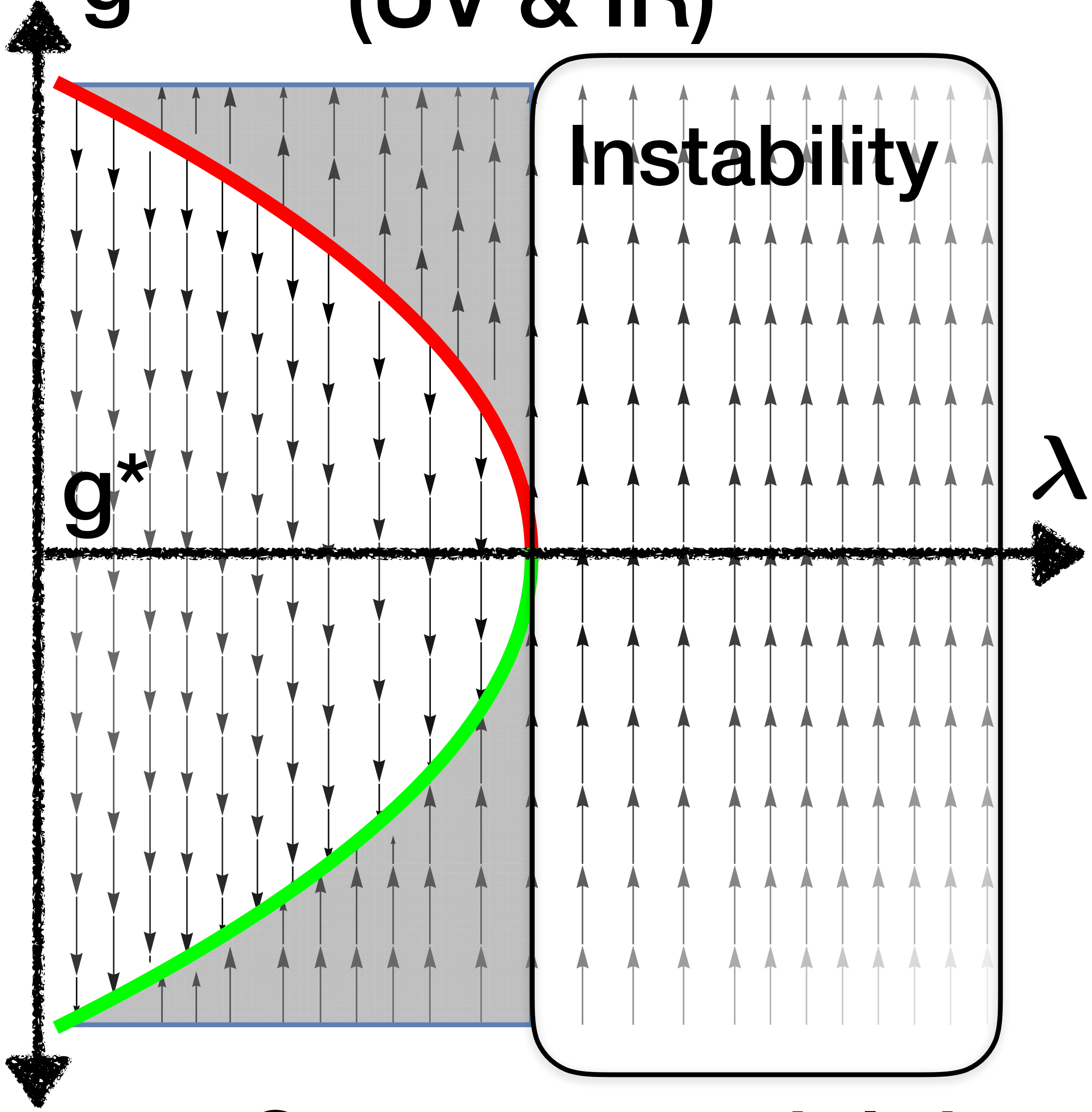
Side Notes: Important!

- Light scalars don’t have to be, and aren’t likely to be, techni-dilatons
  - not natural - requires flat direction in addition to walking = FINE TUNED
  - Higgs looks nothing like a dilaton - couplings set by restoring CI non-lin.
- Spectrum in low energy theory is function of bulk mass
  - Near BF bound, scalar drops down out of spectrum of KK states/~CFT composites
    - Alex Pomarol Planck 2017:
      “Light scalars: From lattice to the LHC via holography”
      no slides and no paper… but also rel. work by Vecchi: arXiv:1012.3742
  - light 0++ scalar may just be consequence of being near boundary of conformal window where operator scaling dim. near critical
  - much easier phenomenologically to interpret light state as Higgs, if that is the aim

Flows

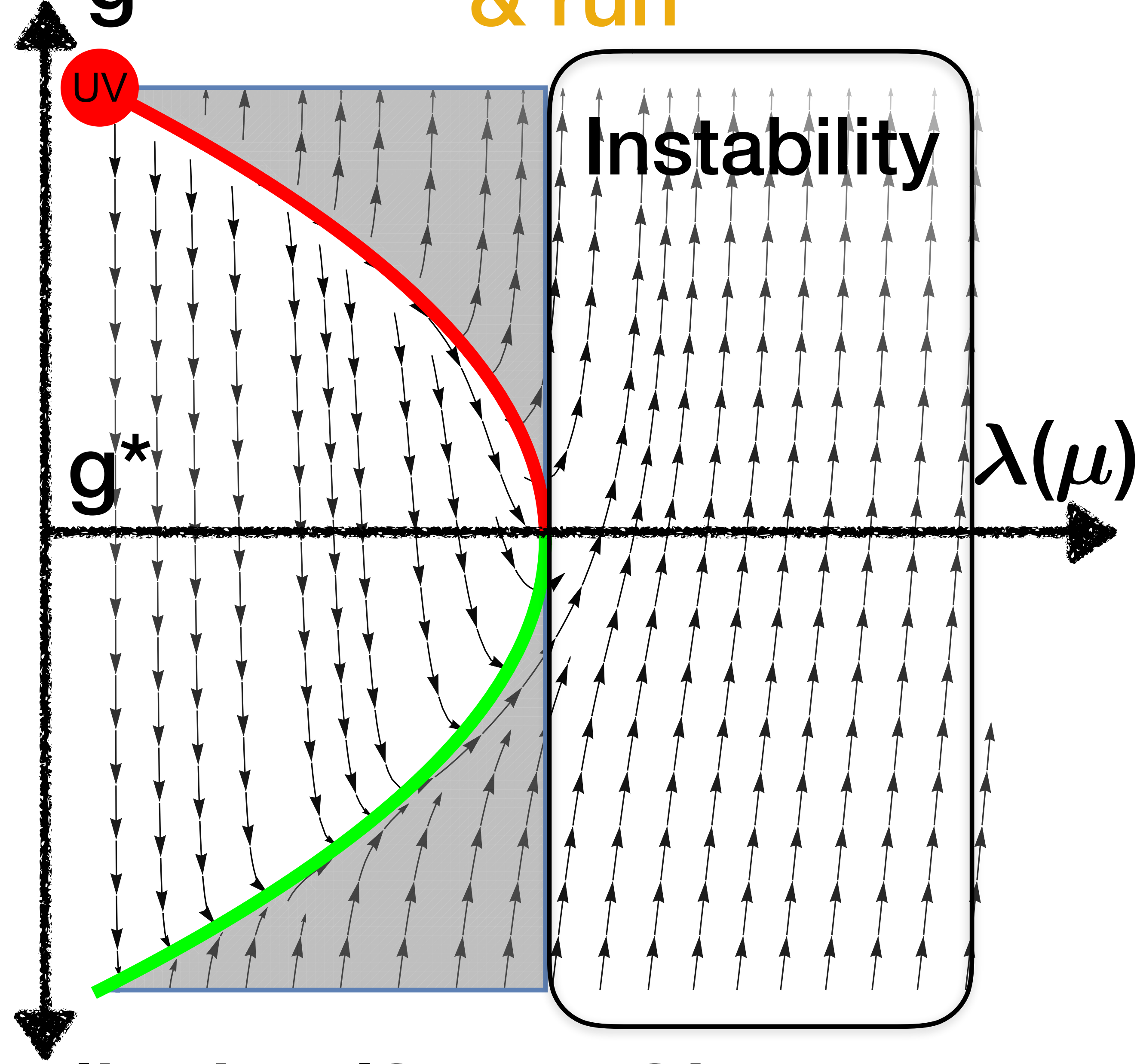
External Descriptor λ:

Line of FP's
(UV & IR)



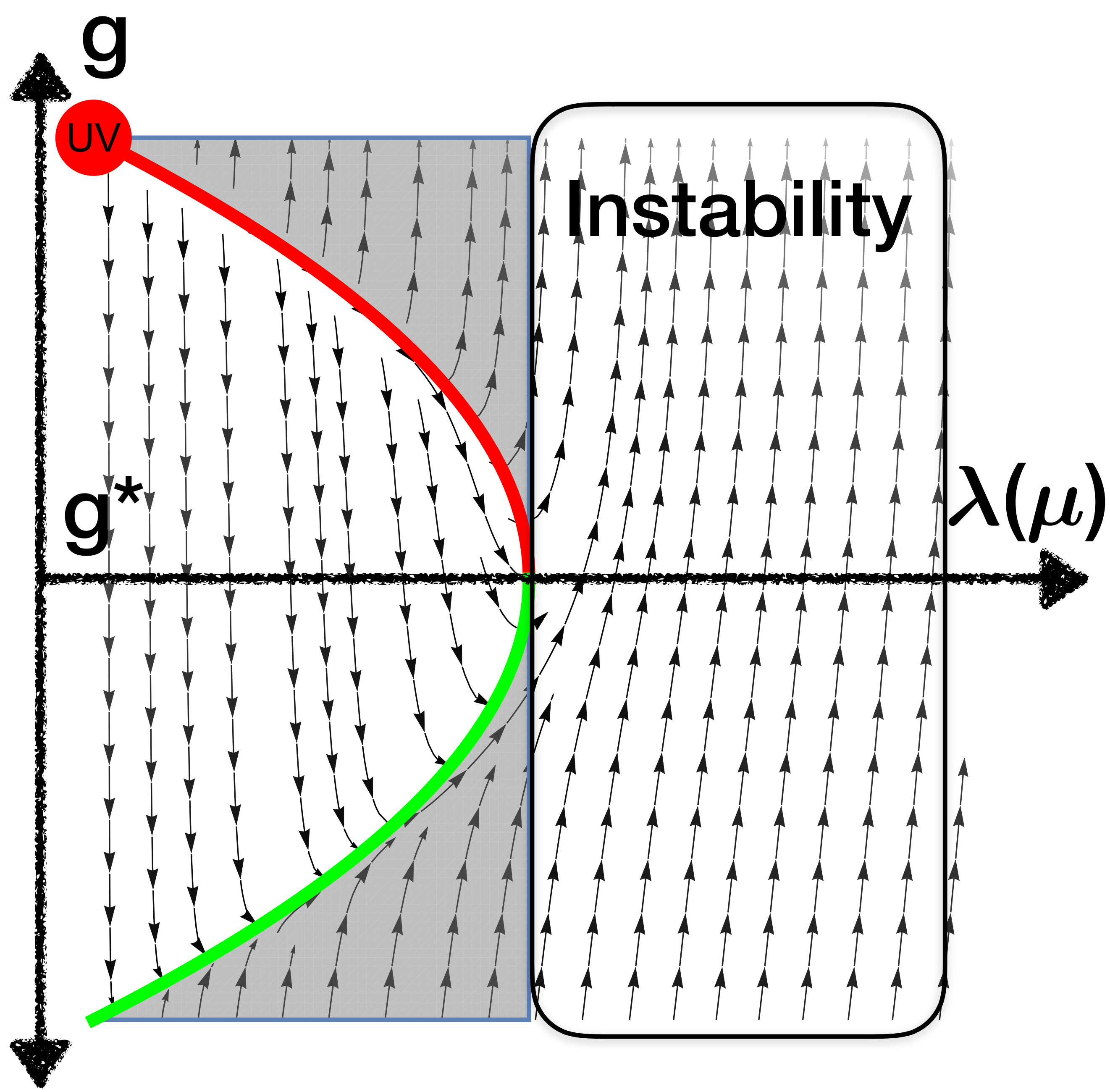
Dynamical λ:

Start @ healthy UV FP
& run



Create a model that walks itself out of its own ~conformal window

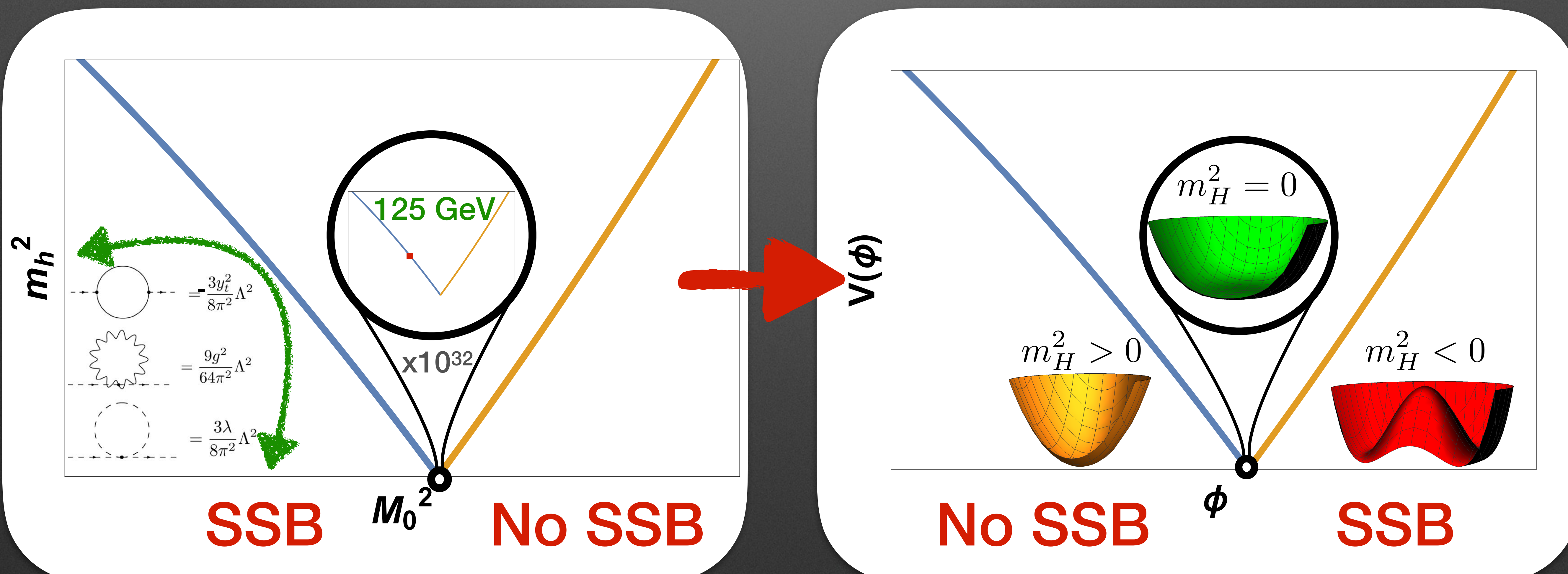
Attractive IR trajectory



Many flows join single ~IR fixed line (near green)
Many flows find the tachyon
How does theory resolve it?

SOC = Self Tuned Phase Transition

We seek some solution to the hierarchy problem similar to that of axion solution to strong CP - axion “self-tunes” effective theta angle to zero at min of instanton potential



Modulus field with potential energy minimum at point where $m_H^2=0$ (condensate in ~CFT)

Randall-Sundrum

Original setup unstable

$$S = \int d^4x dz \sqrt{g} \left[\frac{6k^2}{\kappa^2} - \frac{1}{2\kappa^2} R \right]$$
$$- \int d^4x \sqrt{g_{\text{ind}}(z_0)} T_0 - \int d^4x \sqrt{g_{\text{ind}}(z_1)} T_1$$

Solution to Einstein equations - constant neg. curvature:

AdS metric: $$ds^2 = \left(\frac{1}{kz} \right)^2 (dx_4^2 - dz^2)$$

Higgs lives near z_1
warping makes higgs light
but close to KK scale
w/o tuning

Can integrate over z putting in classical solution - effective potential for brane locations:

$$V_{\text{eff}} = \frac{1}{z_0^4} \left[T_0 - \frac{6k}{\kappa^2} \right] + \frac{1}{z_1^4} \left[T_1 + \frac{6k}{\kappa^2} \right]$$

Branes **move**
unless $T_0 = -T_1 = 6k/\kappa^2$
Tuning

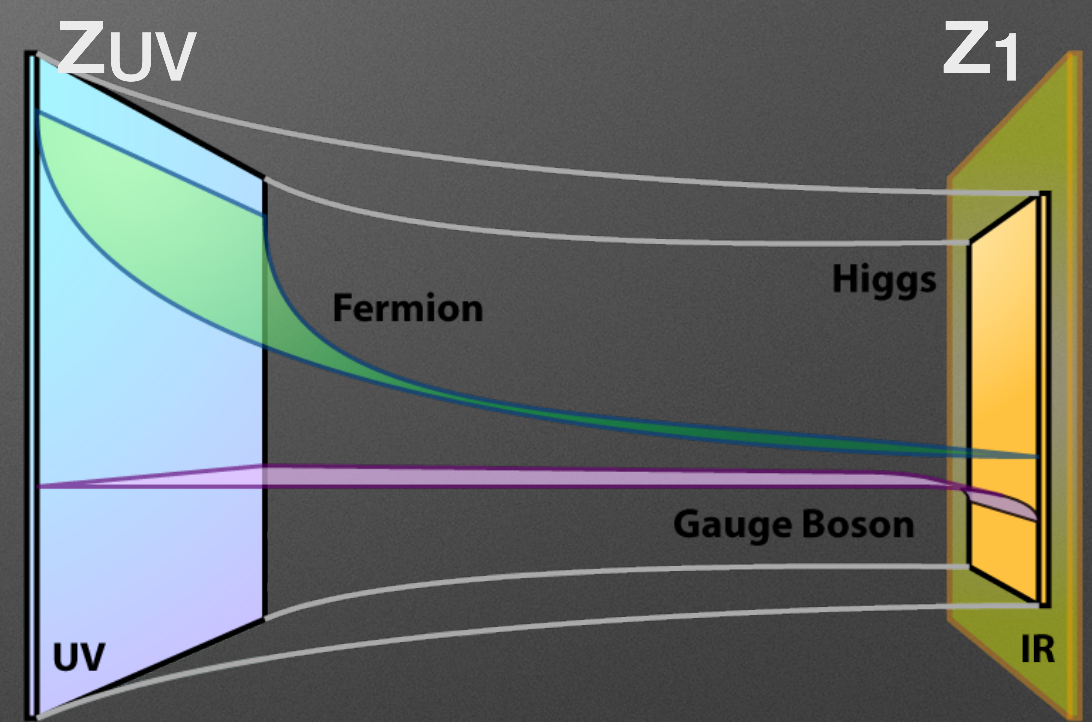
Wrong metric ansatz - e.g. time dependence in ds^2

Holographic Model

AdS/CFT: Coordinate $\Longleftrightarrow$ scale: $z \Longleftrightarrow 1/\mu$

Geometric warping creates large hierarchies

A great toolkit for solving many problems of SM (fermion masses, flavor, etc)



- **Higgs is in the bulk** of the extra dimension
- Radion/modulus is fluctuations in distance between UV and IR branes
  - IR brane = condensates / confinement scale (SB-CFT)
- **Slowly changing 5D Higgs mass (the slow driving force of SOC)**

Stabilization

- Many interesting ways to do this - new fields in 5D:
  - Casimir energies - quantum balances classical
  - Scalar vev's: Goldberger-Wise and related
    - dimensional transmutation
    - additional terms in classical E.E.'s
    - backreaction on geometry feeds into potential
    - can balance pure tensions, alleviate one fine tuning

Of course Higgs is a scalar, and gets a vev
Can Higgs stabilize it?

Answer is yes, although setups slightly different than GW

A Simple 5D Model

5D theory with gravity and a scalar field

5D CC: k=1 5D G_N

$$S = \int d^4x dz \sqrt{g} \left[|\partial_M H|^2 + \frac{6}{\kappa^2} - m^2(z)|H|^2 - \frac{1}{2\kappa^2} R \right]$$

$$- \int d^4x z^{-4} m_0^2 |H|^2|_{z\to 0} - \int d^4x z^{-4} V_1(|H|)|_{z\to z_1}$$

UV potential IR brane potential

Fluctuations: $H = \frac{1}{\sqrt{2}}(\phi + h_0 + i\pi)$ $H \to e^{i\alpha} H$

Higgs vev

There is an IR brane at z_1, KK scale $1/z_1$, but no UV brane

(For simplicity - realistic model needs UV brane)

Scalar field is a Higgs, <u>mass depends on z</u>

z_1 is the modulus vev - fluctuations are the radion

Effective Higgs Mass

Explicitly Break AdS isometries
broken conformal invariance ($z \sim 1/\mu$):

$$m^2(z) = -4 + \delta m^2 - \lambda z^\epsilon$$

ε small, but not tiny (0.1 or so) $\Longrightarrow$ log "running"

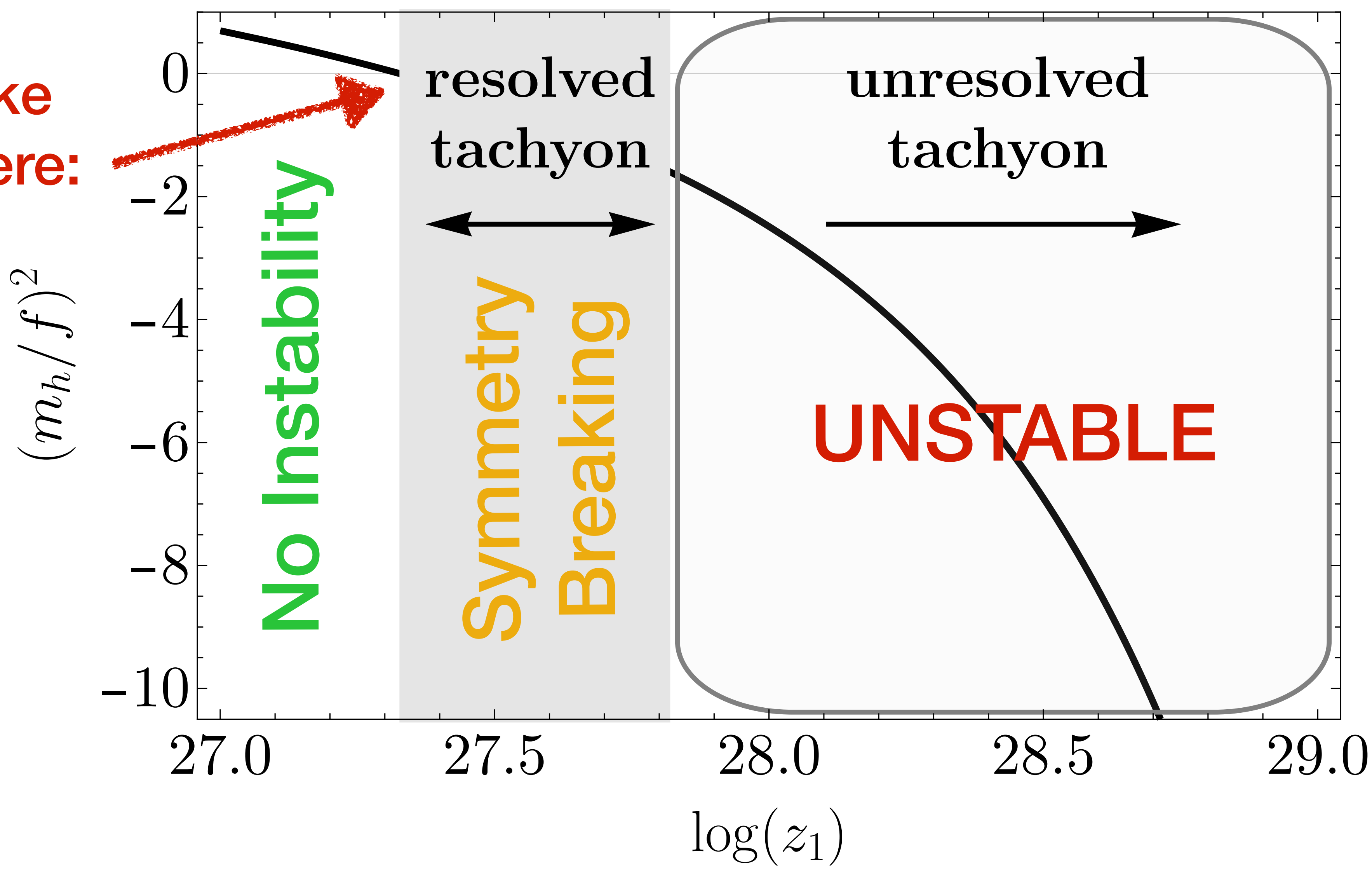
-4 is the Breitenlohner-Freedman bound
(analog of $m^2=0$ in flat space)
δm^2 is positive and O(1)

- There is a 4D low energy EFT with a single Higgs around or below the KK scale
  - Effective Higgs mass² involves both terms in V_1 (IR brane potential) and an integral over z taking into account changing bulk mass
  - Criterion for vev (symmetry breaking) depends on z_1

The Higgs mass^2

Higgs potential changes with brane position
(as function of condensate values in ~CFT)

Would like
to stop here:



There is region where Higgs itself resolves the tachyon

Need to know what most attractive channel is
would prefer to avoid unstable region

Effective potential for the separation

Metric is AdS, up to backreaction in G(z):

$$ds^2 = \frac{1}{z^2}\left[dx_4^2 - \frac{dz^2}{G(z)}\right]$$

Bunk, Jain, JH 2017

If scalar field takes on vev, backreaction on geometry:

$$G = \frac{\frac{-\kappa^2}{6}V(\phi)}{1 - \frac{\kappa^2}{12}(z\phi')^2}$$

If ϕ is zero, G = 1

G(0) = 1

Action is pure boundary term (only IR brane contributes)

$$V_{\text{eff}} = \frac{1}{z_1^4}\left[V_1(\phi) + \frac{6}{\kappa^2}\sqrt{G}\right]$$

Also UV contribution, but just bare CC (tune to zero)

Bellazzini Csàki JH Serra Terning 2014

Effective Potential

Can take the weak gravity limit:

$$V_{\text{eff}} = \frac{1}{z_1^4}\left[V_1(\phi) + \frac{6}{\kappa^2} - \frac{1}{4}m^2(z_1)\phi^2(z_1) + \frac{1}{4}z_1^2\phi'^2(z_1)\right]$$

Scalar Equation of motion (can solve for vev analytically):

$$\phi'' - \frac{3}{z}\phi' - \frac{1}{z^2}\frac{\partial V}{\partial \phi} = 0$$

$$\phi = \phi_+ z^2 J_{\frac{2\sqrt{\delta m^2}}{\epsilon}}\left(\frac{2\sqrt{\lambda}}{\epsilon}z^{\epsilon/2}\right)$$

Also - solution with UV tuning or SUSY

Asymptotics

In the UV (small z) region, solution is scaling

$$\phi \propto z^{\Delta} \text{ with } \Delta = 2 \pm \sqrt{\delta m^2}$$

Choice of delta depends on z=0 boundary action

In the IR (large z), solution develops log-periodic behavior:

For small epsilon: $$\phi \propto z^{2-\epsilon/4} \cos(\sqrt{\lambda} \log z + \gamma)$$

The Boundary Condition:

Action principle determines boundary condition:

IR brane potential:

$$V_1(|H|) = T_1 + \lambda_H |H|^2 \left(|H|^2 - v_H^2\right)$$

IR brane tension

Sets BC:

$$z\phi'|_{z=z_1} = -\frac{1}{2}\frac{\partial V_1}{\partial \phi}\bigg|_{z=z_1}$$

For small z_1, ϕ is just zero, but vev develops if z_1 is larger:

$$\frac{1}{\epsilon}\left(\lambda_H v^2 - 4\right) > \frac{2\sqrt{\lambda} z_1^{\epsilon/2}}{\epsilon} \frac{J'_{\frac{2\sqrt{\delta m^2}}{\epsilon}}\left(\frac{2\sqrt{\lambda} z_1^{\epsilon/2}}{\epsilon}\right)}{J_{\frac{2\sqrt{\delta m^2}}{\epsilon}}\left(\frac{2\sqrt{\lambda} z_1^{\epsilon/2}}{\epsilon}\right)}$$

Equality/**Criticality** at $z_1 = z_0$

Radion Potential Near Criticality

$z_1 \approx z_0$

Barely past criticality, Higgs vev is linear in z

$$\phi(z_1)^2 \approx \sigma^2(z_1/z_0 - 1)$$

For $z_1 > z_0$
0 for $z_1 < z_0$

(σ^2 positive)

$$\sigma^2 = \frac{-4m^2(z_0) + \lambda_H v^2 \left(\lambda_H v^2 - 8\right)}{2\lambda_H}$$

Gives positive contribution to radion potential - also linear:

$$V_{\text{radion}} \approx \begin{cases} \frac{1}{z_1^4}\delta T_1 & z_1 < z_0 \\ \frac{1}{z_1^4}\left[\delta T_1 + \frac{\lambda_H}{8}\sigma^4(z_1/z_0 - 1)\right] & z_1 > z_0 \end{cases}$$

Radion Potential Near Criticality

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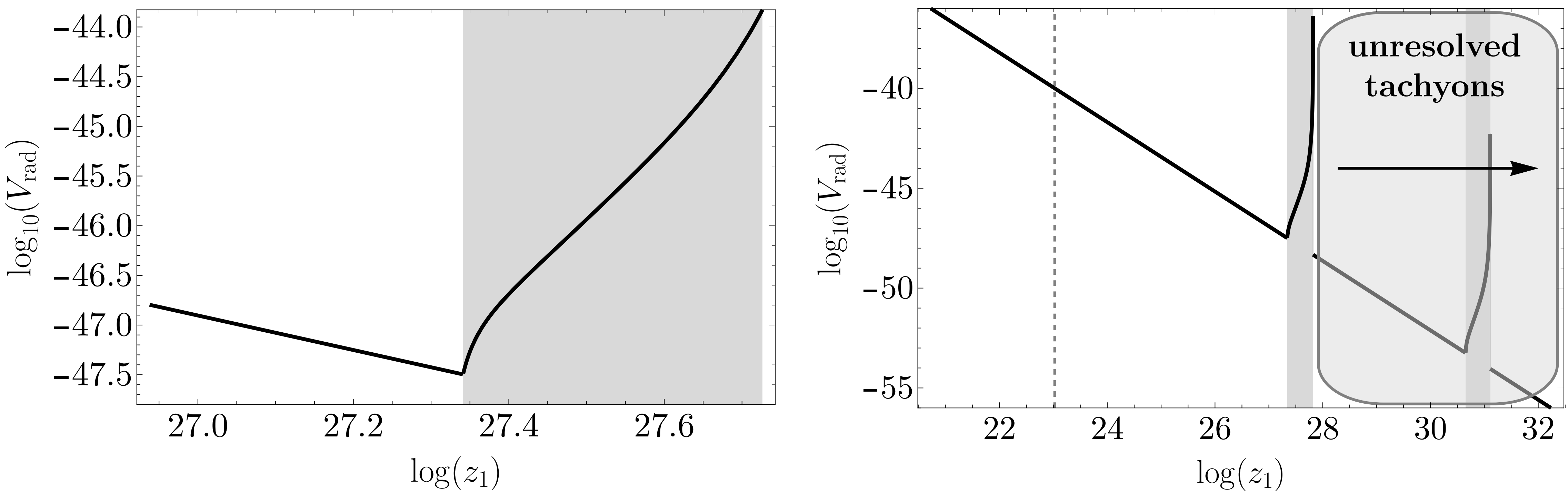
Radion potential has kink singularity at $z_1 = z_0$

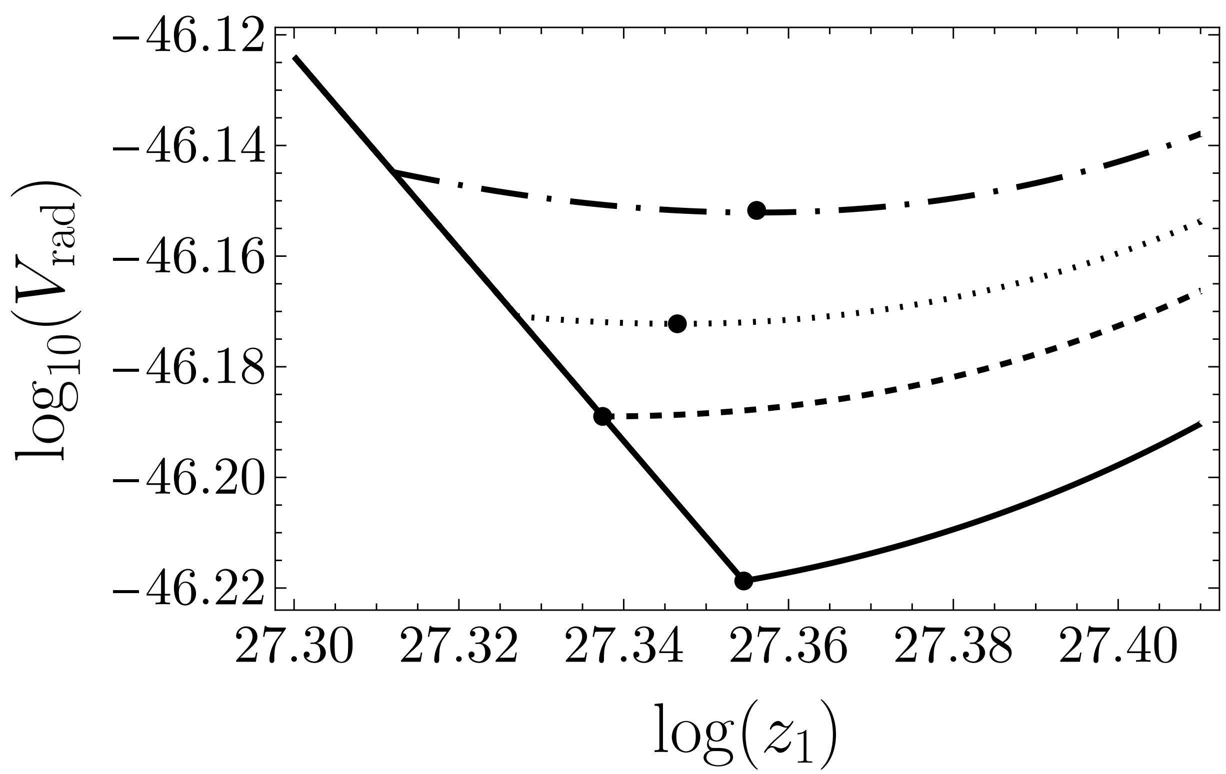


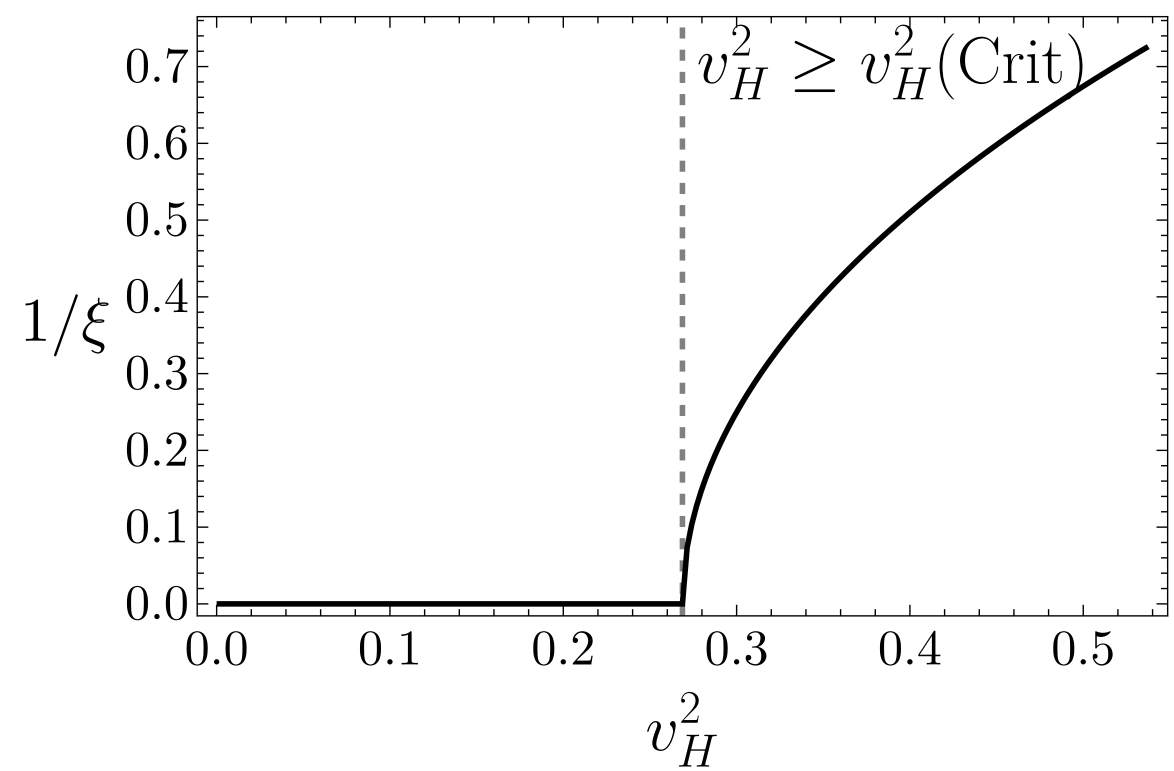
The kink is generic to this construction

Without extra dynamics, metastable

<O$_4$> vs <O$_H$>

In ~CFT, operators that pick vevs can be Higgs operator, or other marginal/near marginal operators





$v_H{}^2$ controls 4D brane Higgs Tachyon

massless Higgs region when AdS tachyon dominates

Determined by boundary conditions in IR

not clear what dual picture is

A Minimum at the kink

We need to have the derivative change sign at z_0

$$0 < \delta T_1 < \frac{1}{128\lambda_H}\left[4m^2\left(z_c^1\right) - \lambda_H v_H^2\left(\lambda_H v_H^2 - 8\right)\right]^2$$

Quartic mis-tune can't be **huge**, but fine for reasonable range of mistune (in units of curvature)

Higgs brane mass plays important role

So we have a model which supplies us with all criterion sufficient to create a potential with minimum where Higgs mass^2 is exactly zero

Required external explicit breaking of AdS isometries = external explicit violation of scale invariance

Quantum Corrections and Fine Tuning

- You have play in the dials, with a reasonably large region of parameter space where Higgs mass is **exactly** zero
- Divergent quantum corrections absorbed into local 5d parameters
  - effect a change in where the kink is, but not its presence, and masslessness of Higgs fluctuation there
- Non-local quantum corrections - don't undo kink, small contribution to brane potential

Metric Ansatz

Gibbons-Hawking-York Condition:

$$\left. \frac{6}{\kappa^2}\sqrt{G} + V_1(\phi) \right|_{z_1} = 0$$

This is a consistency condition on our metric ansatz - flat 4D slices:

$$ds^2 = \frac{1}{z^2}\left[dx_4^2 - \frac{dz^2}{G(z)}\right]$$

With this radion potential, this condition is <u>never</u> satisfied

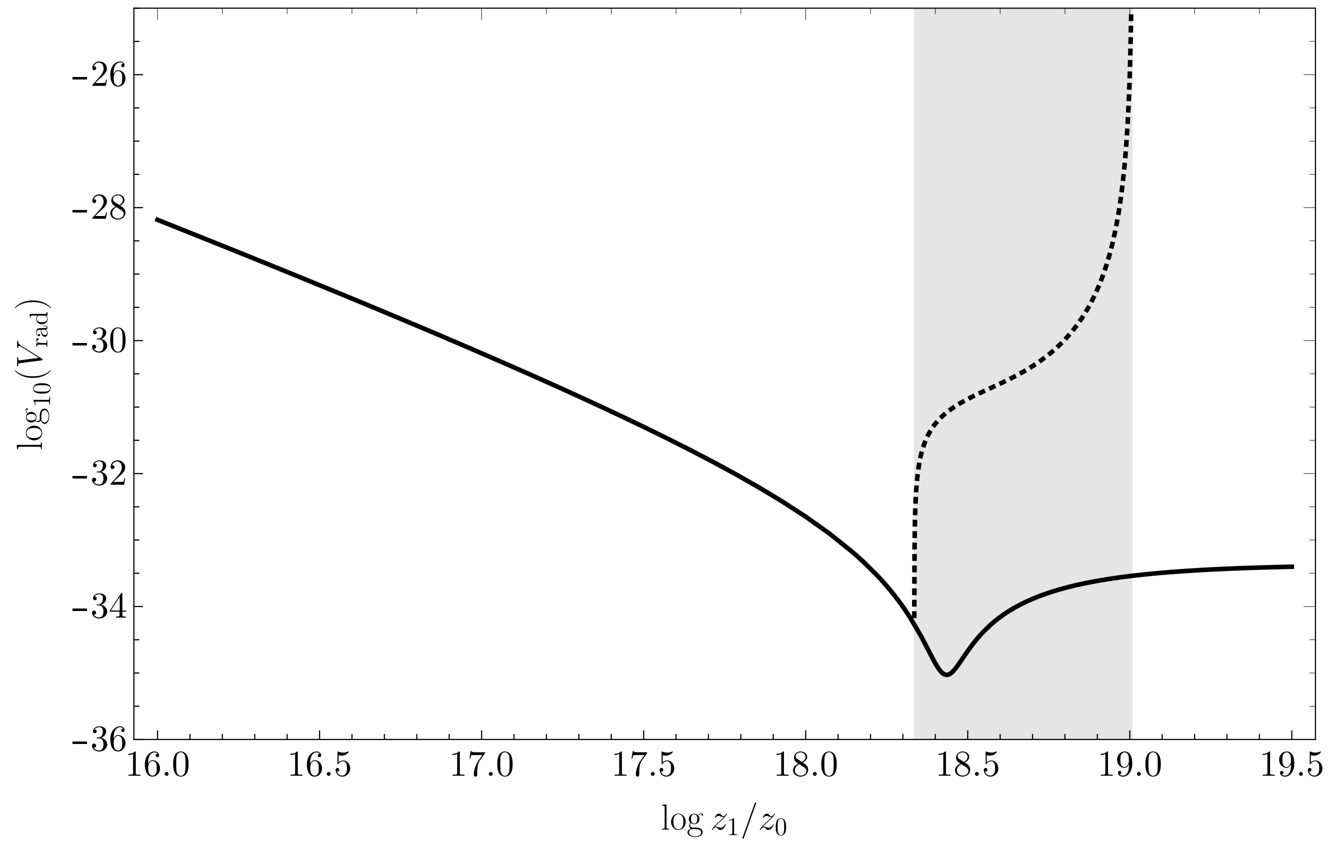
$$V_{\text{radion}} \approx \begin{cases} \frac{1}{z_1^4}\delta T_1 & z_1 < z_0 \\ \frac{1}{z_1^4}\left[\delta T_1 + \frac{\lambda_H}{8}\sigma^4(z_1/z_0 - 1)\right] & z_1 > z_0 \end{cases}$$

The geometry <u>must</u> be moving (4D slices are not flat)

Non-trivial cosmology is an output

seems like “trapped” radion oscillations

Adding Goldberger-Wise potential Stabilize the Kink



It is not difficult to get the first kink to be a global minimum - no metastability issues

If you go to the kink, bulk thinks it is displaced from GW potential minimum - will oscillate

Spontaneous Lorentz breaking “striped phase”

Summary

- tried to mock up components of SOC in holographic model - Conformality lost turned dynamical - some similarities to ongoing work in BSM Lat. community
- hard breaking of AdS isometries/scale invariance
    - drives theory out of a (quasi) conformal window
- holographic model has large range of parameters where low energy Higgs theory exactly at critical point
- Seems to require non-trivial cosmology, perhaps radion oscillations?

Open ???'s

- Have broken 5D diffs in the IR region of AdS - is there a sensible field theory interpretation?
- What might be a dual picture? E.g. sequence of fermion masses explicitly breaking SI, moving the FP's of KSS together? Something more exotic?
- What precisely is the non-trivial 4D effective cosmology?