Comparing staggered and domain wall fermions at a Conformal Fixed Point

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BSM Workshop, Boulder Co

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Flatirons - Boulder Colorado

Sources:

The results I discuss here are from two publications:

- staggered fermions : A.H, D. Schaich, ArXiv:1610.10004
- domain wall fermions : A.H, C. Rebbi, O. Witzel, ArXiv:1710.11578

as well as some new and yet unpublished results.

The domain wall calculations would not have been possible without the continuous help we received from the developers of GRID,
Peter Boyle, Guido Cossu, Antonin Portelli, and Azusa Yamaguchi

Conformal systems

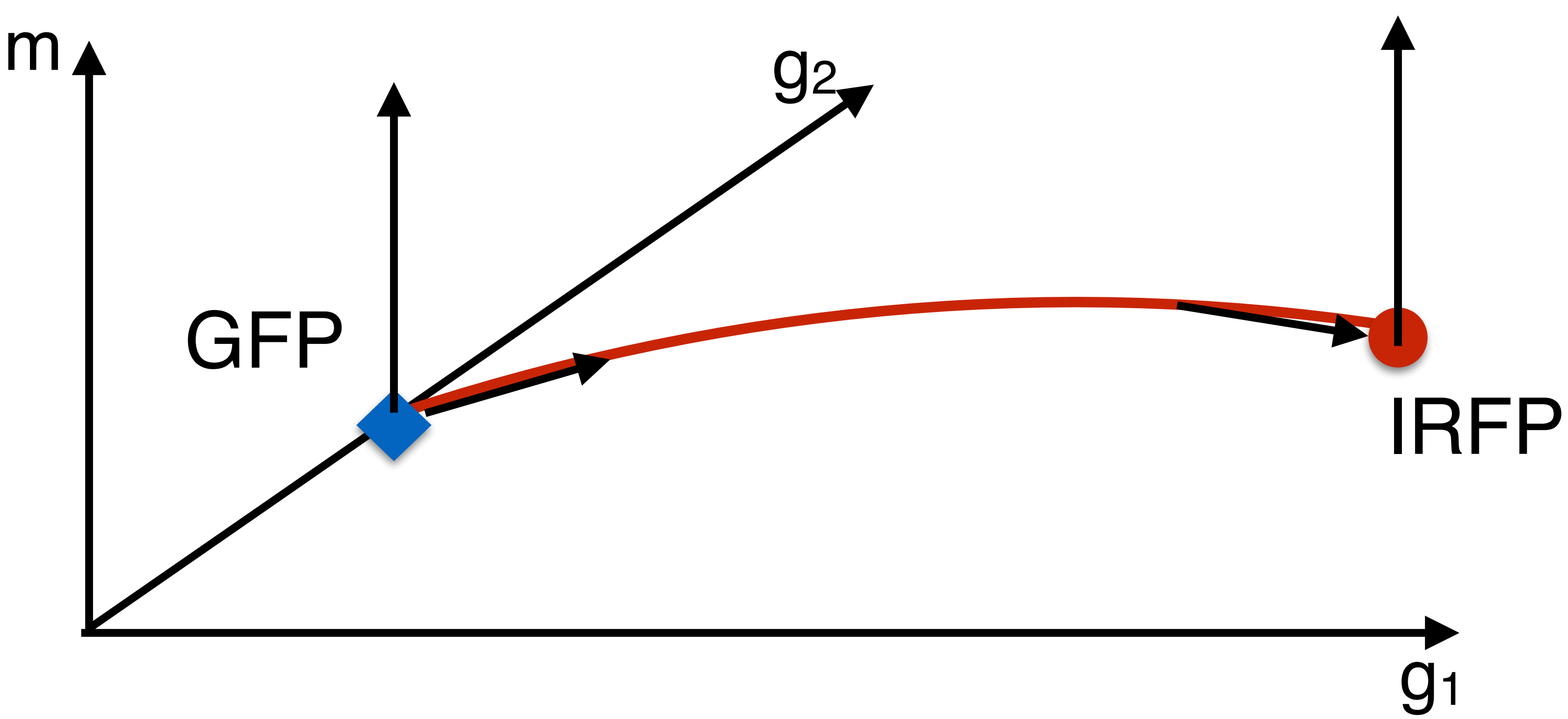
Conformal systems are important

- interesting on their own right as QFT with a non-Gaussian FP
- phenomenological models can be built on the IRFP (O. Witzel)

Conformal IRFPs are very different from QCD ; our QCD intuition is misleading:

gauge coupling is irrelevant and not tuned

⇒ "continuum limit" is $m \to 0$, $g^2 \not\to 0$

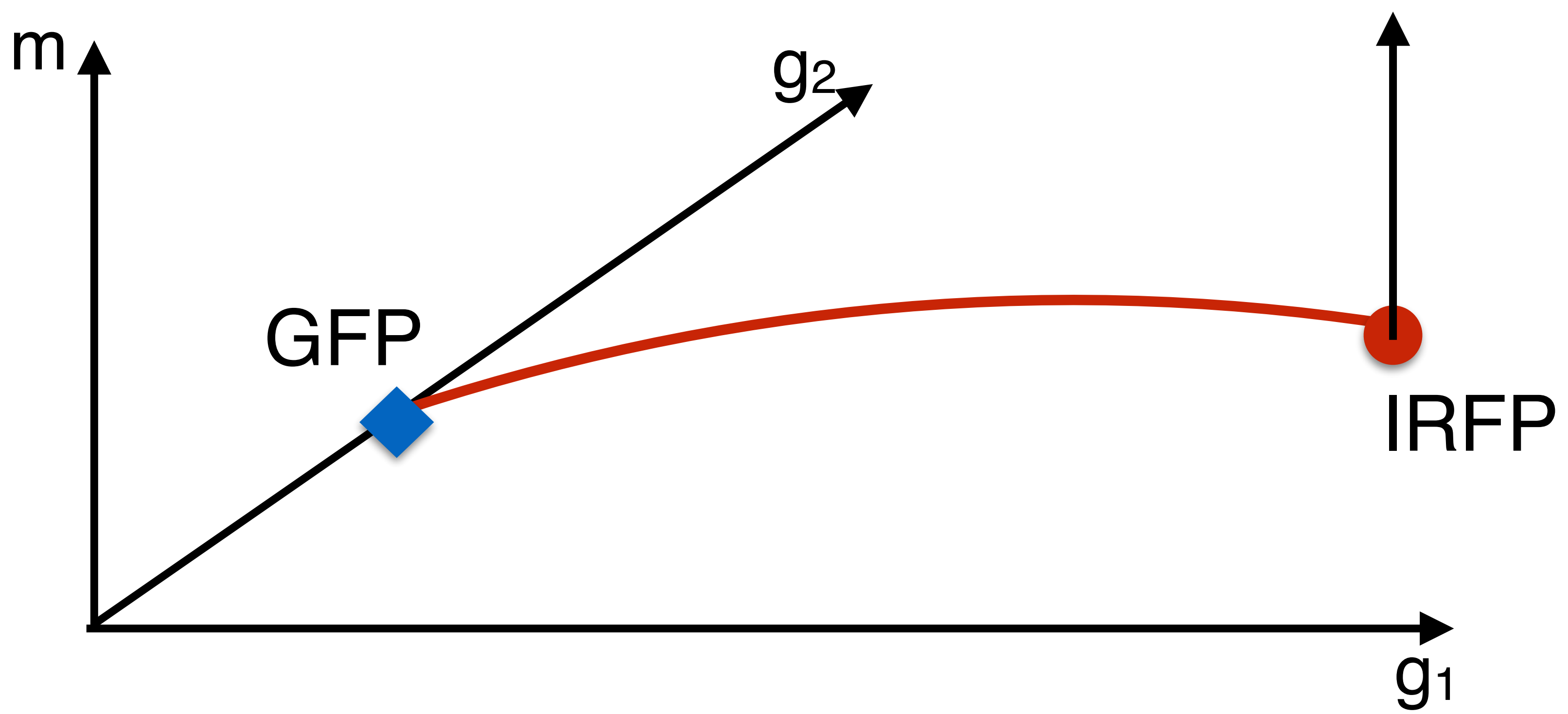


Universality of the continuum limit

The concept of universality is the driving principle of LQCD as it ensures that lattice simulations (with different lattice actions, discretizations, etc.) study the same continuum physics

We expect universality, i.e. universal critical behavior, in systems

- *with identical field content & dimension*
- ***identical symmetries***
- *at criticality (basin of attraction of a FP)*



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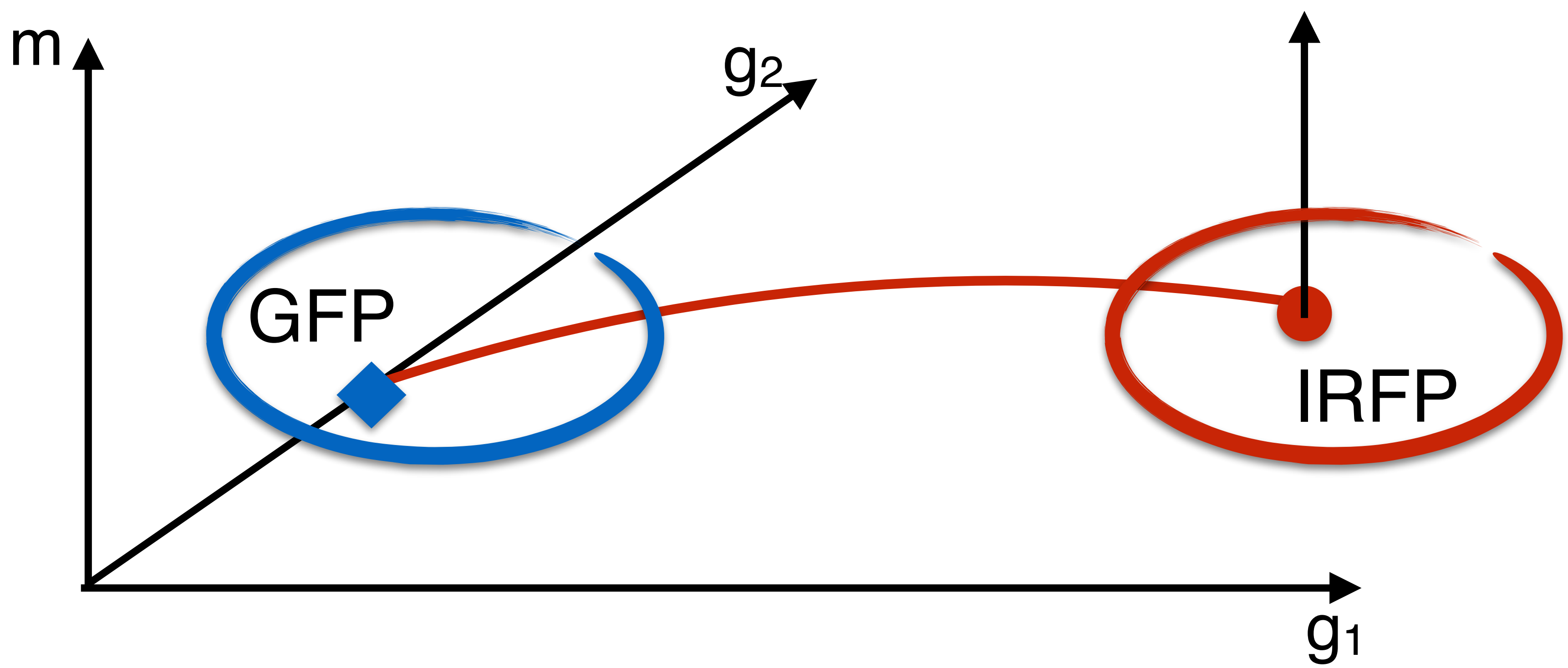


Universality & Staggered fermions

Continuum fermions with N_f flavors exhibit $SU(N_f)xSU(N_f)$ flavor symmetry

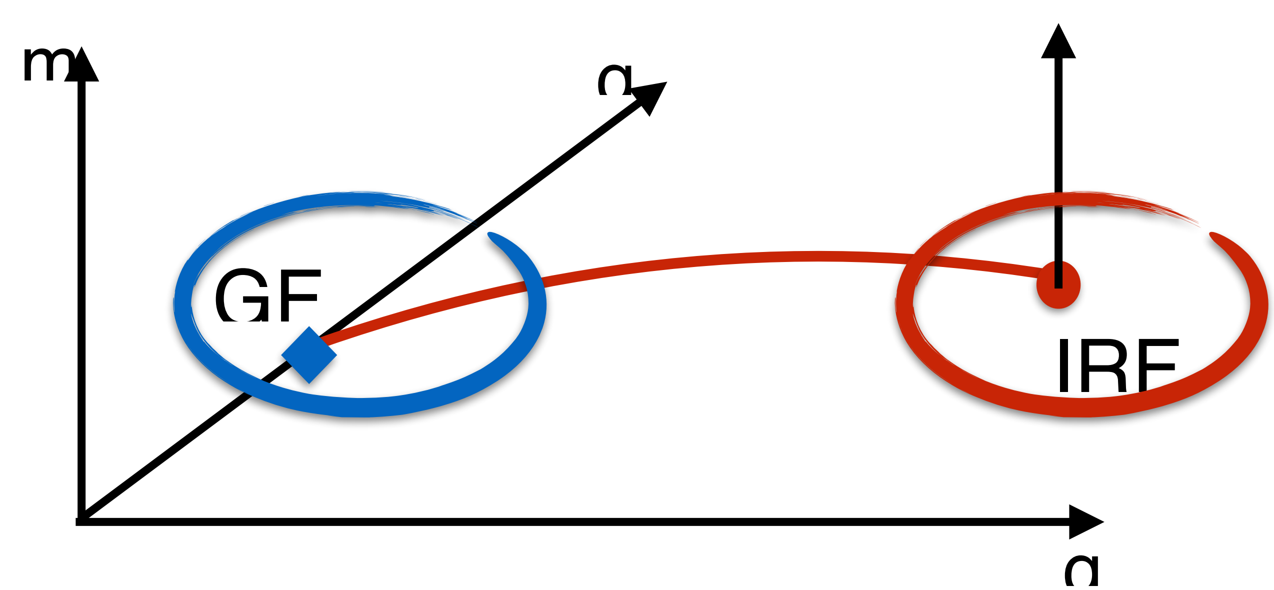
Staggered fermions brake taste to $SU(N_f/4)xSU(N_f/4)$

Taste symmetry is recovered only as $g^2 \rightarrow 0$: GFP✓

Staggered fermions are not in the continuum universality class

unless taste symmetry is restored in the basin of attraction of the IRFP

- Is the possible **without** a phase transition ?
- Recall that RG transformation does not change the IR spectrum

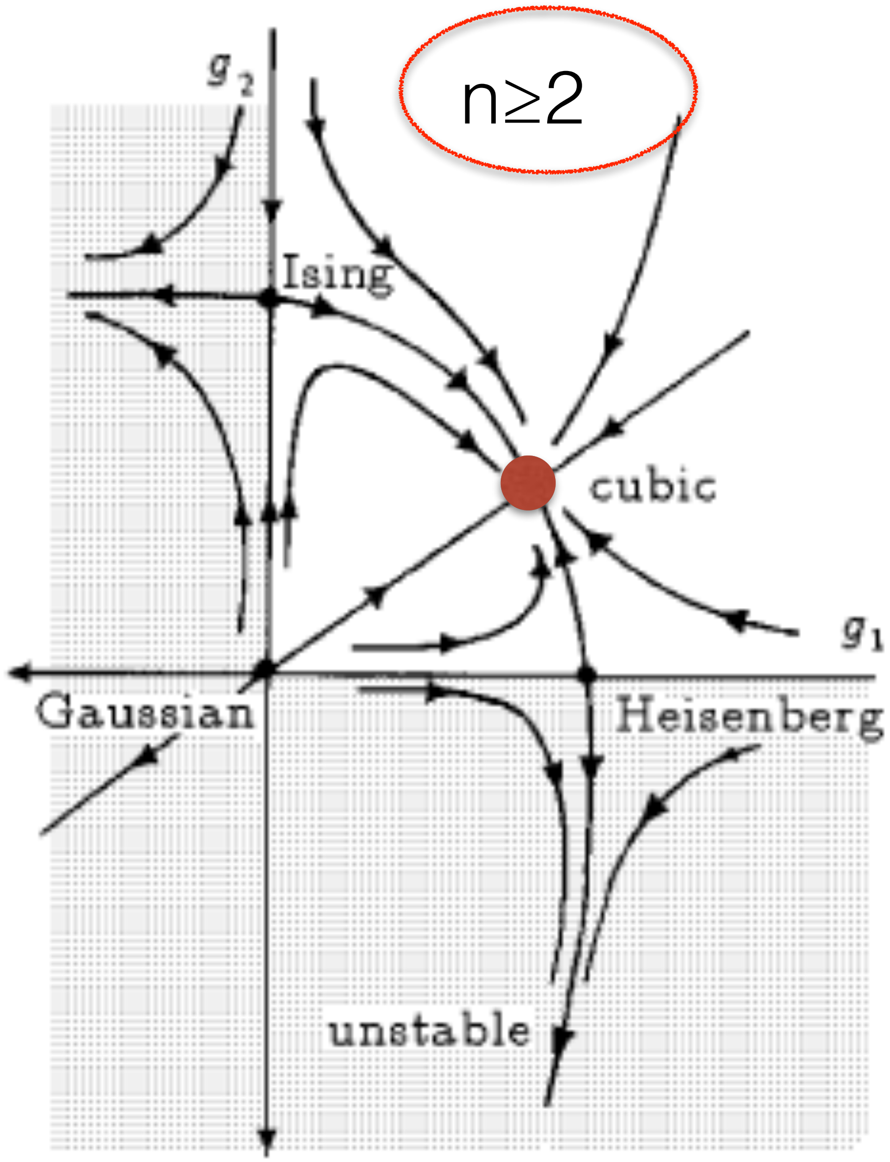


Universality in 3D: O(n)→Z_2 scalar model

Kleinert et al studied a system with O(n)→Z_2 symmetry

$$V = \frac{1}{2} m^2 \phi^2 + g_1 (\phi^2)^2 + g_2 (\sum_\alpha \phi_\alpha^4)$$

Kleinert, Schulte-Frohlinde
cond-mat/9503038



The stable fixed point in neither the O(n), nor the Ising one, but a new FP!

Based on 5th order ε expansion

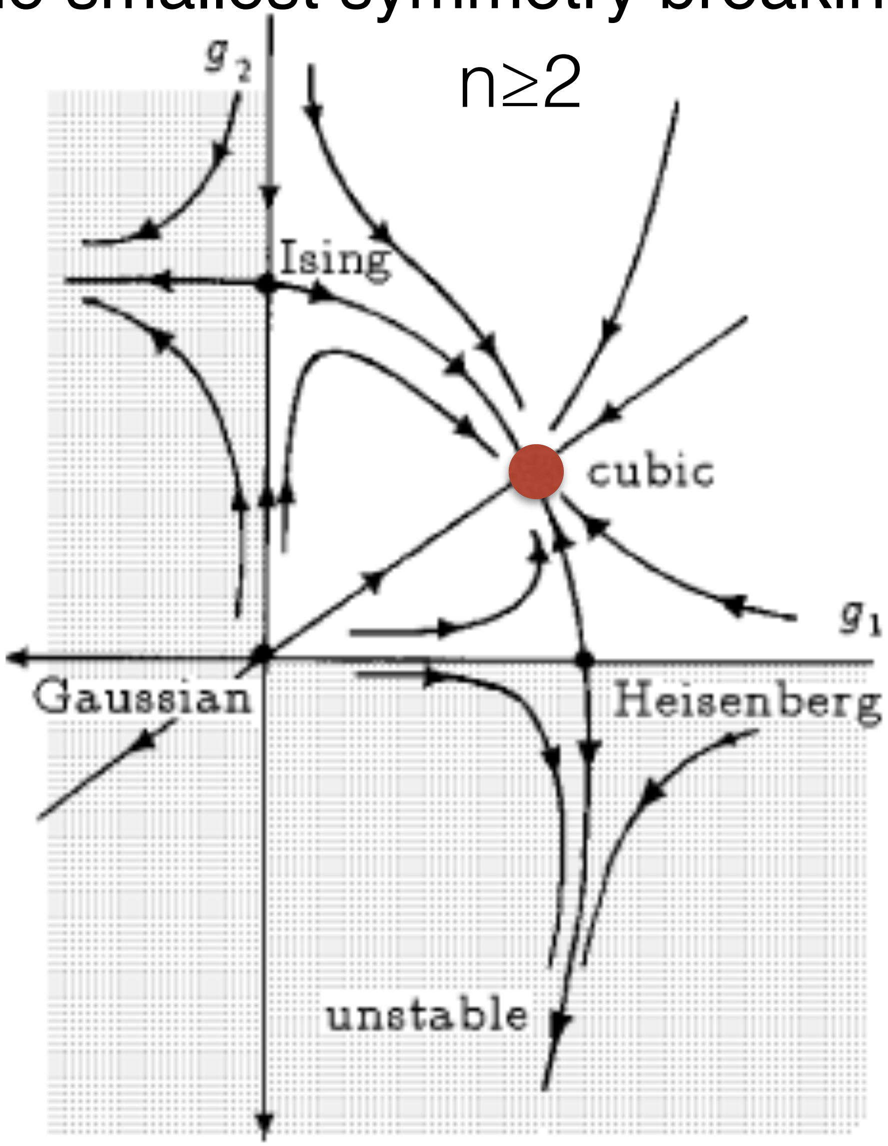
Possible objection:

Staggered action taste breaking terms are only $O(g^2)$ different from the continuum action

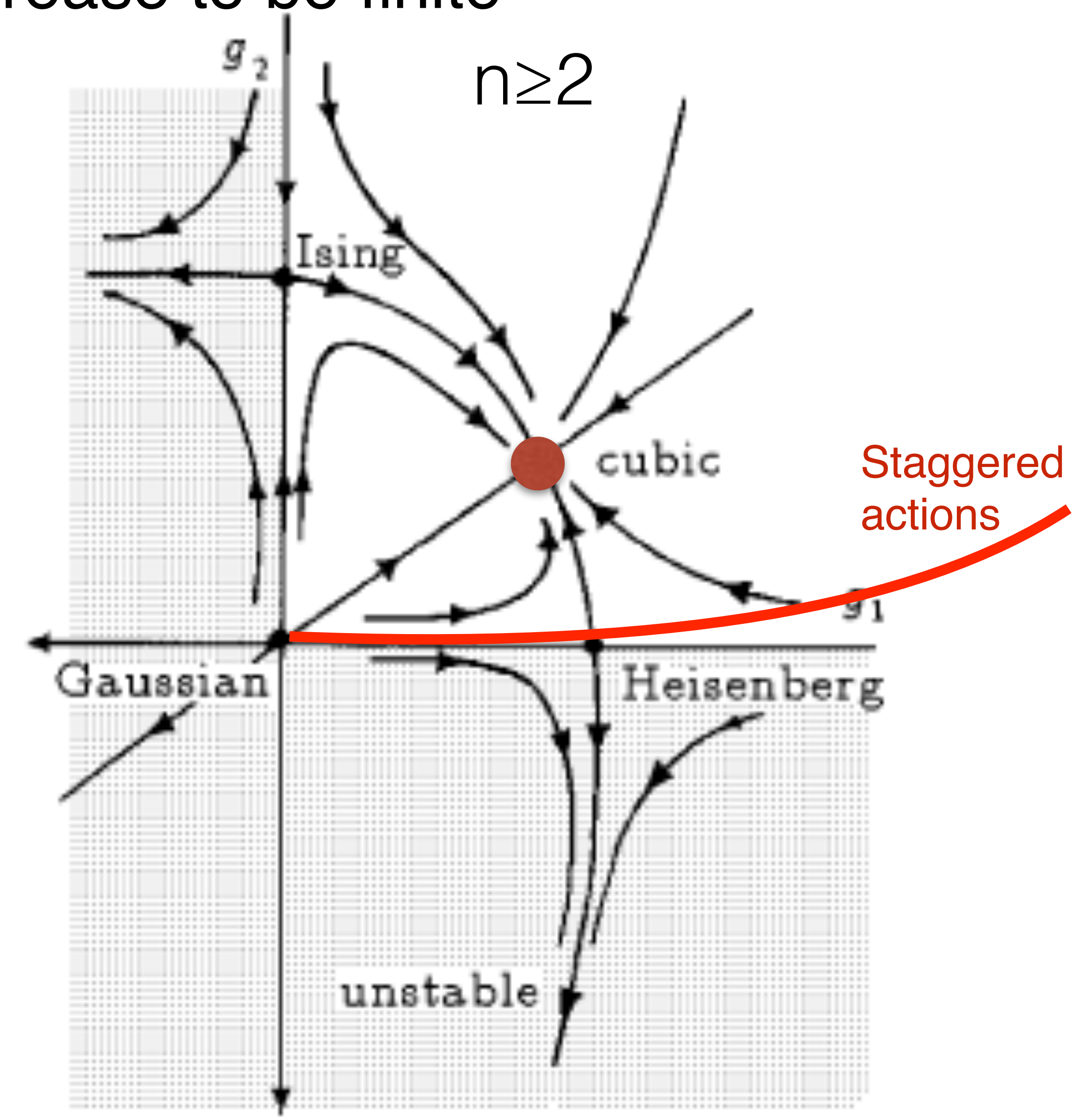
- there is no stand-alone action with “staggered FP” like “Ising FP”

Counter:

Taste breaking terms vanish at $g^2=0$ but the IRFP is at finite g^2 ;
even the smallest symmetry breaking term will increase to be finite



⇒



Numerical test

Compare the **renormalized** step scaling functions (discrete β funct) in the same gradient flow renormalization scheme

RG β-function from gradient flow

Gradient flow:

- continuous and invertible transformation
- the flow time is related to energy scale $\mu = 1/\sqrt{8t}$
- renormalized coupling is defined as

$$g_c^2(\mu) = \frac{128\pi^2}{3(N^2-1)} \langle t^2 E(t) \rangle$$

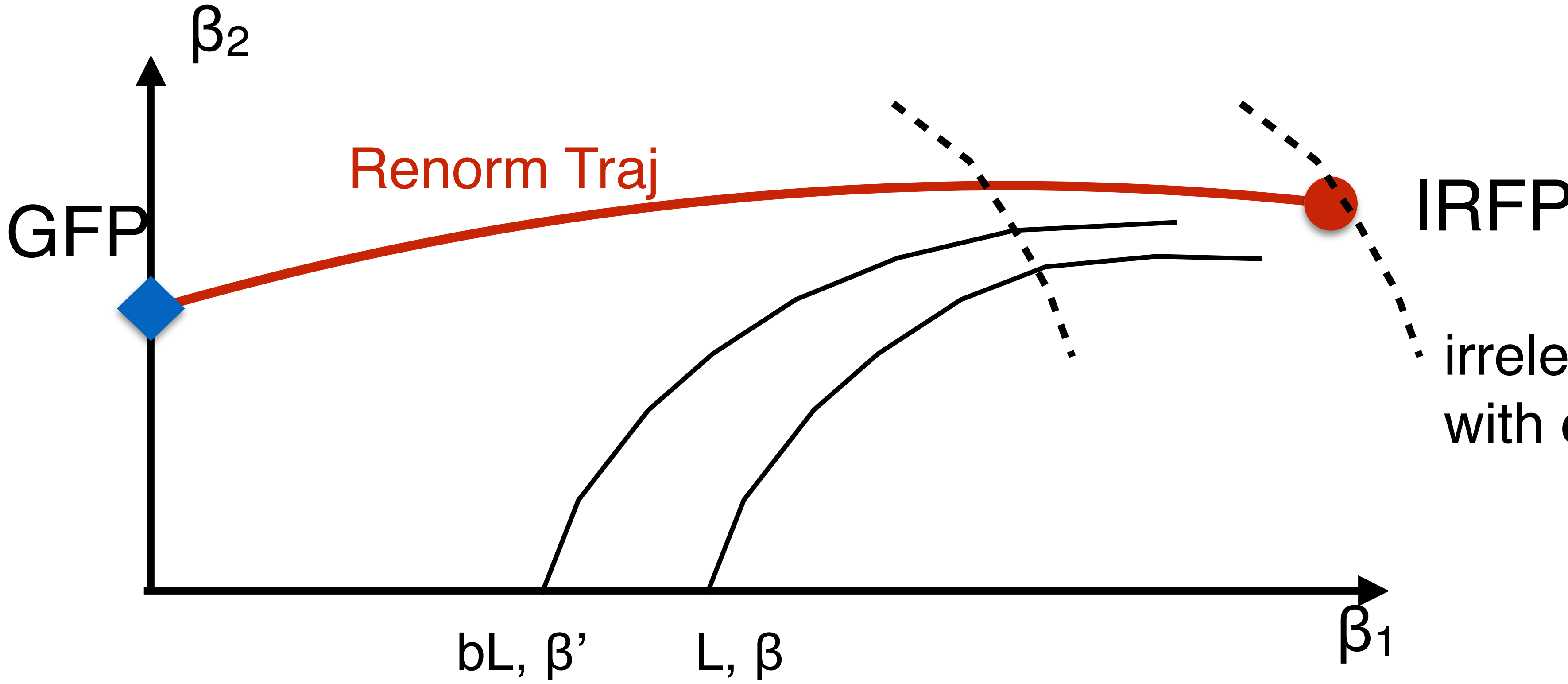
On the lattice at criticality (m=0)

- volume sets the scale: $\mu^{-1} = \sqrt{8t} = cL$, c=const
- step scaling function

$$\beta_s(g_c^2;L) = \frac{g_c^2(sL;\beta) - g_c^2(L;\beta)}{\log(s^2)}$$

- at finite L $\beta_s(g_c^2;L)$ contains lattice artifacts

Lattice artifacts:



- There are no lattice artifacts on the RT ;
- Away from the RT : lattice artifacts ~ distance from RT

"Continuum limit" is $L \to \infty$;
It is approached with the critical exponent of the irrelevant operator α

$$\beta_s(g_c^2; L) = \beta_s(g_c^2; L = \infty) + \kappa L^{\alpha}$$

Around the GFP α = -2 ; Around the IRFP α is unknown!

Infinite volume limit

Infinite volume limit as

$$\beta_s(g_c^2;L)=\beta_s(g_c^2;L=\infty)+\kappa L^\alpha$$

Determining the exponent α is difficult

➠ consider different flows and operators with the same α and $\beta_s(g_c^2;L=\infty)$ to constrain the fit :

Action: W plaquette or Symanzik (hard to change)
Flow: W plaquette, Symanzik or Zeuthen
Operator: W plaquette, Symanzik or Clover

S Z S	optimized perturbatively (Ramos, Sint)
W W C	large cancellations of lattice artifacts
S S C	poor perturbatively but might be good at strong coupling (LatHC)

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Is there a combination that removes most artifacts making L→∞ straightforward?

S, Z, mS + (1-m)W

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S, Z, mS + (1-m)W

S, m*S+(1-m)W, C

(LatHC)

N_f = 12 fundamental flavors

The IR nature of the system is controversial despite extensive

- spectrum studies
- finite size scaling studies
- step scaling function investigations

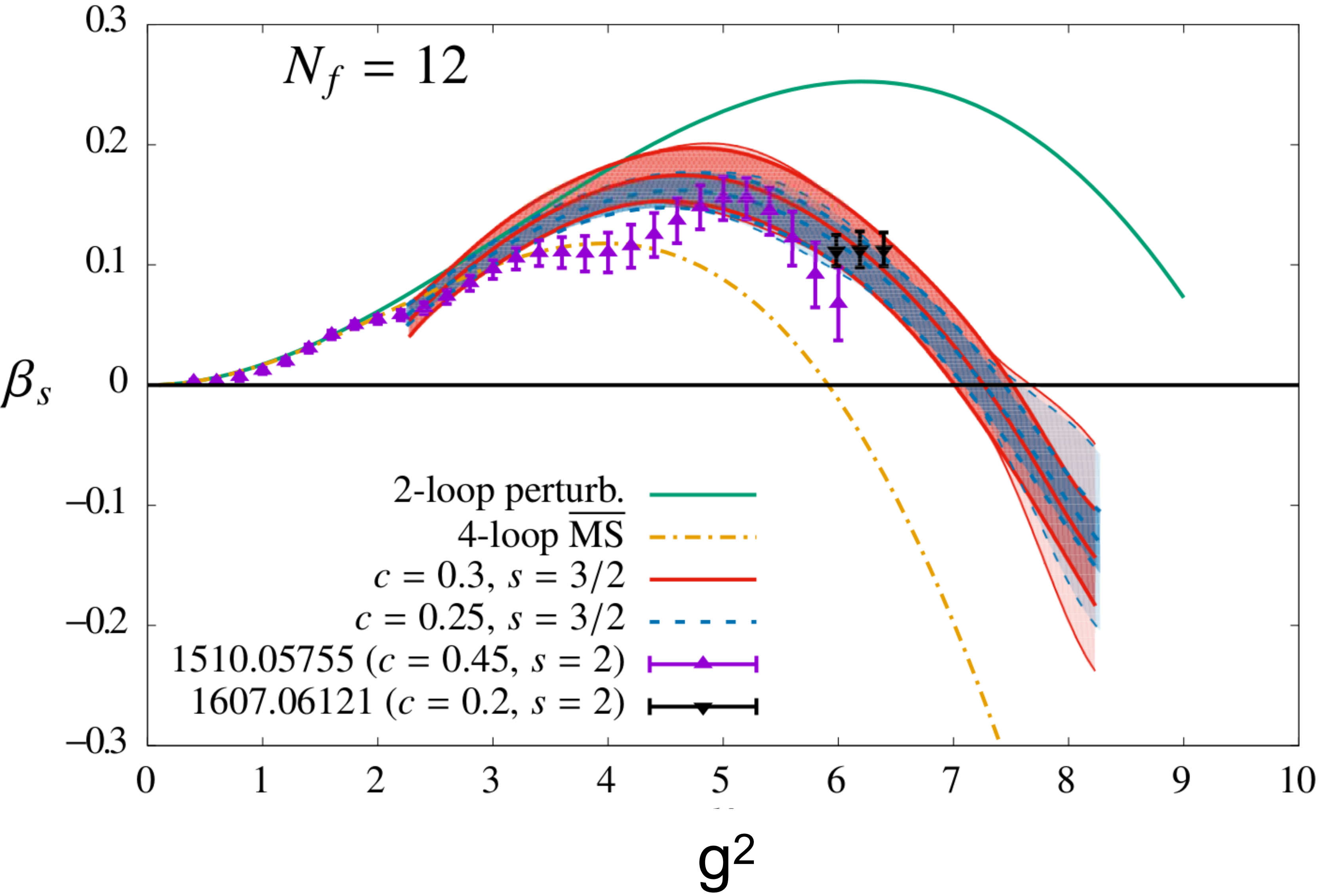
by several groups (LSD, LatHC, LatKMI, Boulder, etc)

The N_f = 12 controversy

SU(3) gauge, 12 fundamental staggered fermions

A.H., D. Schaich, ArXiv:1610.10004

gradient flow step scaling



red/blue band: AH,Schaich

purple: Lin, Ramos

black: Fodor et al (2016)

All staggered, but different actions, flows, fits.

Remarkable consistency even between different c values for $g^2 < 6.5$

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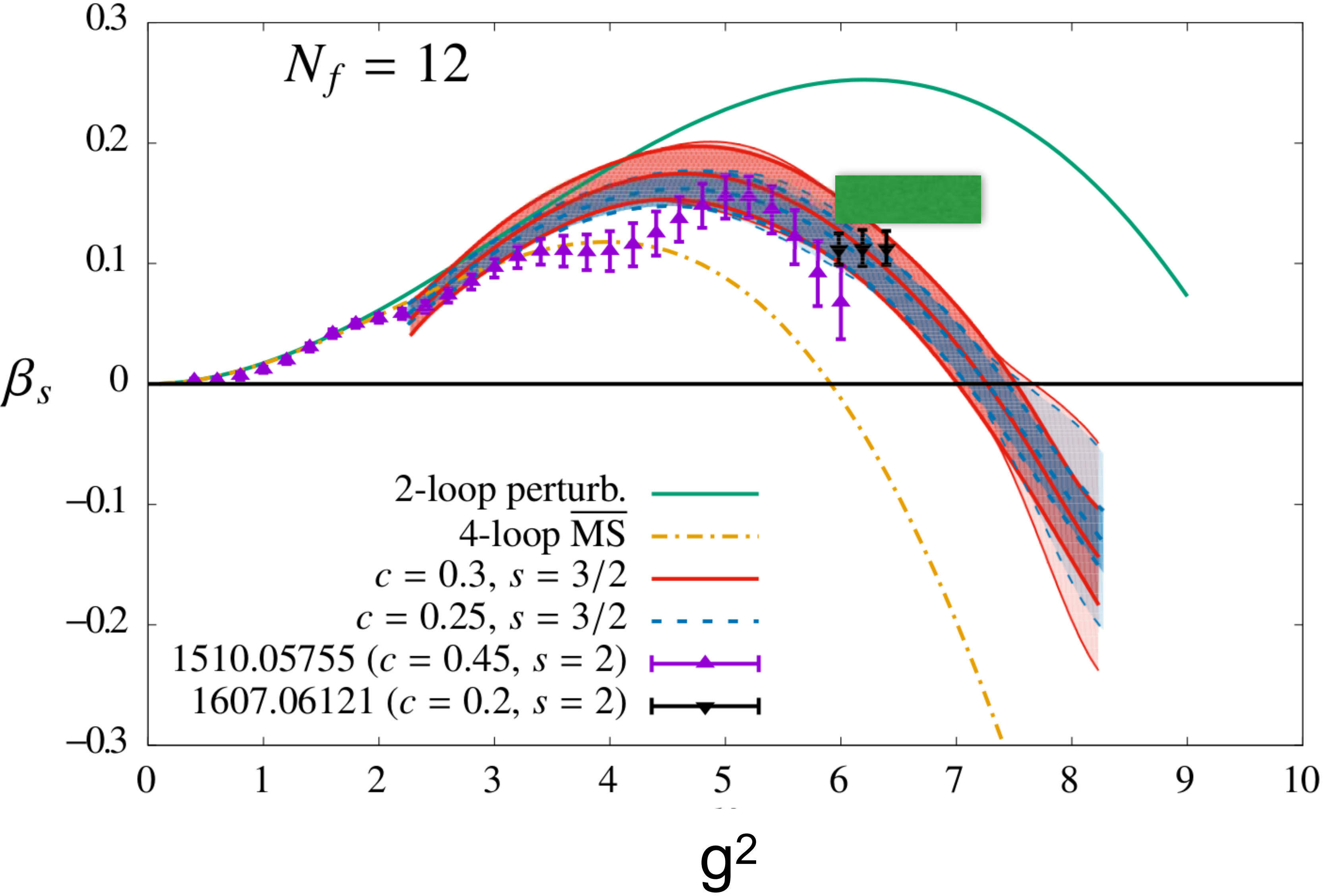

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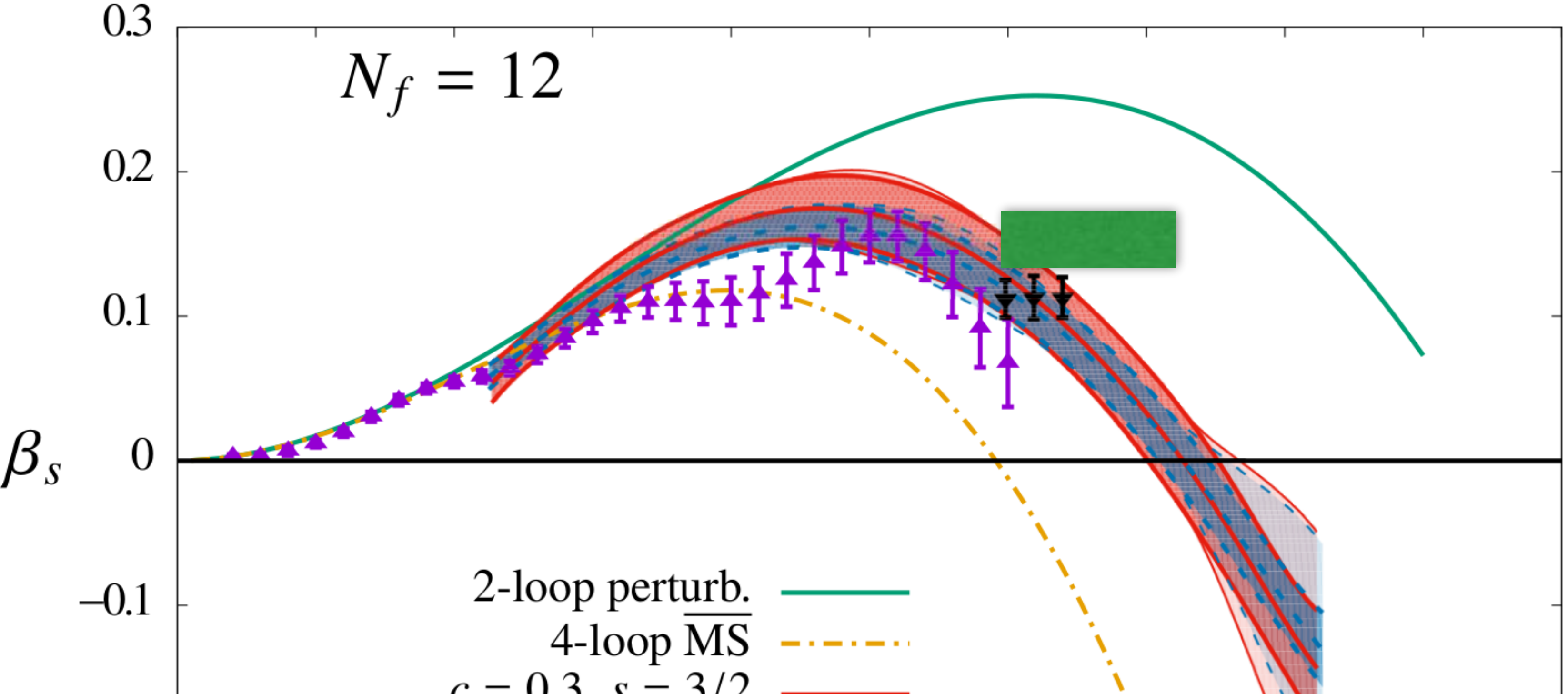

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All staggered, but different actions, flows, fits.

These calculations used $1/L^2$ continuum extrapolation
— not justified near an IRFP

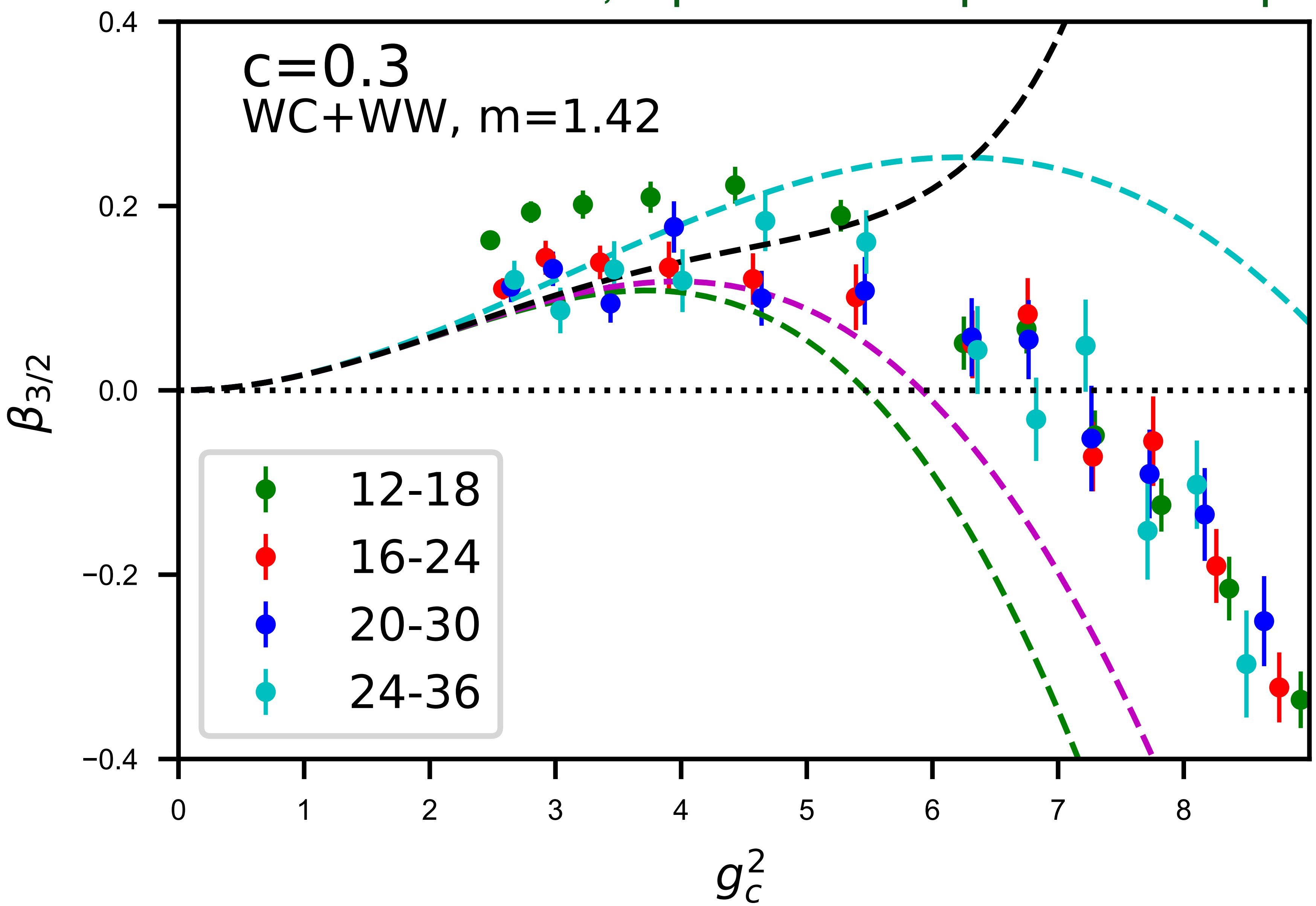
Staggered step scaling - revisited

Fitting an exponent is hard;

- reduce/remove L dependence so ω has no effect

Mixing of operators or flows or both can do just that

Wflow; Op = 1.42*Cop - 0.42 *Wop



Dashed lines:
Cyan: 2-loop
Green: 3-loop
Magenta:4-loop
Black: 5-loop
perturbative

This is raw data:
no interpolation,
no extrapolation,

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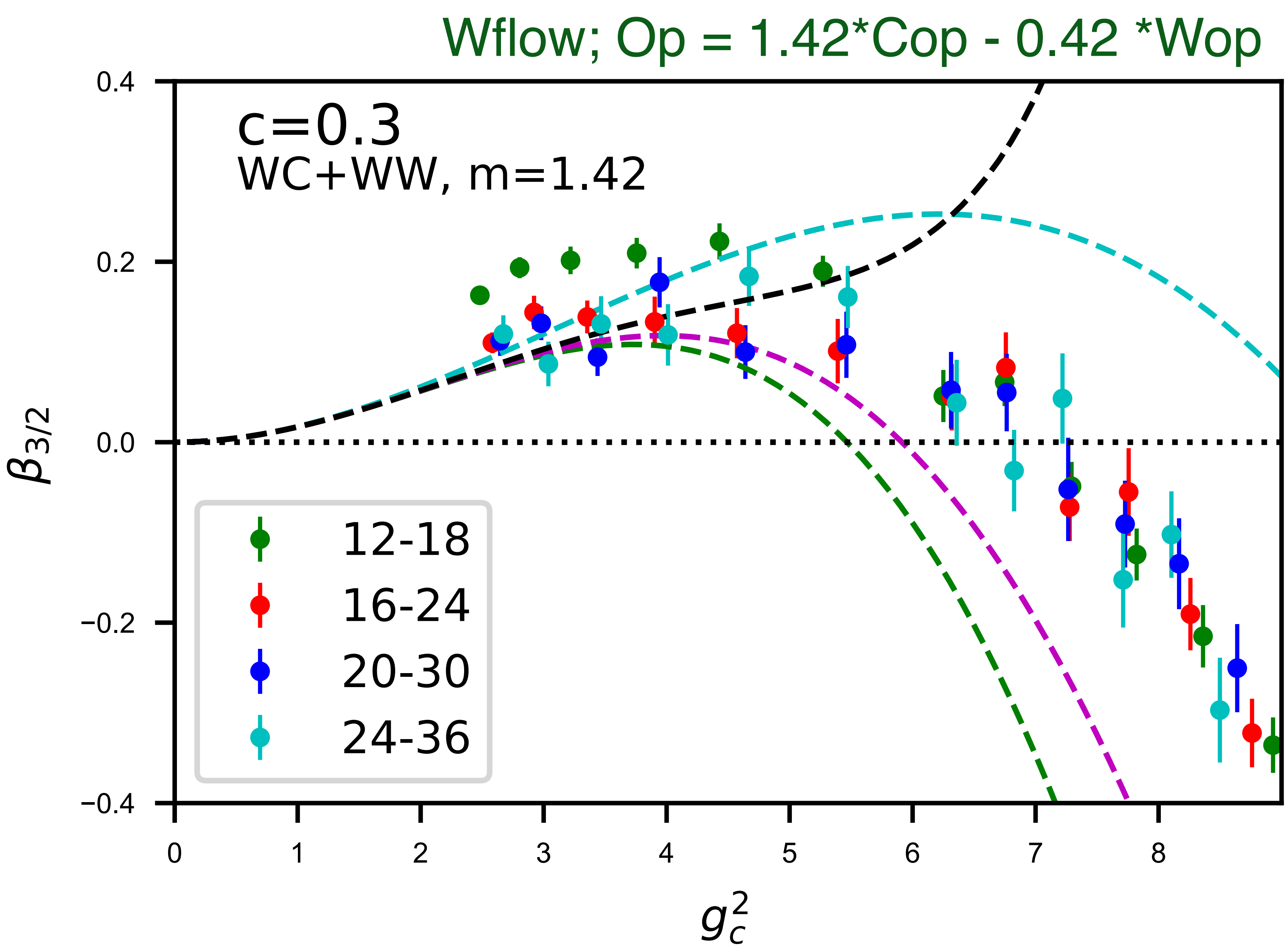

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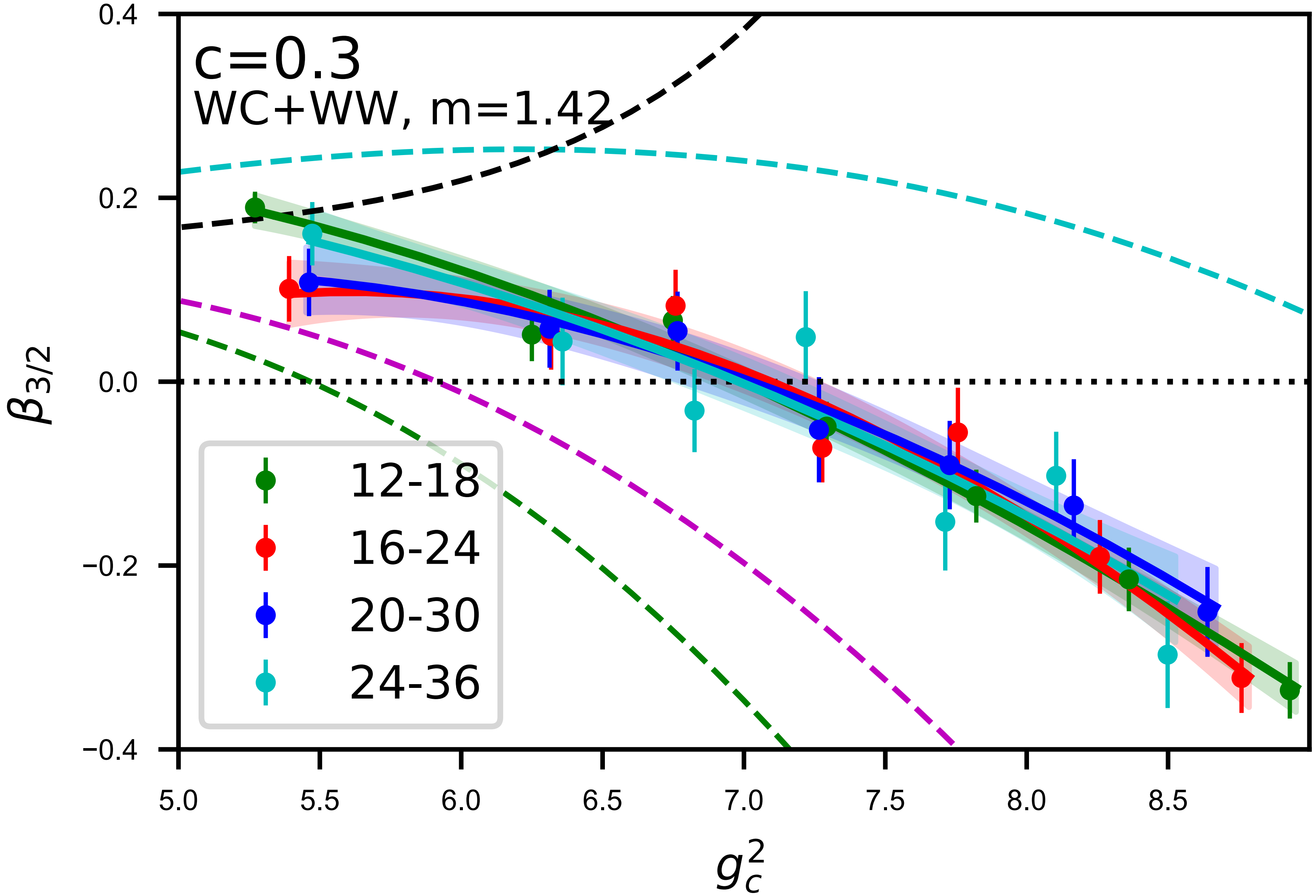
Staggered step scaling - revisited

Prediction of finite volume step scaling function

- No significant volume dependence
- large region with negative step scaling function



Mixing parameter near optimal in $6.5 < g^2 < 8.0$ range

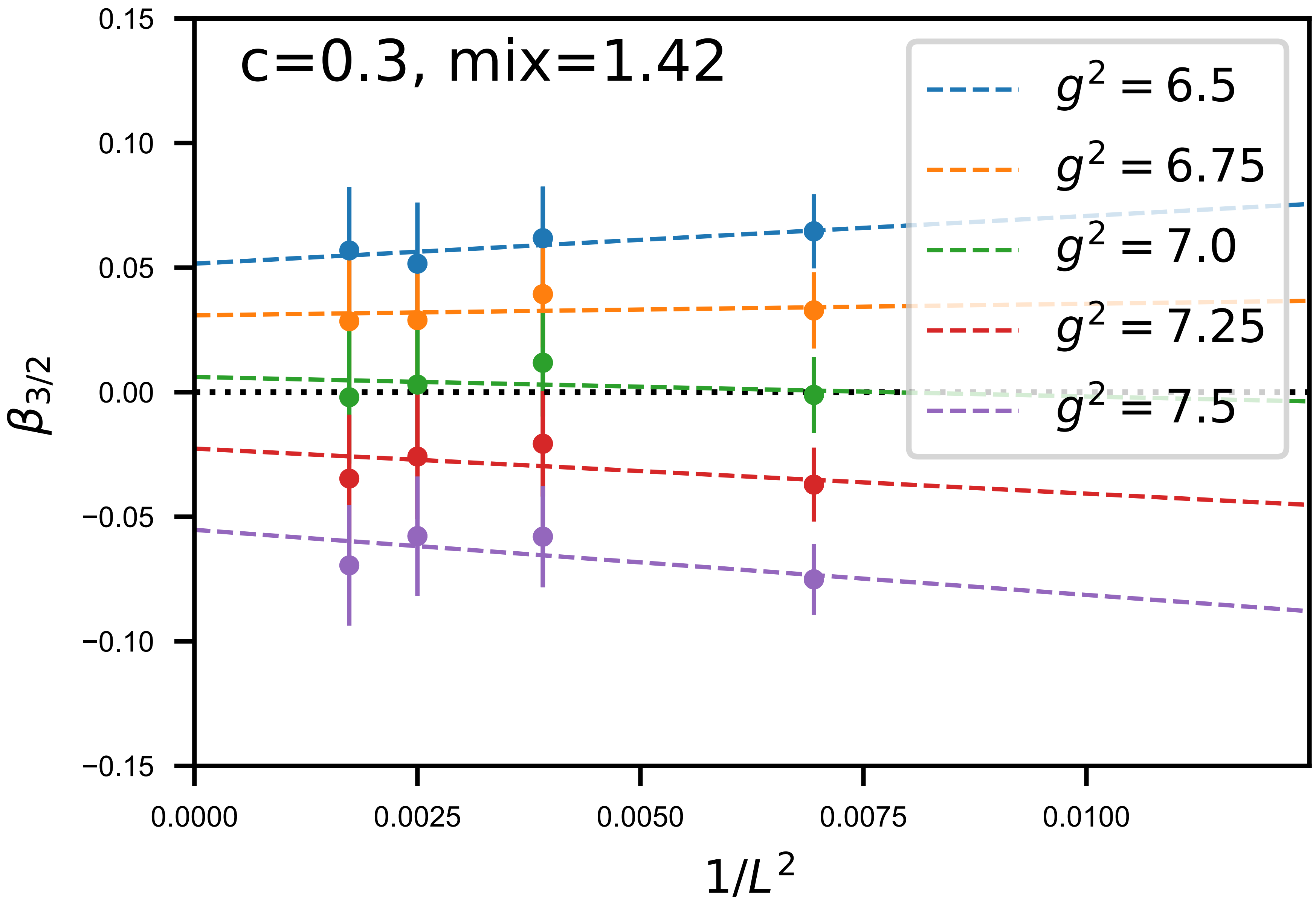
Staggered step scaling - revisited

L → extrapolation:

- No significant volume dependence :

  $1/L^2$ or $1/L^\omega$ extrapolations are not significantly different

- large region with negative step scaling function



Volume pairs:
12-18
16-24
20-30
24-36

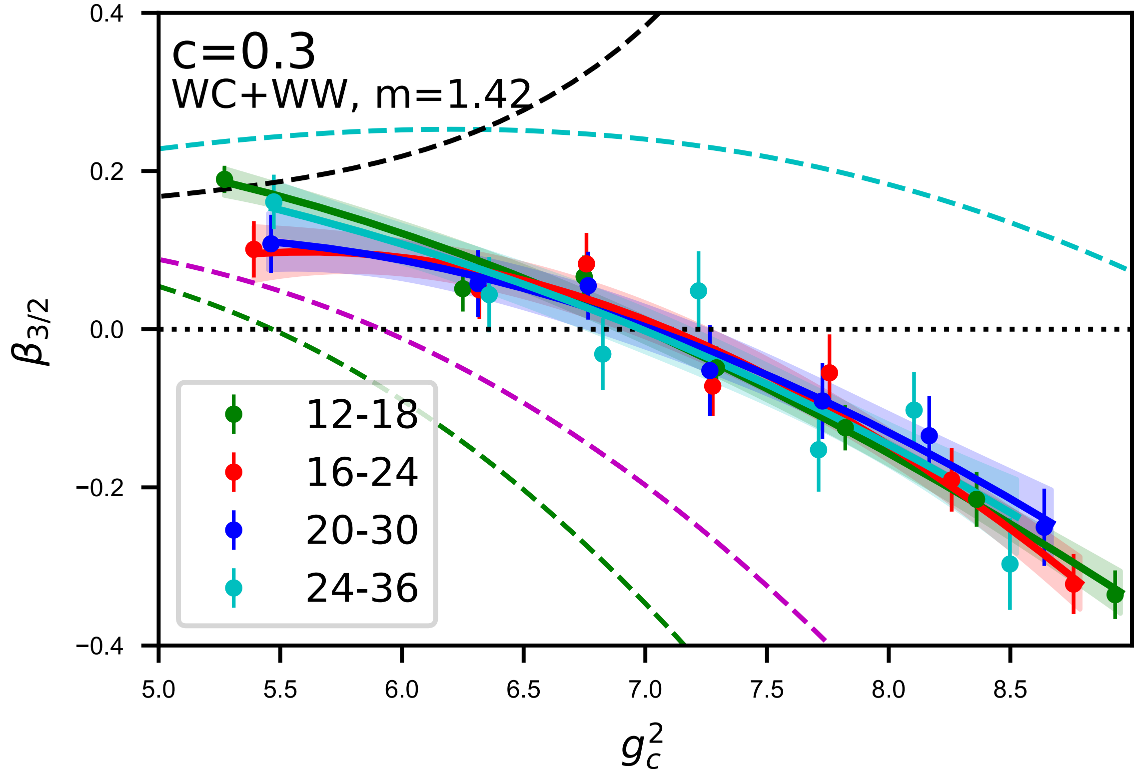
Staggered step scaling - summary

At a conformal IRFP L → extrapolation has an undetermined exponent

- Avoid it by using operator/flow with minimal volume dependence
- Determine it by combining different flows/operators, and require consistency

Neither approach will 'mimic' an IRFP; If the system is not conformal, both approaches are still correct

It is hard to imagine how the present data could avoid a FP



Domain Wall study

Simulations:

- 3-stout smeared Mobius DW fermions
- Symanzik gauge action
- Periodic BC for gauge, antiperiodic in all 4 directions for fermions
- Volumes 8^4 - 32^4
- L_5 = 12 in most cases; 16 in some, 24 and in others : needed to control residual mass $m_{res} < 10^{-5}$

First results: A.H, C. Rebbi, O. Witzel, ArXiv:1710.11578

We greatly appreciate the opportunity to use the new GRID code while still in development and the help we received from
Peter Boyle, Guido Cossu, Antonin Portelli, and Azusa Yamaguchi

Domain Wall study

Are DW and staggered fermion in the same universality class at an IRFP?

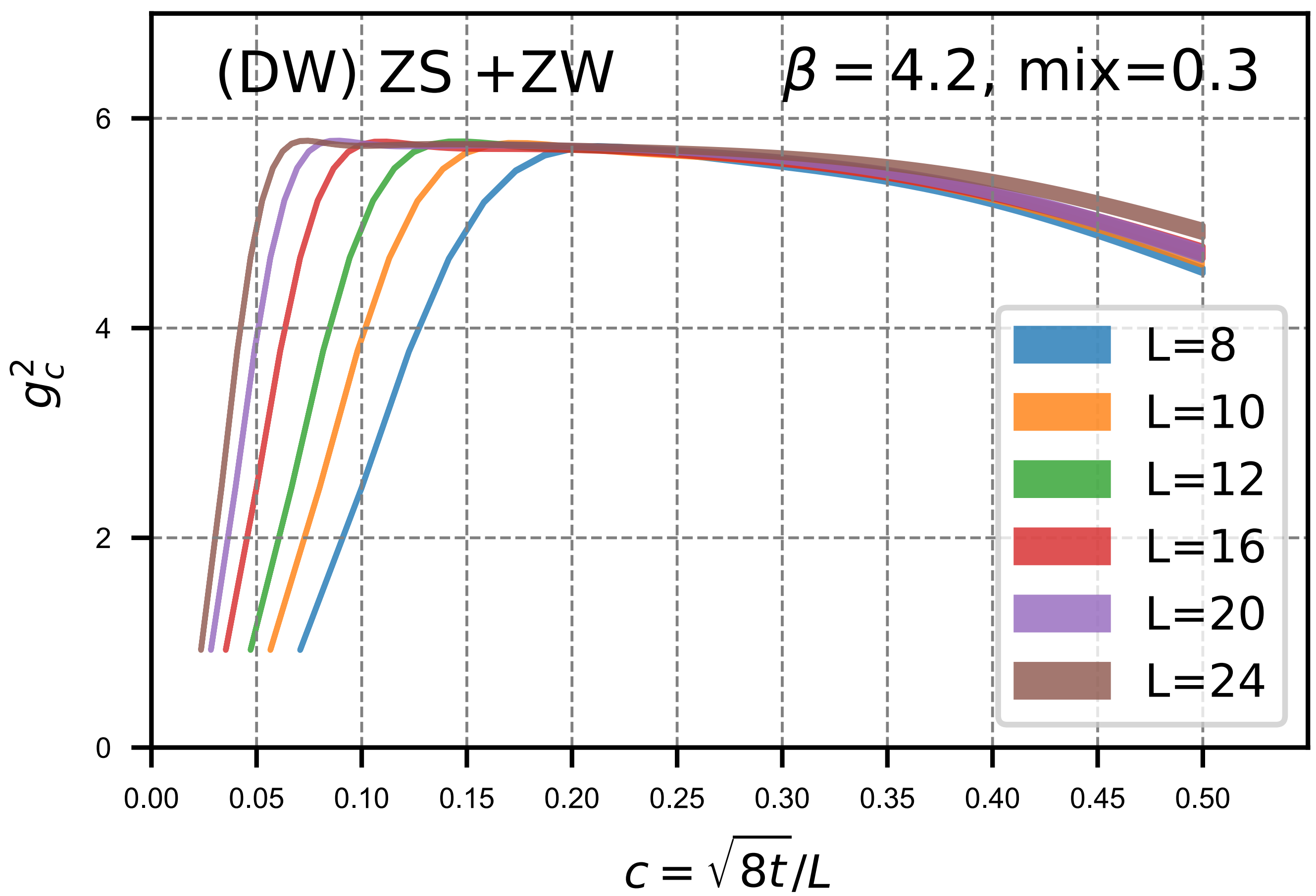
- Compare the renormalized step scaling functions

In ArXiv:1710.11578 we extrapolated in $1/L^2$ - valid in the vicinity of $g^2 = 0$

New results :

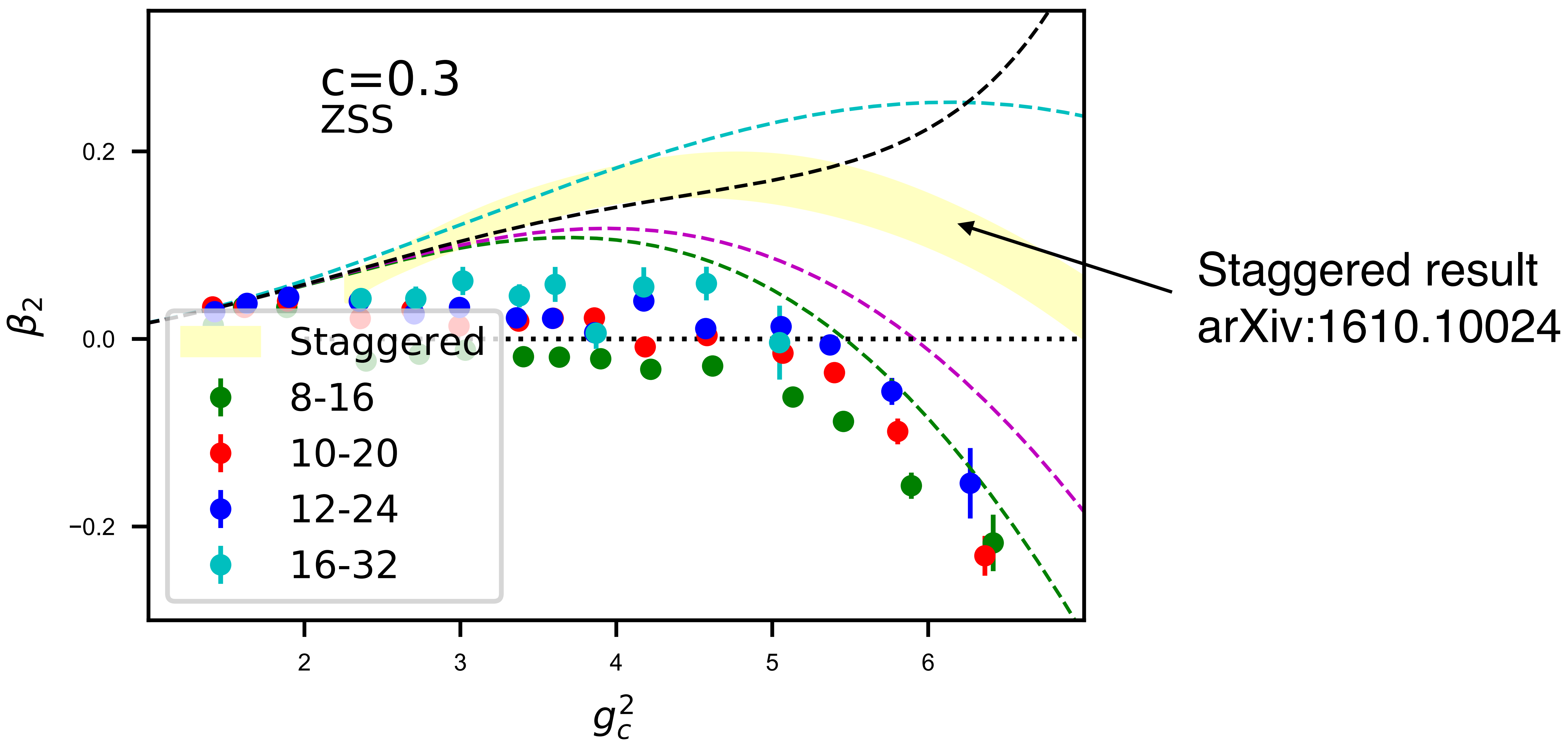
- Extended statistics
- added S and W flows, W,S and C operator

Smaller volumes
(8-16, 10-20, 12-24, 14-28, 16-32)
are sufficient with c=0.3



DW step scaling

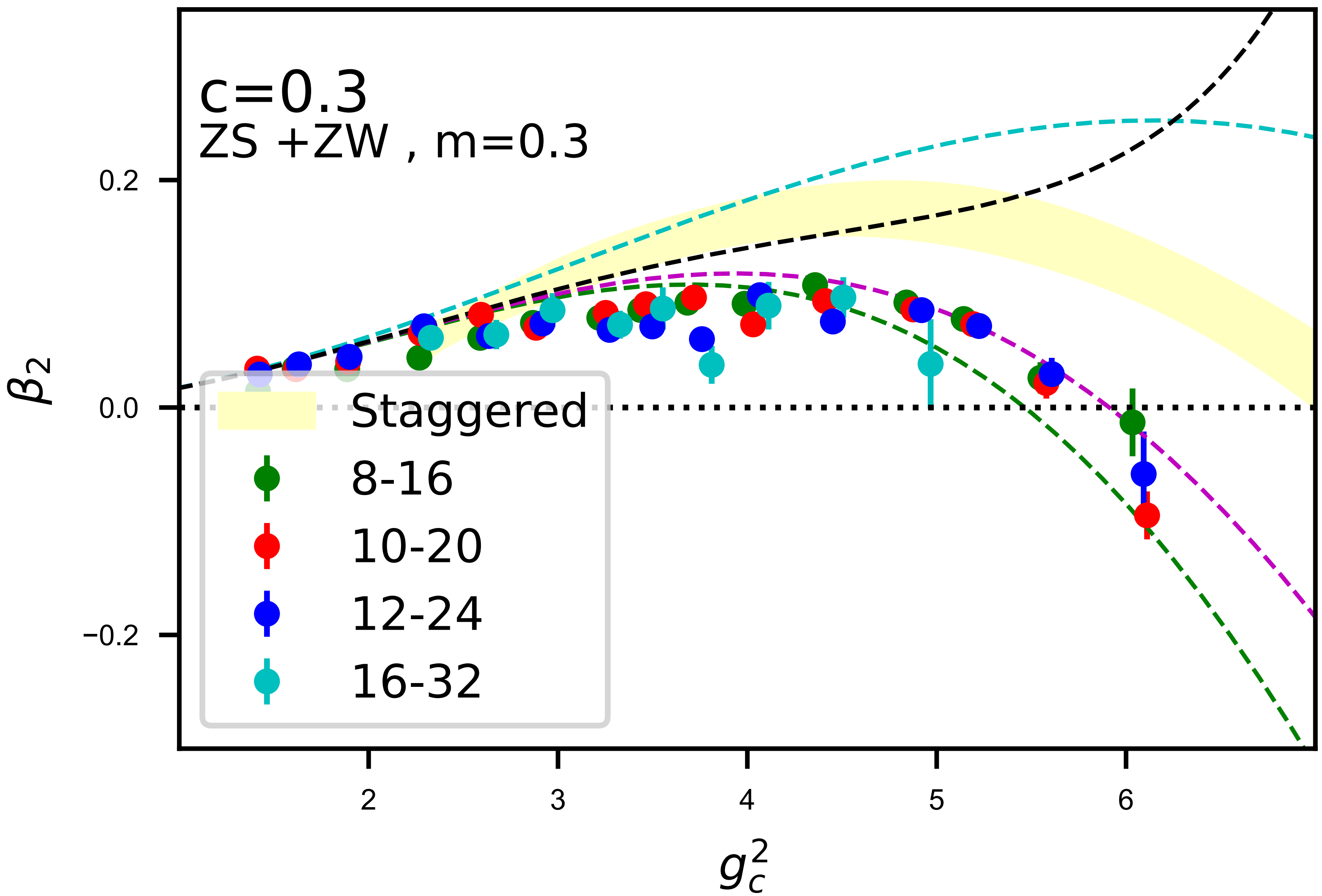
Z-flow, S-operator, (S-action) shows small volume dependence



We have more large volume data - will be published soon

DW step scaling

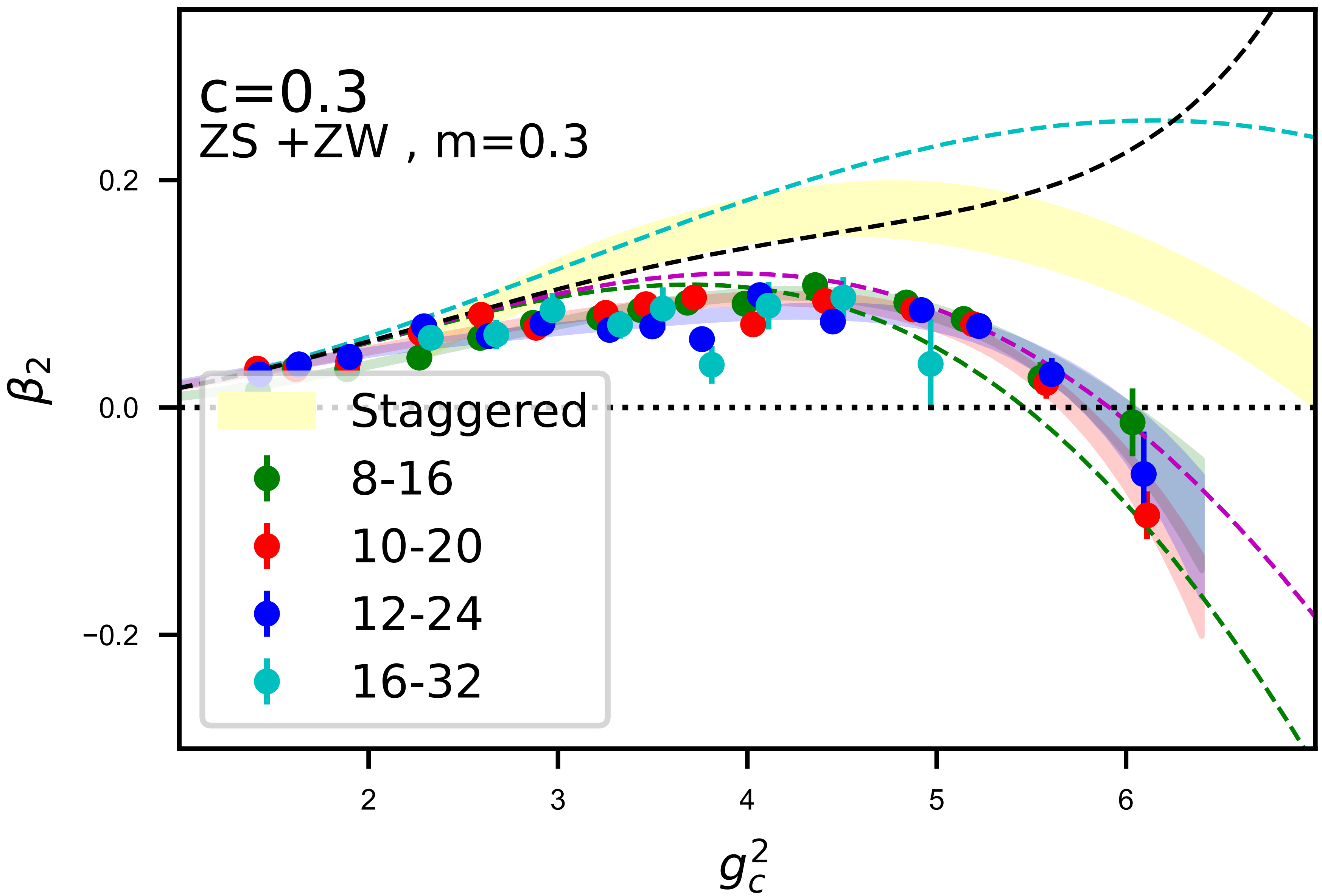
Mixing Op = S * 0.3 + W * 0.7 removes most volume dependence
(mixing W and C would do the same)



This is raw data
no interpolation
no extrapolation

DW step scaling

Fit the finite volume step scaling function directly
(Different from staggered analysis)



motivated by PT:

$$\beta_2 = g^4(a + bg^2 + cg^4)$$

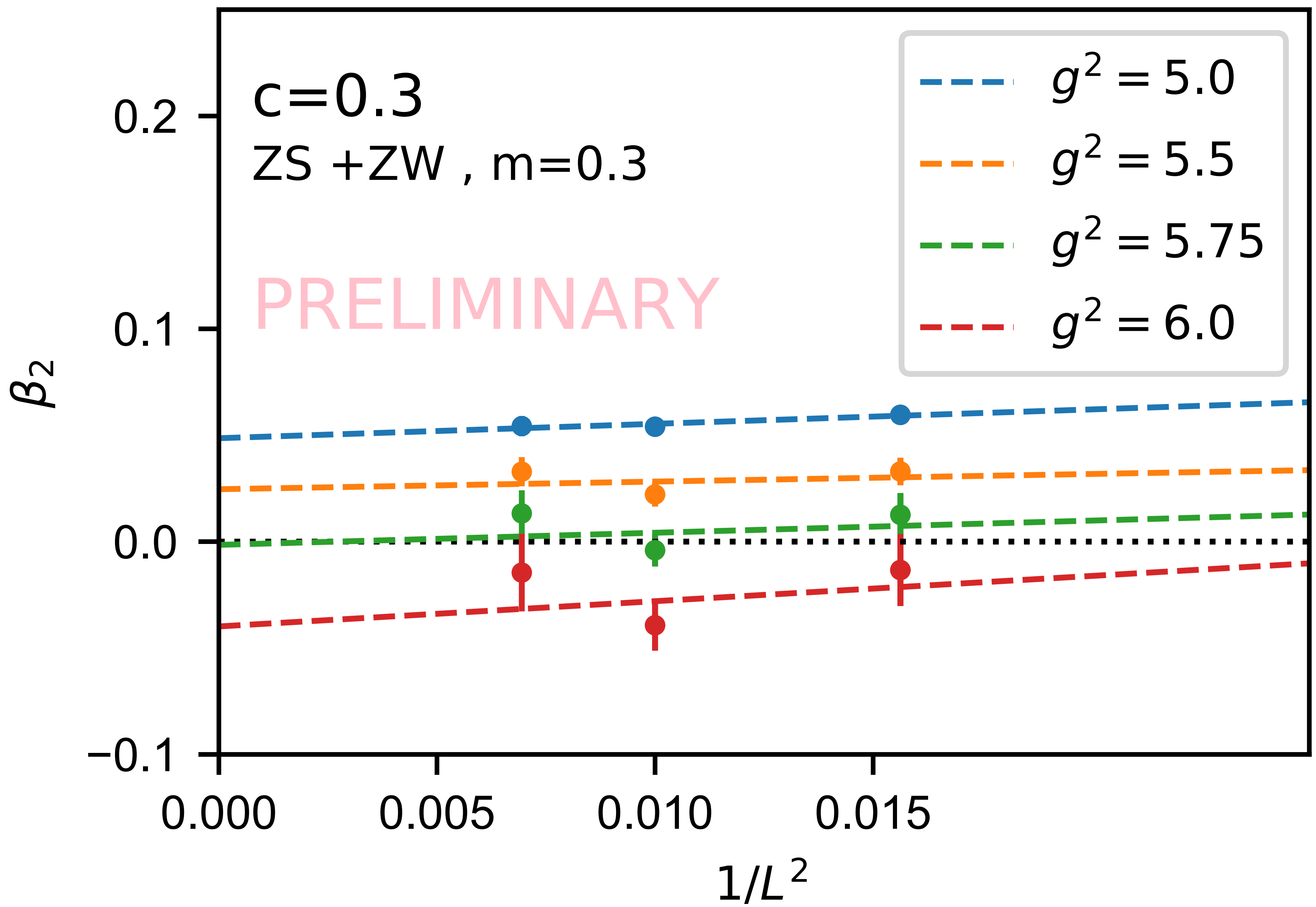
DW step scaling

L → extrapolation:

- No significant volume dependence :

  L extrapolation is not sensitive to the exponent α

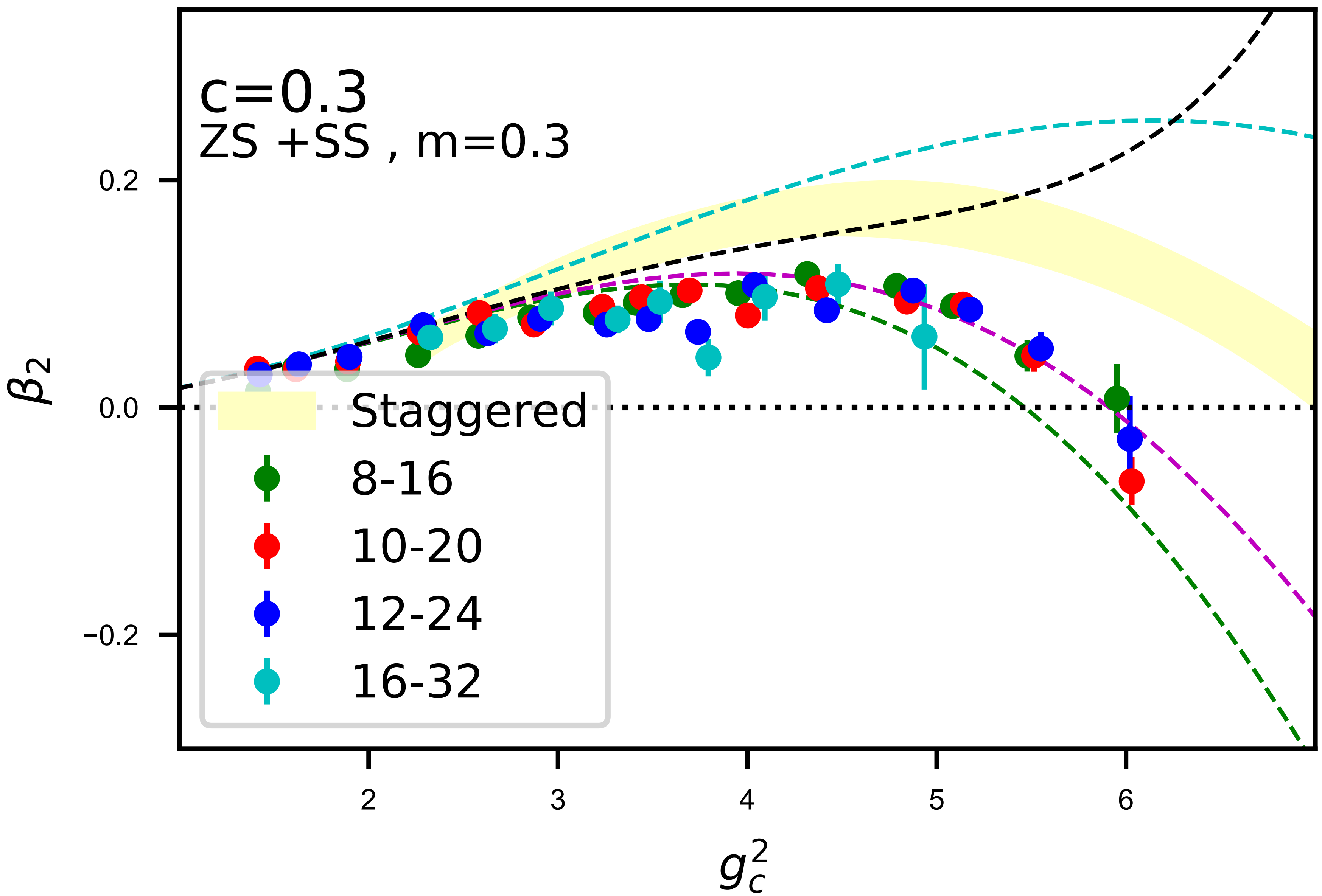
- strong indication of zero/negative step scaling function



Volume pairs:
8-16
10-20
12-24

Combine different flows

Mixing ZFlow=Z * 0.3 + S * 0.7 ; S op, S action;
mixing removes most volume dependence



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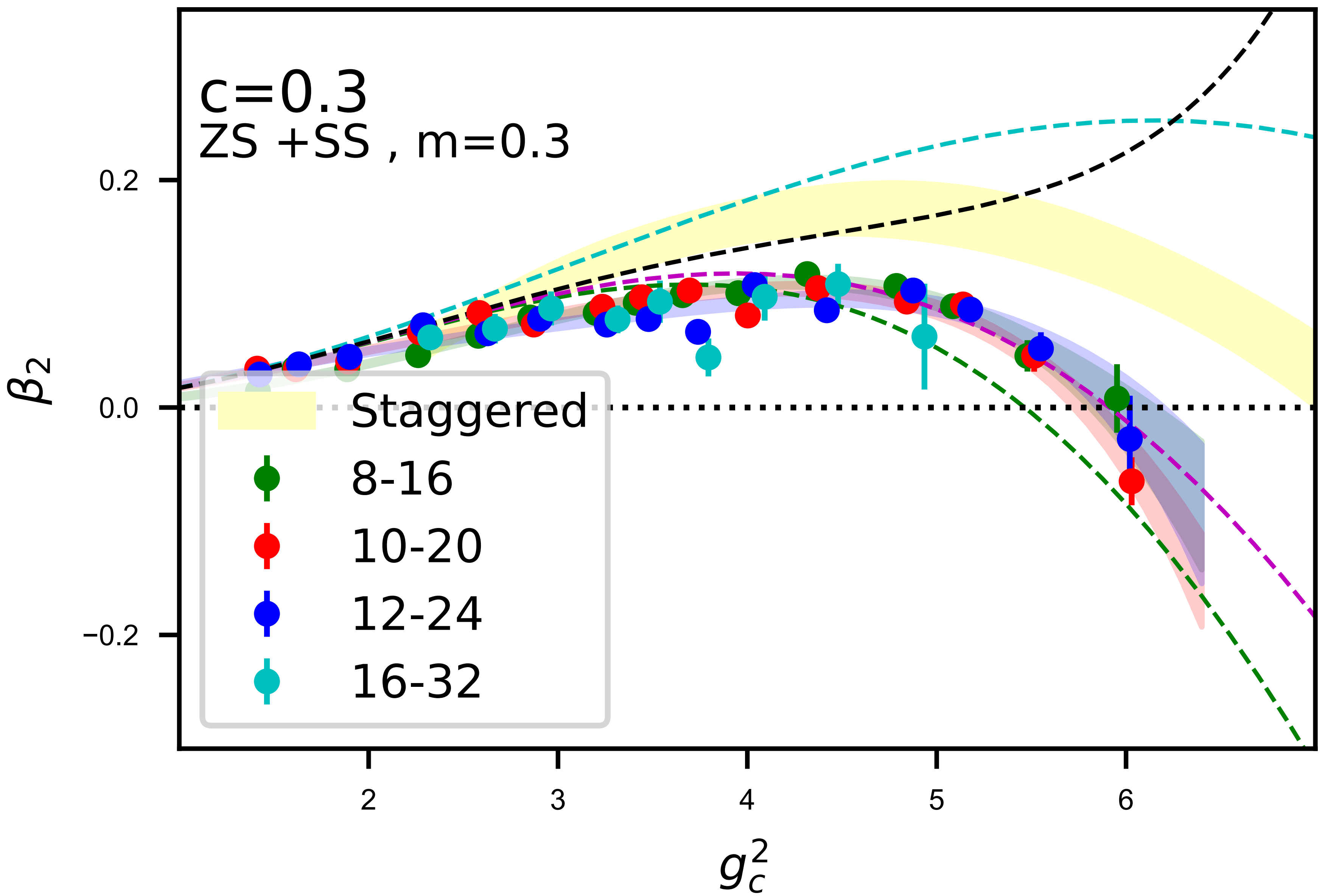
DW step scaling

Fit the finite volume step scaling directly



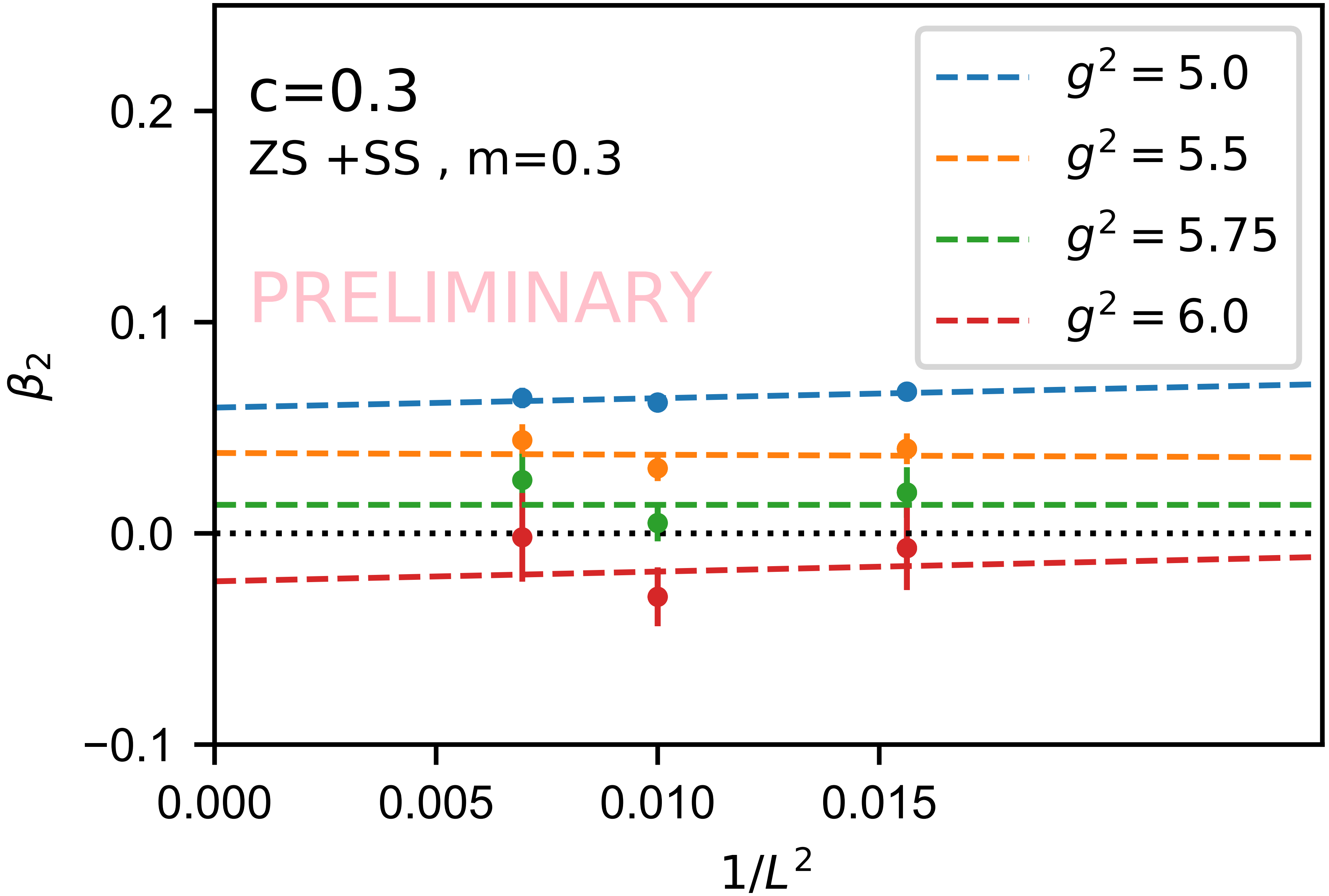
DW step scaling

L → extrapolation:

- No observable volume dependence :

  $1/L^2$ or $1/L^\omega$ extrapolations are not significantly different

- strong indication of zero/negative step scaling function



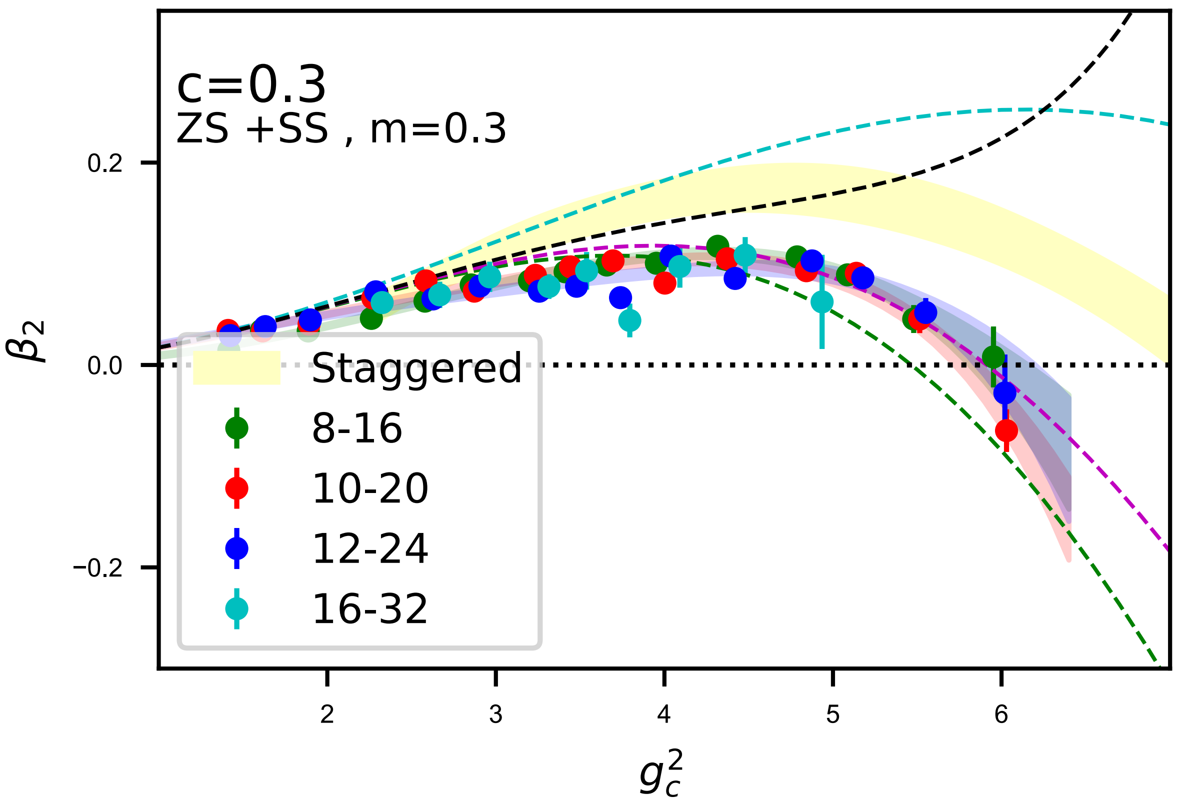
Volume pairs:
8-16
10-20
12-24

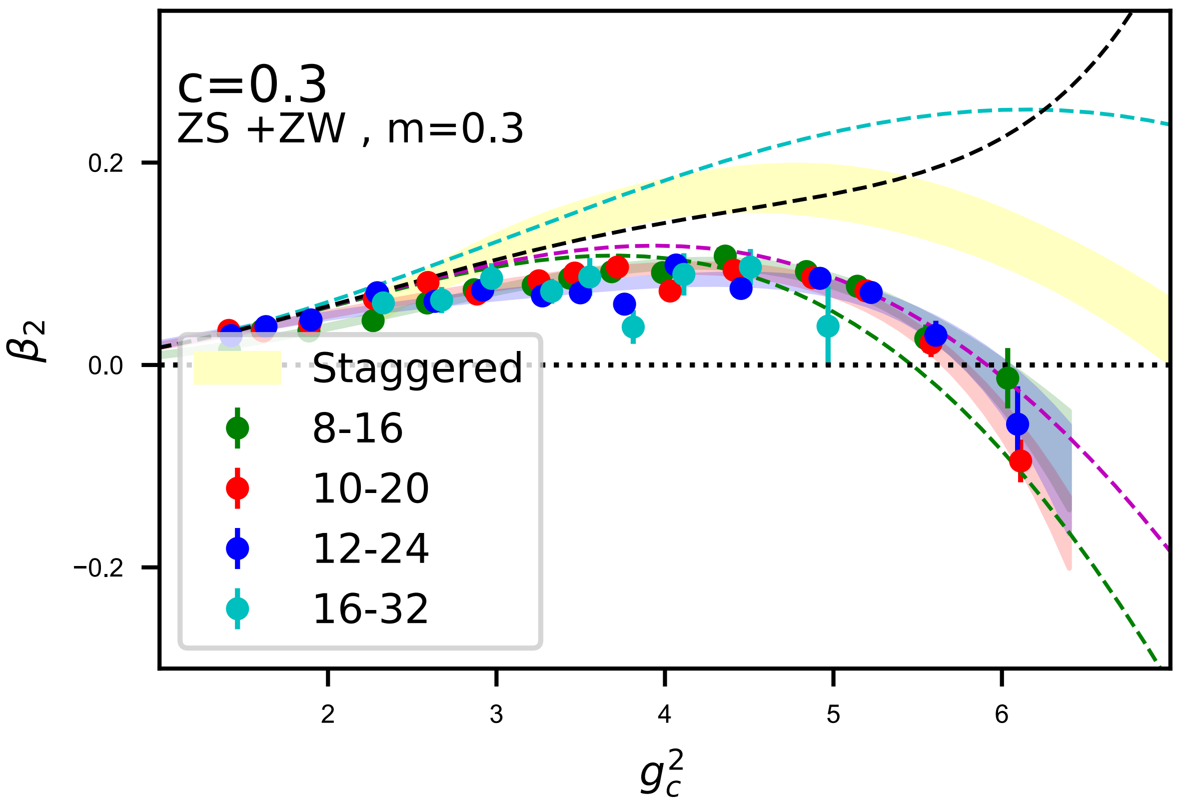
Compare:

Mixing operators:
ZS * 0.3 + ZW* 0.7

Mixing flows:
ZS * 0.3 + SS * 0.7

Both predict the same continuum result





Staggered and DW predictions are significantly different, suggesting that staggered fermions are not in the same universality class as DWF (or continuum) at the conformal IRFP

Conclusion & Summary

- It is (perhaps) not surprising that lattice fermions with different chiral symmetries have different conformal fixed points. I illustrated this via the step scaling function
  - N_f=12 in ArXiv:1710.11578 & new results now
  - N_f=10 in ArXiv:1710.11578
  - Old: SU(3) with 2-flavor sextet: Wilson vs staggered: ArXiv:

- Disagreement between various staggered simulations at strong coupling might be resolved by correct $L\to\infty$ extrapolation. Combination of different flows, operators can be used to determine the leading irrelevant exponent

- Consequence for lattice studies:
  - Models that rely on a conformal IRFP should be simulated using DWF (or Wilson) fermion
  - Expensive, but necessary; Improved chiral properties help

Extra slides

$N_f = 10$

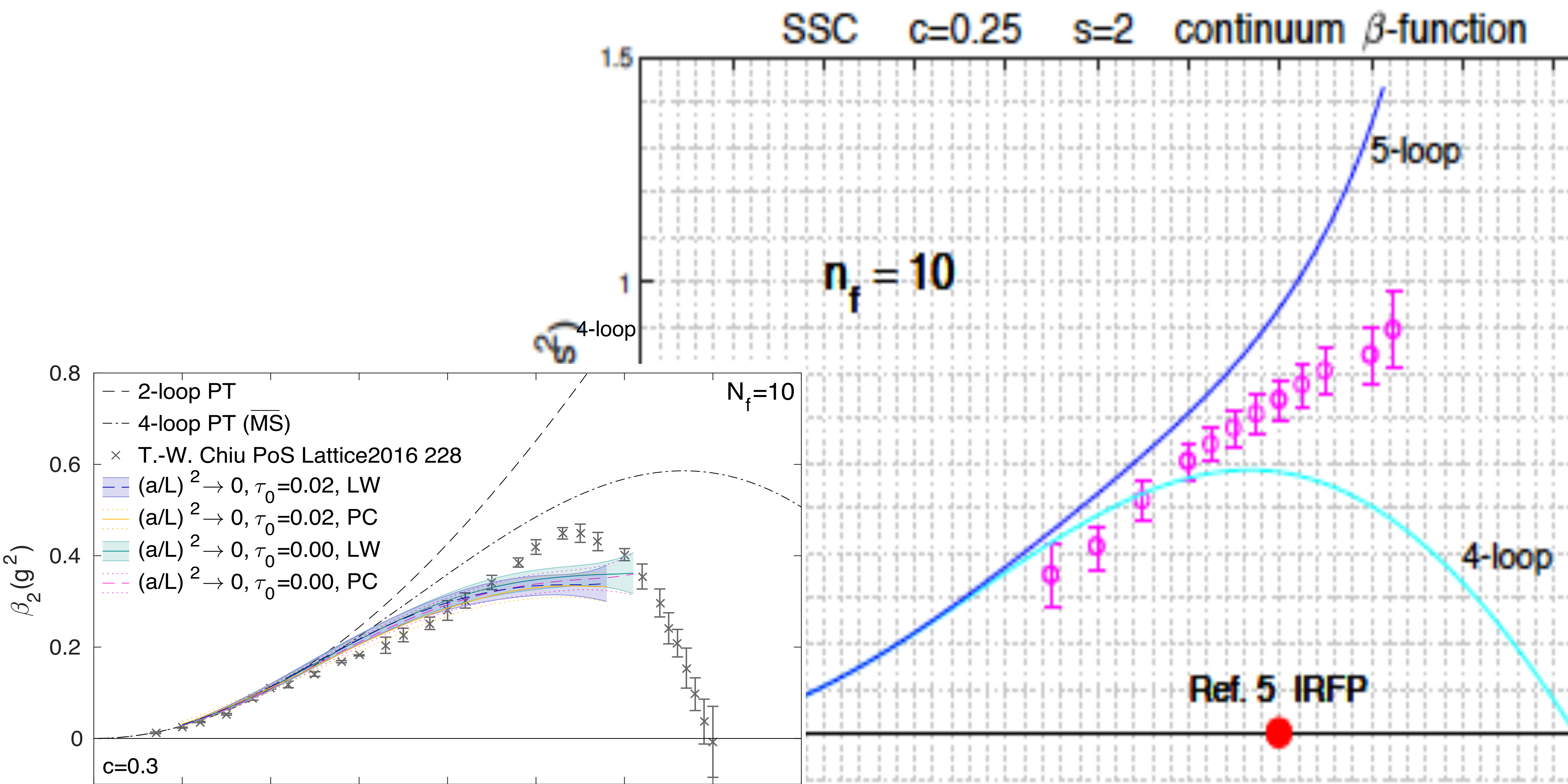
DW :

A.H, C. Rebbi, O. Witzel, ArXiv: 1710.11578

T-W Chiu, ArXiv:1603.08854

Staggered : (rooted):

Fodor et al,ArXiv:1710.09262



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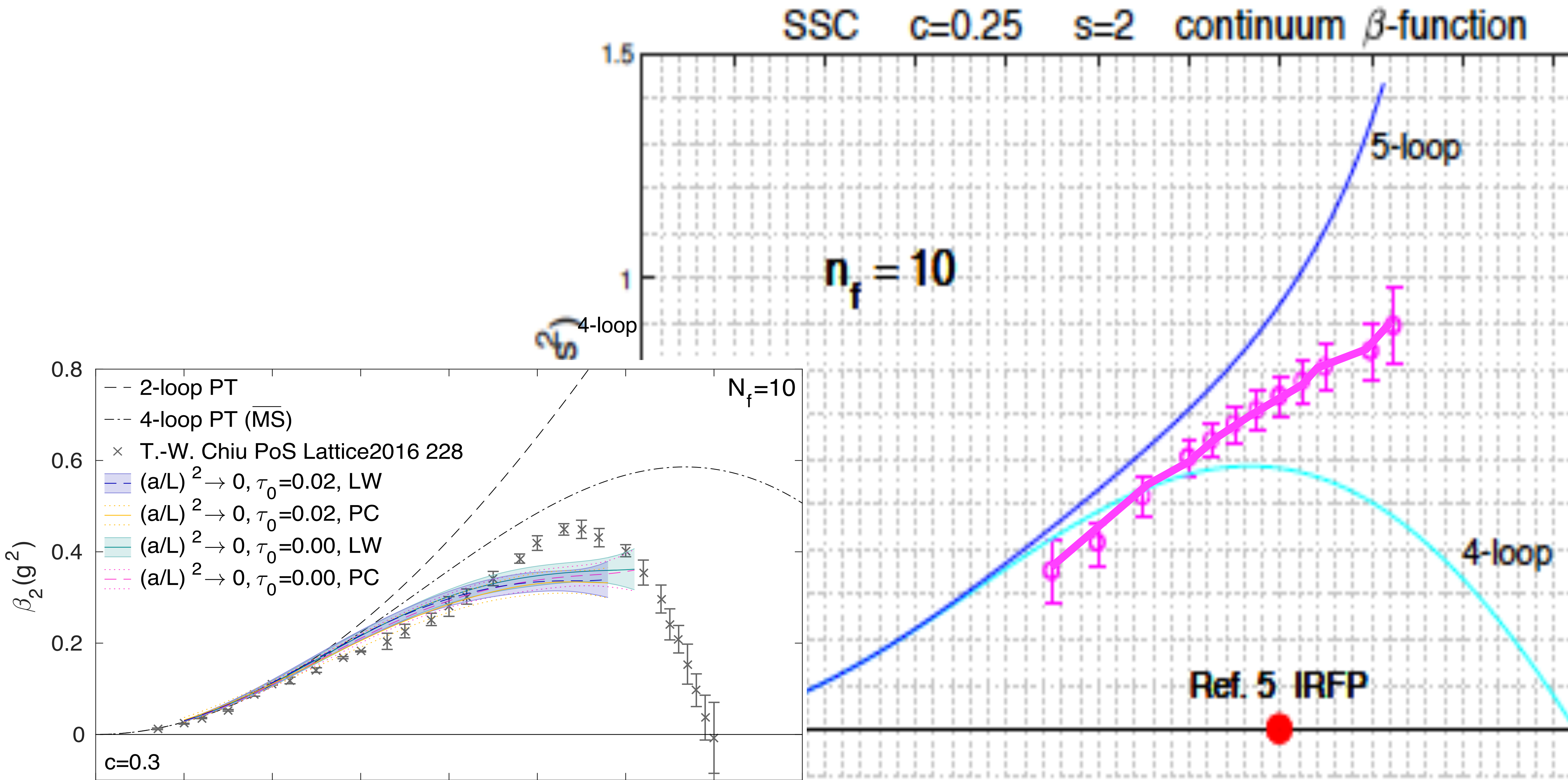

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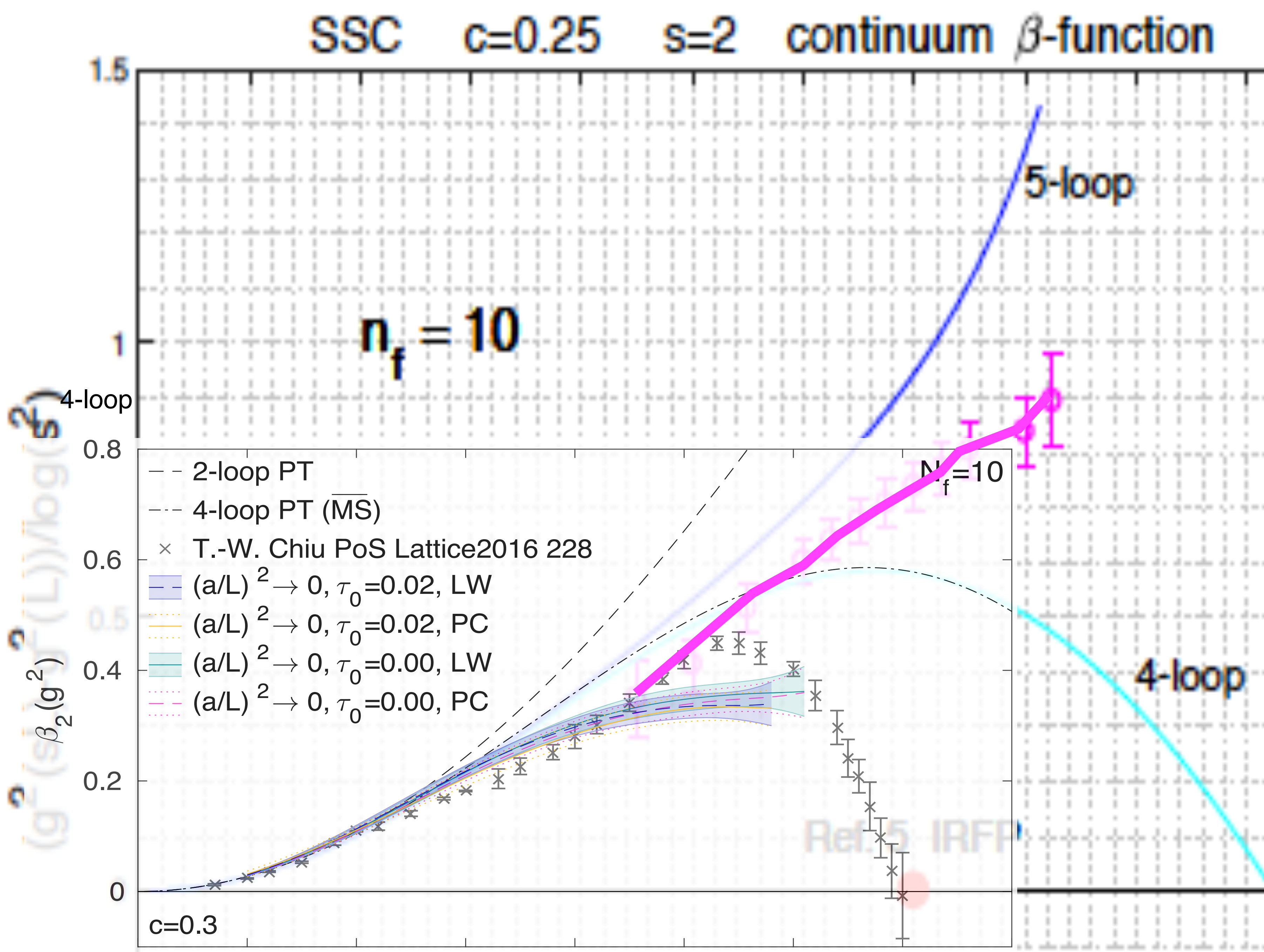
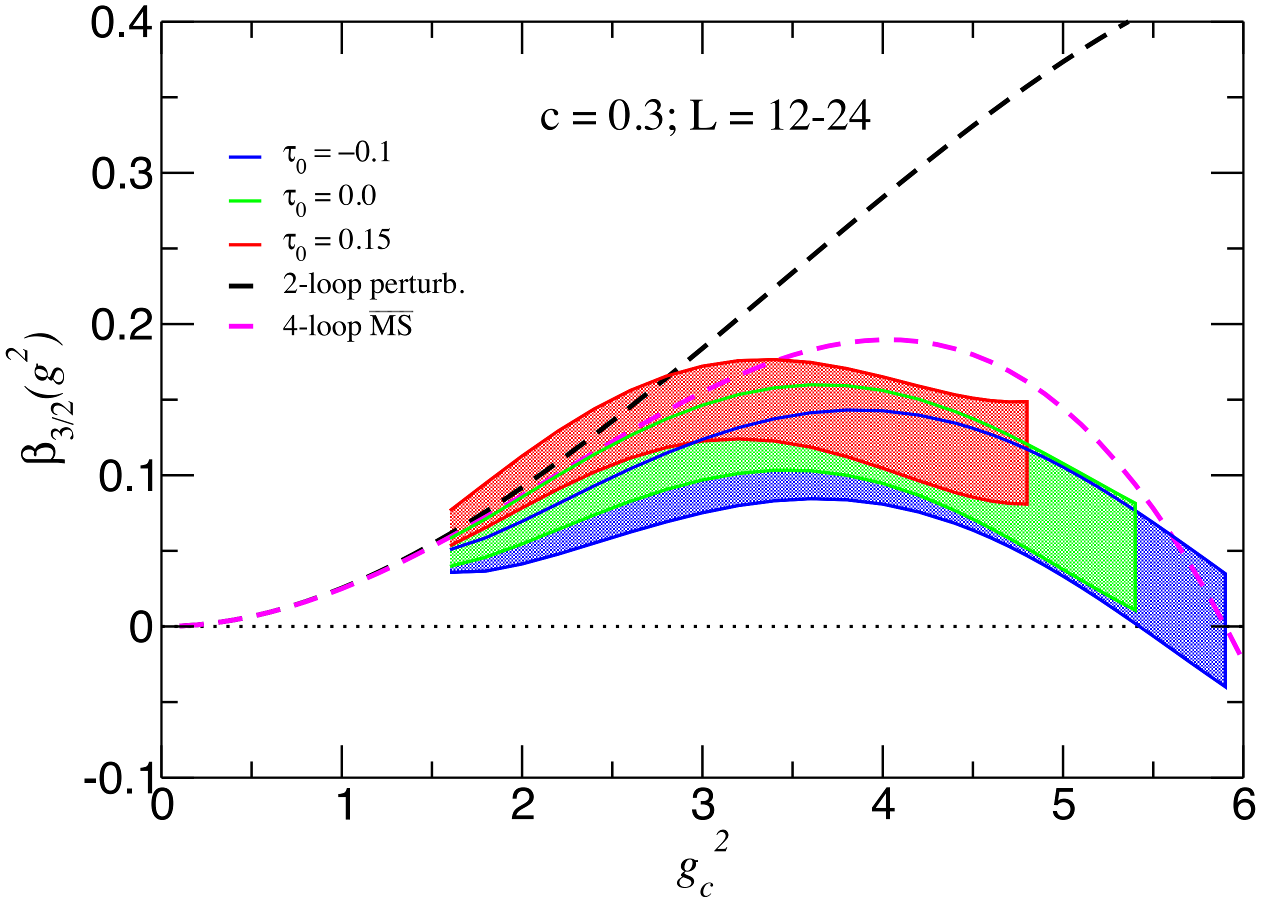


SU(3) N_f=2 sextet

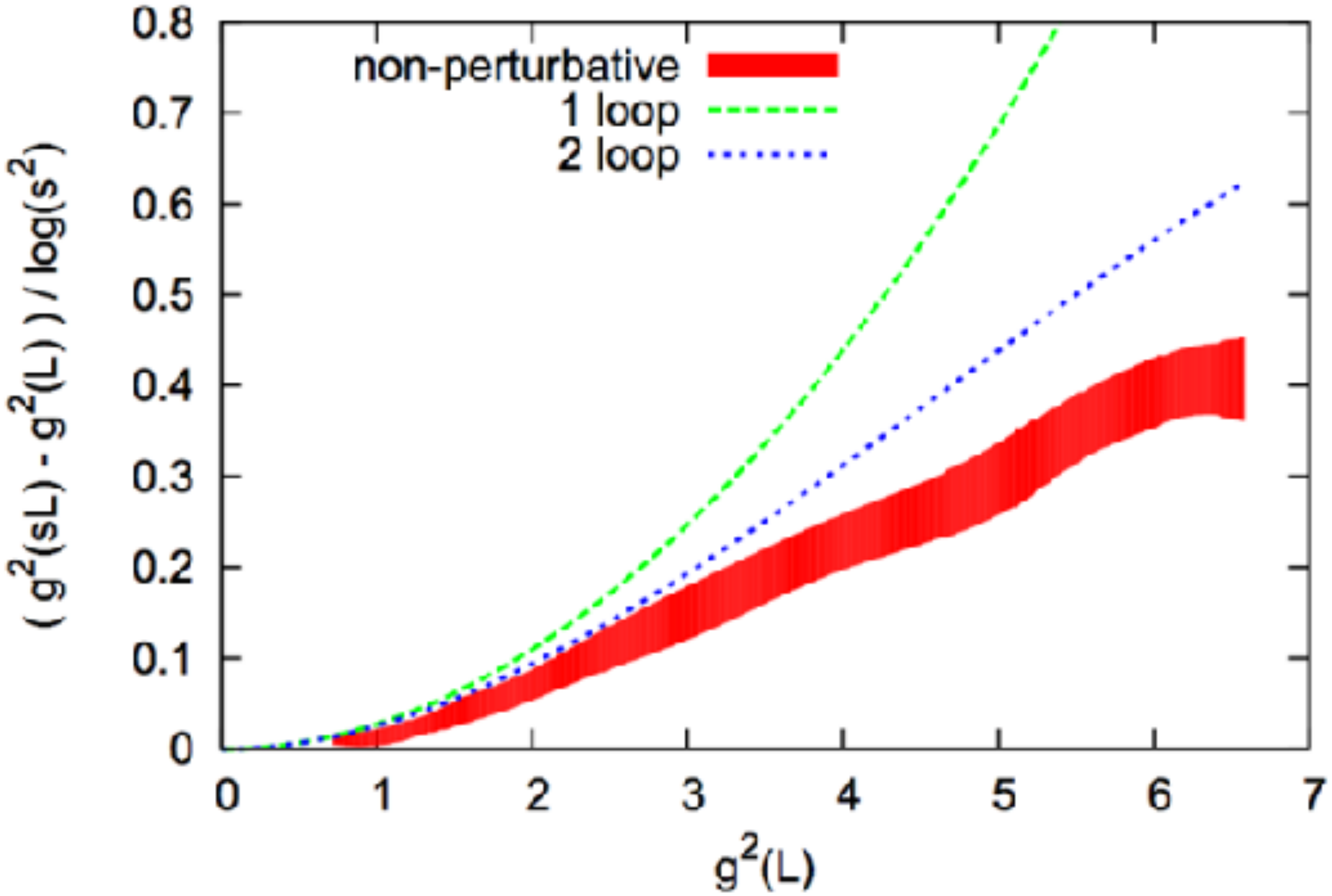
Wilson

(A.H., Y. Liu)



Staggered

(Fodor et al)



- Wilson and staggered results are not consistent at large g²