

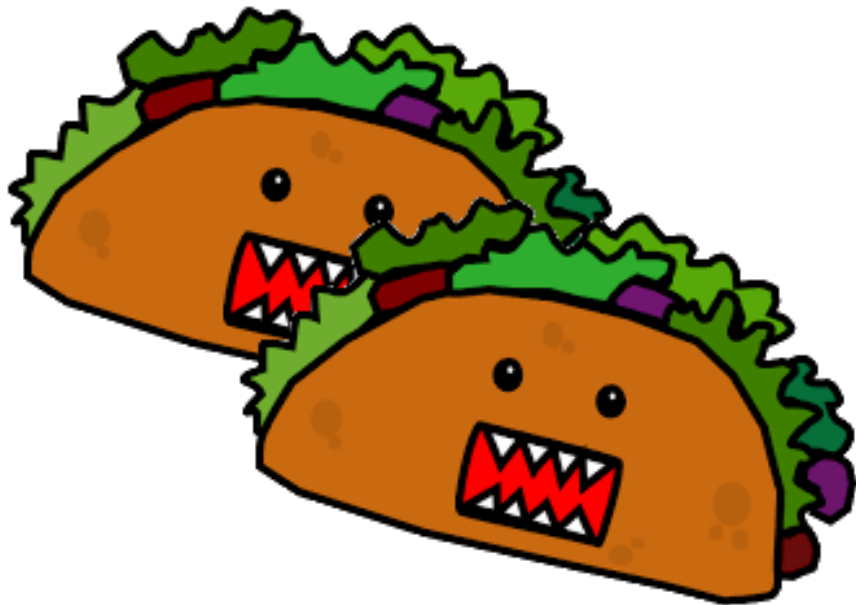
# Partial compositeness on the lattice: SU(4) gauge theory with fermions in multiple representations

Presented by Daniel Hackett

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# Overview

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## Ferretti's model & our lattice deformation

Composite Higgs, partially composite top quark

Only fermions and gauge bosons; no fundamental scalars; no SUSY

Multiple fermion representations: “multirep theory”

First ever lattice investigation of a multirep theory [w/o SUSY, in 4D]

## Results:

Zero-temperature spectrum

Pseudoscalars, vectors, baryons

Finite-temperature phase structure

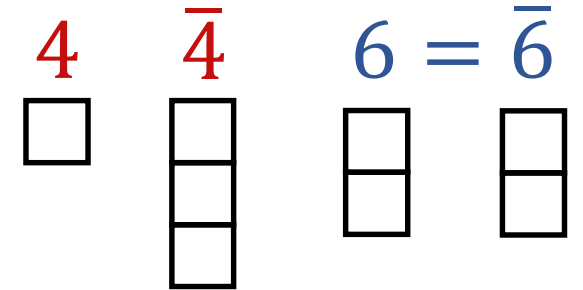
# Ferretti's Model [[arXiv:1404.7137](https://arxiv.org/abs/1404.7137)]

“Hypercolor” SU(4) gauge theory coupled to

$N_4 = 3$  Dirac flavors of fundamental fermion (cf. QCD)  $q$

$N_6^W = 5$  Weyl flavors of sextet (two-index antisymmetric) fermion  $Q$

[Note: 6 is a real irrep of SU(4)]



$\beta$  function  $\rightarrow$  QCD-like

Chiral symmetry breaking pattern

$$SU(3)_L \times SU(3)_R \times U(1)_X \times SU(5) \times U(1)_A \rightarrow SU(3)_c \times U(1)_X \times SO(5)$$

[ $U(1)_A$  a non-anomalous superposition of  $U(1)_{A(4)}$  and  $U(1)_{A(6)}$ ]

Custodial symmetry in unbroken chiral subgroups:

$$SU(3)_c \times SO(5) \times U(1)_X$$

$$\supset G_{cus.} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \quad [\text{Note: } SU(2)_L \times SU(2)_R \simeq SO(4) \subset SO(5)]$$

$$\supset G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$

Gauge  $SU(3)_c \rightarrow$  QCD in Standard Model

Gauge  $SU(2)_L \times U(1)_Y \rightarrow$  Electroweak force in Standard Model

# Ferretti model hadrons

## Mesons

$\bar{q}q$  fundamental pNGBs, vectors

$QQ, \bar{Q}Q, \bar{Q}\bar{Q}$  sextet pNGBs, vectors

Ferretti limit  $m_6 \rightarrow 0$ : Higgs is massless sextet NGB

Higgs potential from SM interactions

Fermion masses from quadratic mixing  $u\bar{u}H \rightarrow u\bar{u}QQ$

Non-anomalous  $U(1)_A \rightarrow$  axial singlet pNGB ( $\zeta$  meson)

## Baryons

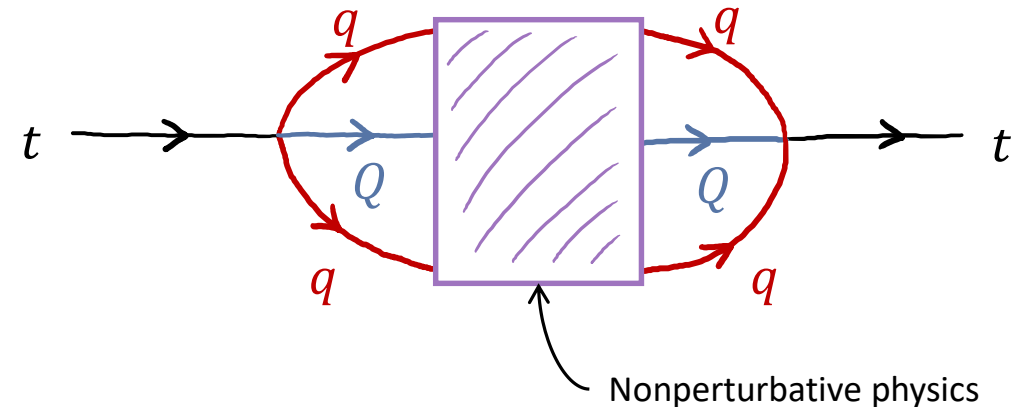
Fundamental  $qqqq$  [Boson]

Sextet  $QQQQQQ$  [Boson]

Chimera  $Qqq$  [Fermion]

$t$  partner: Mixes linearly with  $t$  via  $tQqq = t\mathcal{O}_{PC}$

$Qqq$  mass  $\sim \Lambda_{HC} > \Lambda_{SM} \Rightarrow$  Large mass for  $t$



# Ferretti's model on the lattice

**Goal:** Investigate (semi-quantitatively) strong dynamics

**Simulated theory:** “Lattice-deformed Ferretti model” or “the multirep theory”

SU(4) gauge theory coupled to  $N_4 = 2$  Dirac flavors of fundamental fermion

$N_6 = 2$  Dirac flavors ( $N_6^W = 4$  Weyl flavors) of sextet fermion

Easier flavor content for lattice,  $\sim$  same physics

Same (types of) states as Ferretti model:  $\zeta$  axial singlet pNGB,  $Qqq$  chimera baryon

Lattice action

Wilson gauge action

+ nHYP Dislocation Suppressing (NDS) term [DeGrand, Shamir, Svetitsky 2014]

Clover-improved Wilson fermions with nHYP smearing

3D bare parameter space:  $\beta, \kappa_4, \kappa_6$

# Technical details

Simulate with Multirep MILC [Shamir]

## Spectroscopy

Extract masses by fitting two-point functions

Measure fermion masses with Axial Ward Identity (AWI)

$$\partial_\mu \left\langle A_\mu^{(r)}(x) P^{(r)}(0) \right\rangle = 2m_r \left\langle P^{(r)}(x) P^{(r)}(0) \right\rangle$$

Pseudoscalar, vector decay constants from

$$\left\langle 0 \left| A_\mu^{(r)} \right| P^{(r)} \right\rangle \sim p_\mu F_P \quad [F_\pi = 130 \text{ MeV convention}]$$
$$\left\langle 0 \left| V_i^{(r)} \right| V_j^{(r)} \right\rangle = \delta_{ij} M_{Vr} F_{Vr}$$

## Scale setting

Wilson flow with definitions adjusted for  $N_c = 4$  [DeGrand 2016]

$$\langle t_0^2 E(t_0) \rangle = 0.1 N_c = 0.4$$

[Notation: any quantity without explicit  $a$  has been scaled by appropriate factors of  $t_0$ ]

# Zero-temperature data & analysis

$\mathcal{O}(40)$  ensembles

Volumes:  $16^3 \times 18$ ,  $16^3 \times 32$ ,  $24^3 \times 48$

Masses:  $0.5 \lesssim M_P/M_V \lesssim 0.8$

**General approach:**

Have ensembles at many  $m_4$ ,  $m_6$ ,  $a$

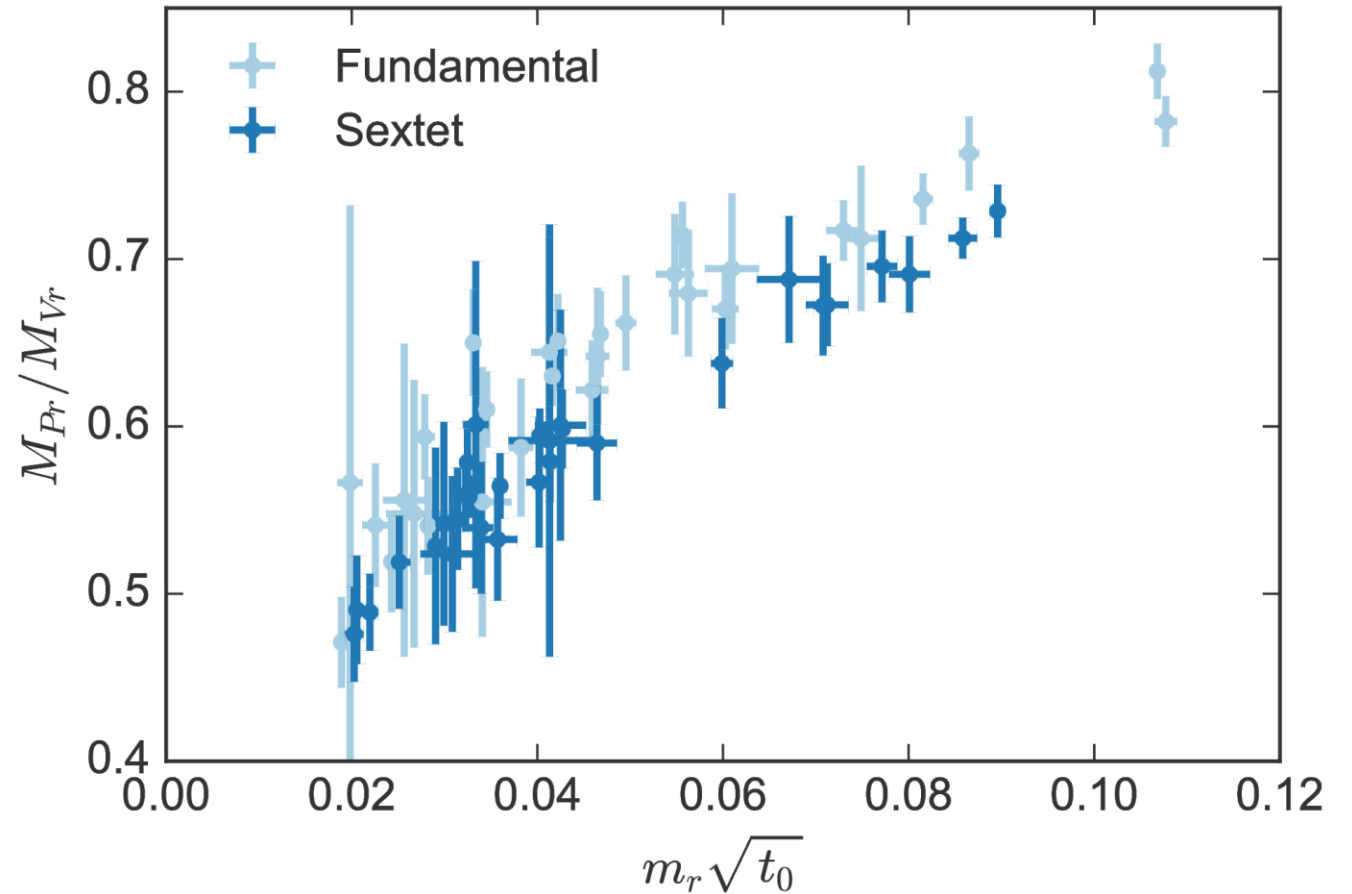
Fit **all** data to model in  $m_4$ ,  $m_6$ ,  $a$

Model aware of  $a$  dependence

→ Can take continuum limit  $a \rightarrow 0$

Model aware of  $m_r$  dependence

→ Can take chiral limits  $m_r \rightarrow 0$



# Modeling the pseudoscalar sector with $\chi$ PT

## Model

Multirep  $\chi$ PT gives expressions for  $M_{P4}$ ,  $M_{P6}$ ,  $F_{P4}$ ,  $F_{P6}$  as a function of  $m_4$ ,  $m_6$   
[[arXiv:1605.07738](https://arxiv.org/abs/1605.07738)]

Wilson fermions break chiral symmetry

Use Wilson  $\chi$ PT to account for lattice artifacts

**Analysis:** Fit lattice measurements of  $M_{P4}$ ,  $M_{P6}$ ,  $F_{P4}$ ,  $F_{P6}$  to measure

$B_4$ ,  $B_6$  [GMOR:  $M_{P_r}^2 = 2B_r m_r + \dots$ ]

$F_4$ ,  $F_6$ ,  $F_\zeta$  [ $\zeta$  sector has its own decay constant]

...and NLO LECs [including LECs for  $a$  dependence]

Chiral fit works:  $\chi^2/\text{dof} = 0.48$  for (172 observations) – (21 fit params) = 151 dof

For more analysis details, see our paper [[arXiv:1710.00806](https://arxiv.org/abs/1710.00806)]

$\zeta$  meson contributes chiral logs to  $M_{P4}^2$ ,  $M_{P6}^2$

→ Chiral fit indirectly measures  $\zeta$  sector! [In practice, use LO  $M_\zeta$  and measure  $F_\zeta$ ]



# $\zeta$ meson mass

Reconstruct  $M_\zeta$  as a function of  $m_4, m_6$  from chiral fit

## Phenomenology:

In  $m_6 \rightarrow 0$  limit,  $M_\zeta < M_{P4}$

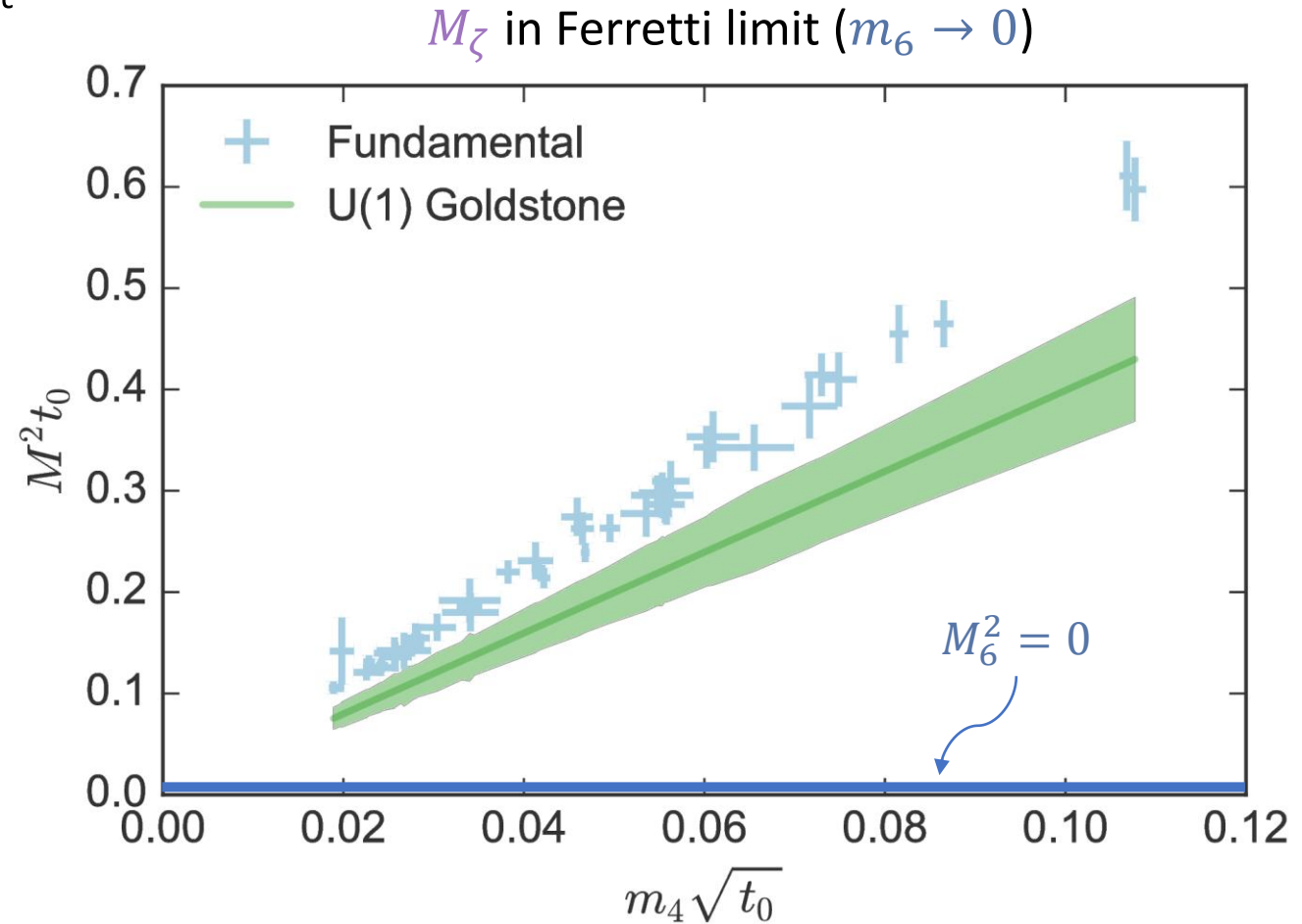
$\Rightarrow \zeta$  meson lightest (massive) state in the spectrum

[Sextet pNGB is exactly massless]

Axial singlets decay to two SM gauge bosons

[Ferretti et al. [arXiv:1610.06591](https://arxiv.org/abs/1610.06591)]

$\Rightarrow$  Experimental constraints?



# Vector meson decay widths from KSFR

Assuming vector meson dominance, predict widths of vector resonances:

$$\frac{\Gamma_{V \rightarrow PP}}{M_V} \approx \frac{M_V^2}{48\pi F_P^2}$$

Works (qualitatively) in QCD

For physical  $\rho$ , predicts  $\sim 0.23$  vs experimental  $\sim 0.19$

**Analysis:** Take  $a \rightarrow 0$  by subtracting lattice artifacts

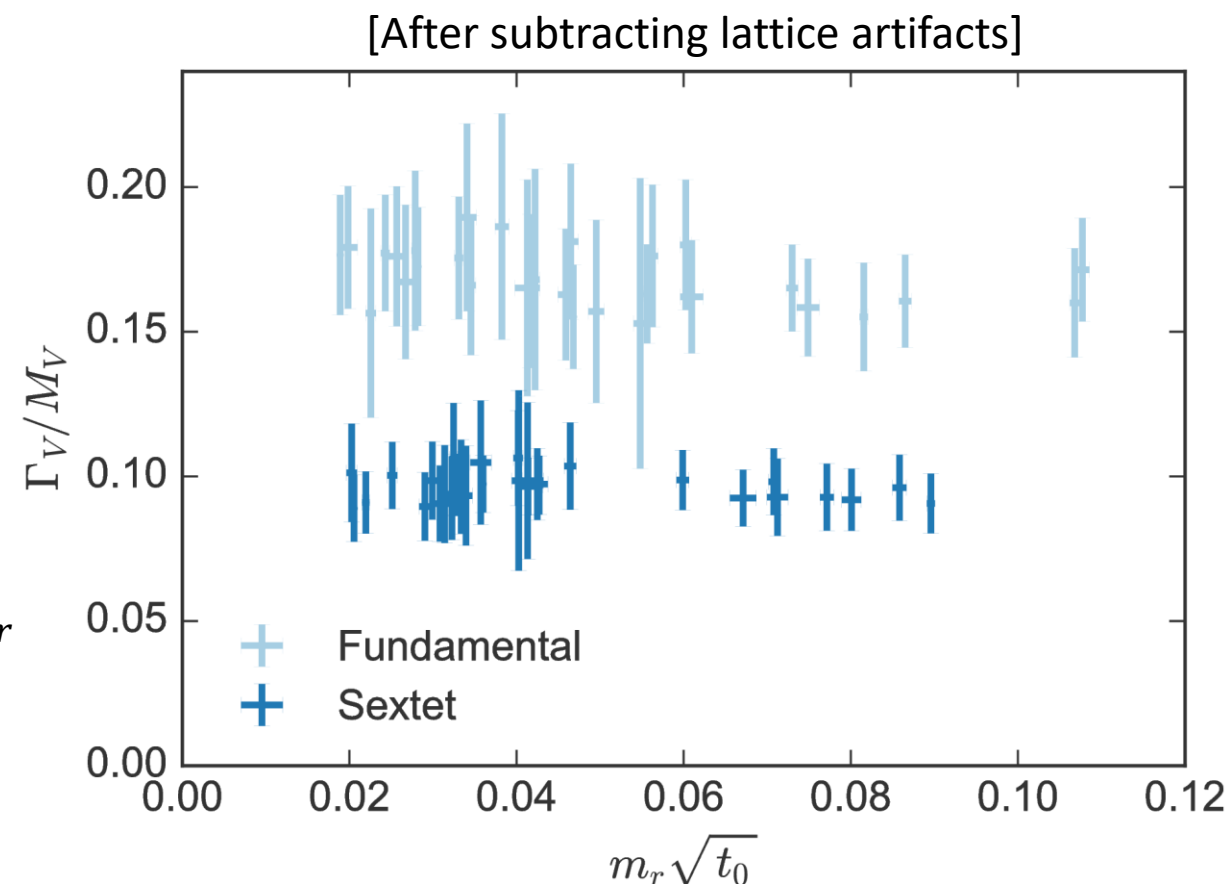
Fit data from 30 ensembles

Empirical model for  $M_{Vr}, F_{Vr}$ : linear in  $a$  and same-rep  $m_r$

**Prediction:** broad vector resonances

[but narrower than QCD]

[KSFR: Kawarabayashi, Suzuki 1966; Riazuddin, Fayyazuddin 1966]



# Baryon spectrum: quark model

Fermions acquire dynamical mass, so define “constituent masses”

$$m_4^{(c)} = C_4 + C_{44}m_4 \qquad m_6^{(c)} = C_6 + C_{66}m_6$$

Baryon masses: constituent masses + rotor splitting [ $J$  is total spin]

$$M_{q^4} = 4m_4^{(c)} + \dots J(J+1) + \dots a$$

$$M_{Q^6} = 6m_6^{(c)} + \dots J(J+1) + \dots a$$

Chimera baryons  $Qqq$  get additional rotor corrections [ $I$  is spin of  $qq$ ]

$$M_{Qqq} = 2m_4^{(c)} + m_6^{(c)} + C + \dots a + \dots J(J+1) + \dots I(I+1)$$

[Can justify more rigorously as  $1/N_c$  expansion. See preprint: [arXiv:1801.05809](https://arxiv.org/abs/1801.05809)]

# Quark model fit

Baryon masses for 12 ensembles

Baryons noisy, difficult to fit

10 baryon masses per ensemble

Sextet  $Q^6$  with  $J = 0,1,2,3$

Fundamental  $q^4$  with  $J = 0,1,2$

Chimera  $Qqq$  with

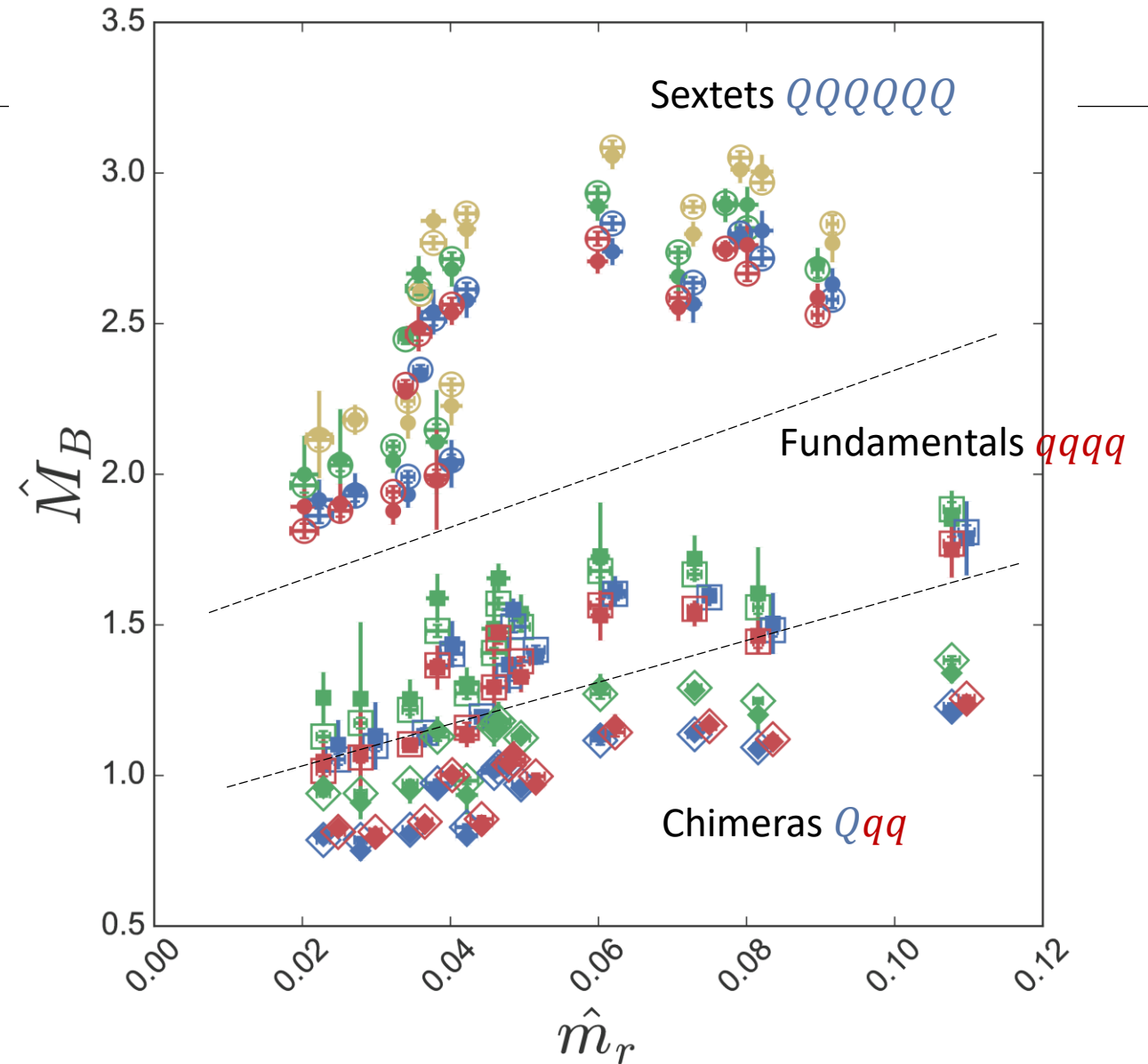
$$(J, I) = \left(\frac{1}{2}, 0\right), \left(\frac{1}{2}, 1\right), \left(\frac{3}{2}, 1\right)$$

↑ Top partner

Simultaneous fit to all 120 baryon masses

120 measurements – 11 fit params = 109 dof

Good fit:  $\chi^2/\text{dof} = 0.85$



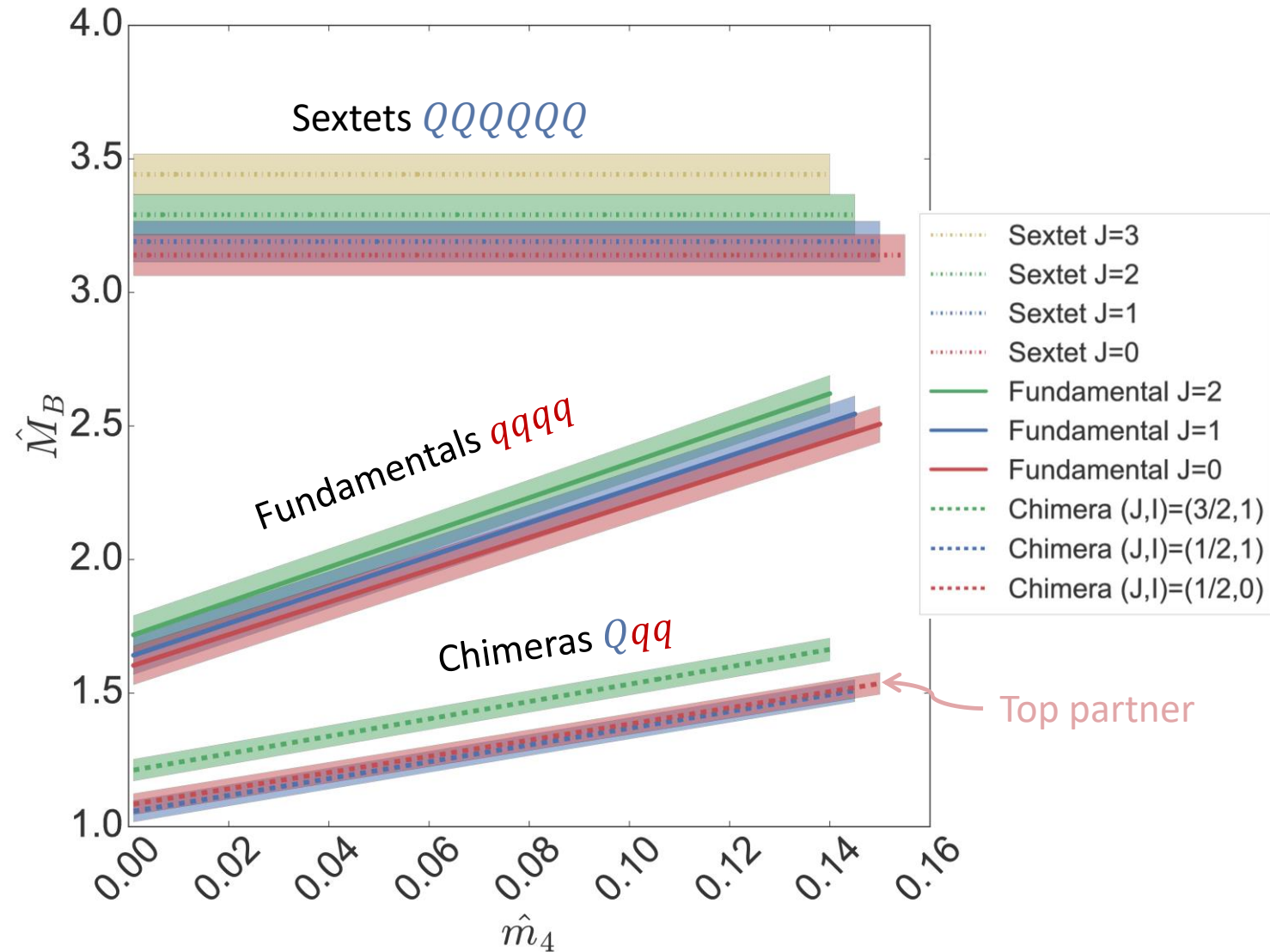
Solid markers: lattice data

Open markers: fit prediction

# Baryon spectrum in Ferretti limit

Use model to take  $a \rightarrow 0$ ,  $m_6 \rightarrow 0$   
 Sextet masses constant by construction

Top partner:  
 ~ degenerate with  $(1/2, 1)$  chimera  
 Lightest states in baryon spectrum



# Spectrum in Ferretti limit

Experimental constraints:

$$F_6 \gtrsim 1.1 \text{ TeV}$$

$$\Rightarrow M \gtrsim 6.5 \text{ TeV for top partner}$$

See our paper for details [[arXiv:1801.05809](https://arxiv.org/abs/1801.05809)]

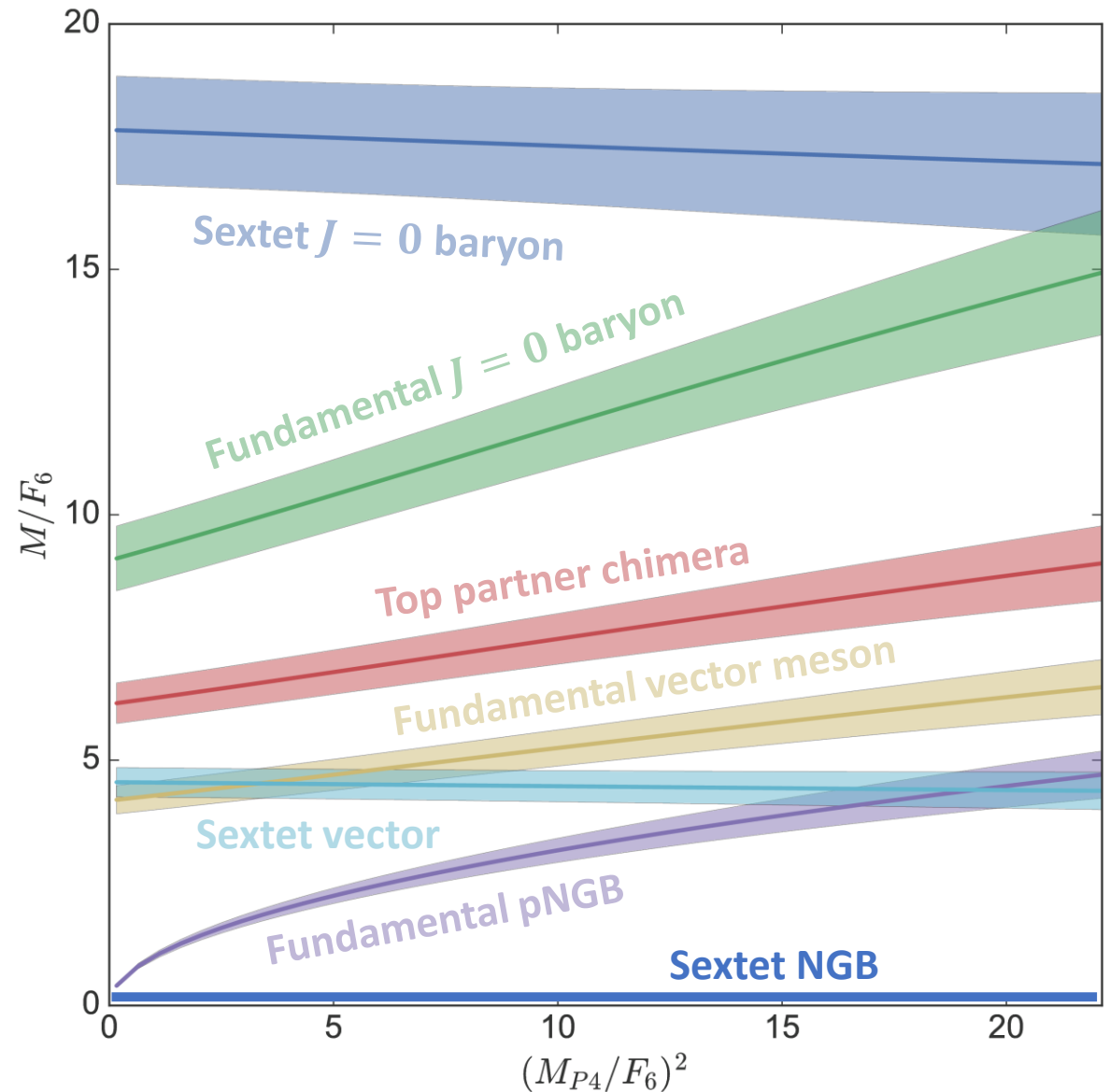
**Summary:** set of models predicts

$M_s, F_s$  for pseudoscalar and vector mesons

Baryon masses

...in the continuum limit, as a function of  $m_4, m_6$

$\Rightarrow$  Measure one mass, predict entire spectrum!



# Thermodynamics

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Zero-temperature results: both fermion species are chirally broken

Theory is asymptotically free

⇒ Both fermion species deconfined at high temperature

## Questions:

How many phase transitions between  $T = 0$  and  $T = \infty$ ?

Tumbling/condensation in to Most Attractive Channel [Raby, Susskind, Dimopolous 1980]

Prediction: sextets condense before fundamentals, intermediate “partially confined” phase

Order of phase transition(s)?

Transition temperature(s)?

[[arXiv:1802.09644](https://arxiv.org/abs/1802.09644)]

# Numerical details

## $\mathcal{O}(500)$ ensembles

Mostly  $12^3 \times 6$  and  $16^3 \times 8$

Mostly at  $\beta = 7.4, 7.75$

## Spectroscopy

Lattices with short temporal extent

→ Measure screening masses

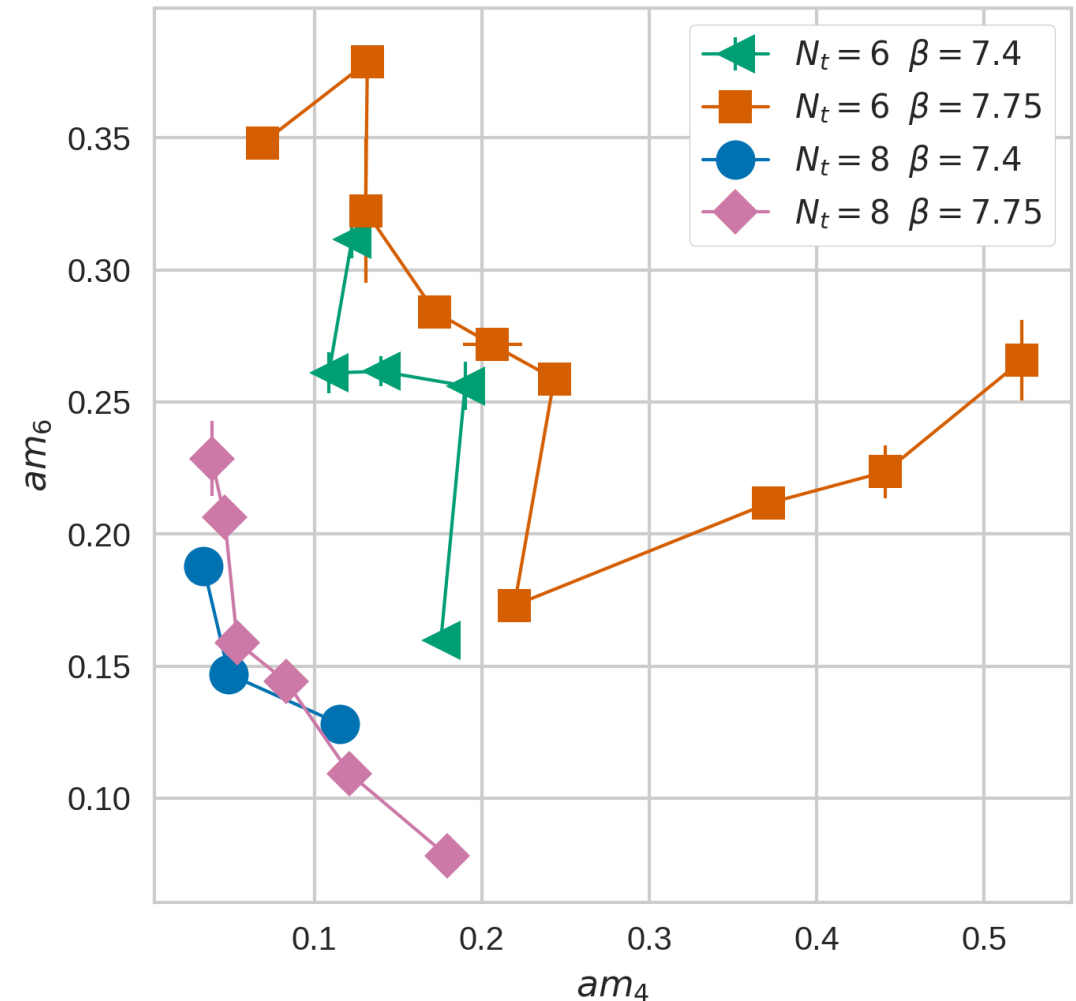
## Scale setting

$t_0$  contaminated by finite- $a$  effects in regions of interest

Instead, use  $t_1: \langle t_1^2 E(t_1) \rangle = \frac{2}{3} \frac{N_c}{3} = \frac{8}{9}$

[Sommer [arXiv:1401.3270](https://arxiv.org/abs/1401.3270)]

Lattice-units fermion masses near transition





# No intermediate phase

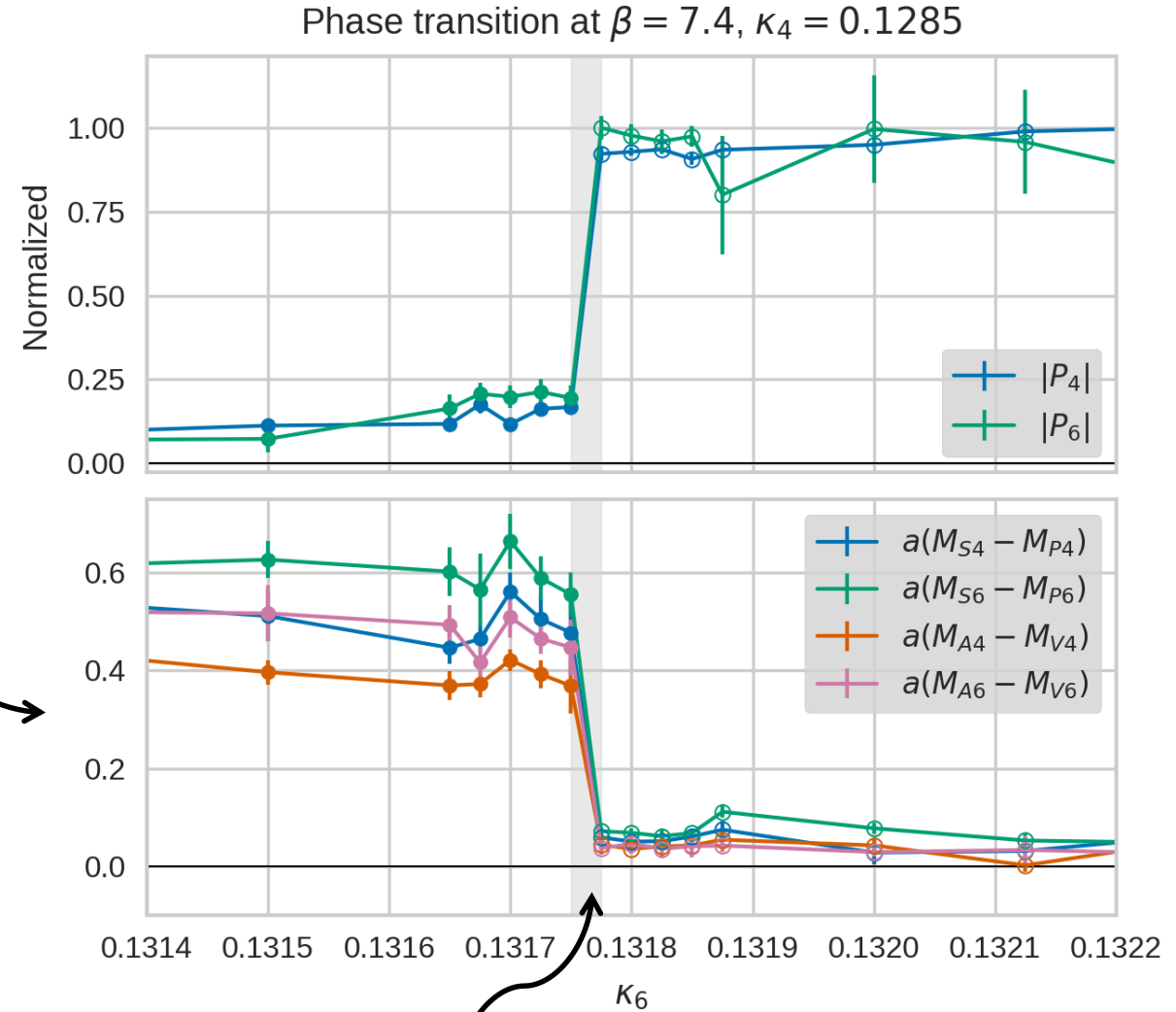
## (Normalized) Confinement Diagnostics

(Polyakov loop for each irrep is small when species is confined, large when deconfined)

## Chiral Transition Diagnostics

(Parity doubling: when irrep is chirally restored, S and PS (V and PV) mesons become degenerate)

Typical slice for explored regions of parameter space  
⇒ Only two phases, like in QCD



Gray band: all transitions occur simultaneously

# Transition is first-order

All observables jump at the transition

Discontinuity is present everywhere

Transition is sharp

Observables are either “confined-like” or “deconfined-like,” with no interpolation

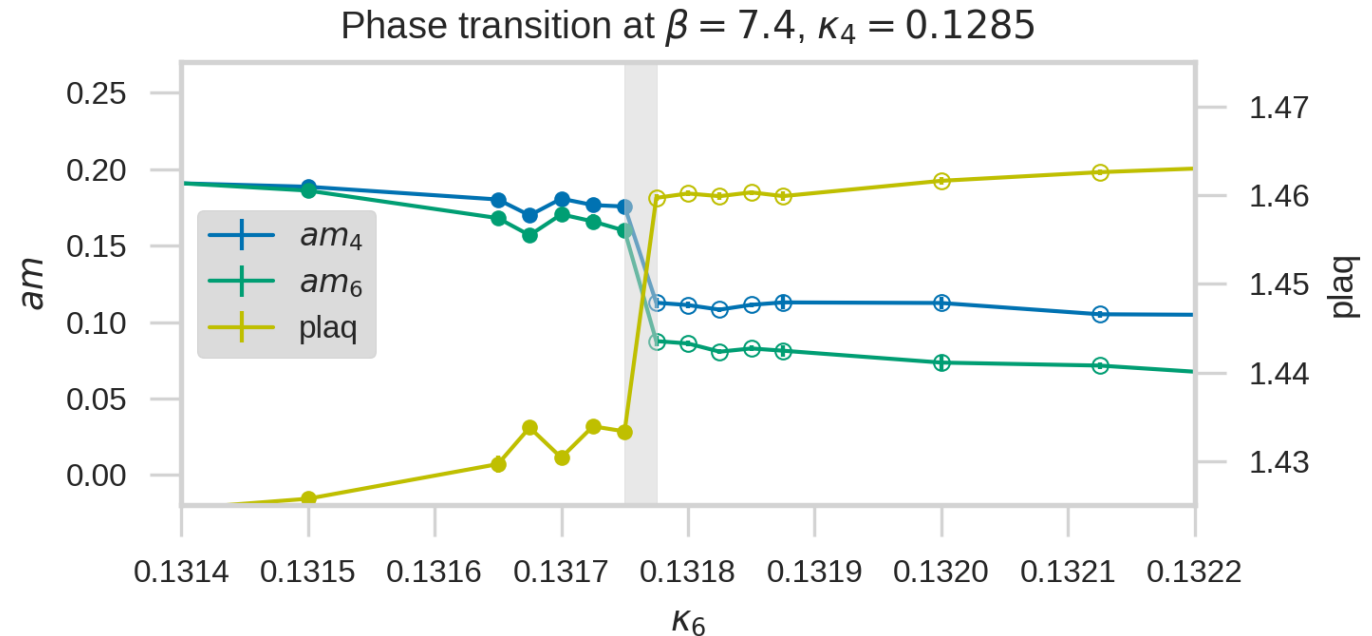
Also observe metastability in equilibration

⇒ (Violently) first-order transition!

**Phenomenology:** first-order transitions in the early universe make gravitational waves

[Schwaller [arXiv:1504.07263](https://arxiv.org/abs/1504.07263)]

[LISA [arXiv:1610.06481](https://arxiv.org/abs/1610.06481)]



Same slice as previous slide

**Left axis:** axial Ward identity quark masses in lattice units

**Right axis:** Plaquette (roughly, energy density of gauge sector)

# Analytics: “multirep Pisarski-Wilczek”

Generalization of calculation by Pisarski and Wilczek [PW 1984]

Recently extended to high order for complex, real irreps [Pellissetto, Vicare 2003, 2005, 2005’]

**Idea:** Does 3D EFT of scalar/pseudoscalar modes have any stable fixed points?

If not, transition must be first order!

**Inputs:**

Chiral symmetry breaking pattern

$$SU(N_4)_L \times SU(N_4)_R \times SU(N_6^W) \times U(1)_A \rightarrow SU(N_4)_V \times SO(N_6^W)$$

Transition occurs simultaneously for 4 and 6 (as observed)

Work to first order in  $\epsilon$  expansion

**Result:** No stable fixed points  $\Rightarrow$  Transition must be first-order

Applies to both Ferretti model and lattice deformation

[\[arXiv:1712.01959\]](https://arxiv.org/abs/1712.01959)

# Transition temperature

Roughly:  $T_c \sim 0.2/\sqrt{t_1}$

Comparison with QCD:

In QCD:  $1/\sqrt{t_0} \approx 1380$  MeV

[MILC [arXiv:1503.02769](https://arxiv.org/abs/1503.02769)]

$\Rightarrow 1/\sqrt{t_1} = 770$  MeV

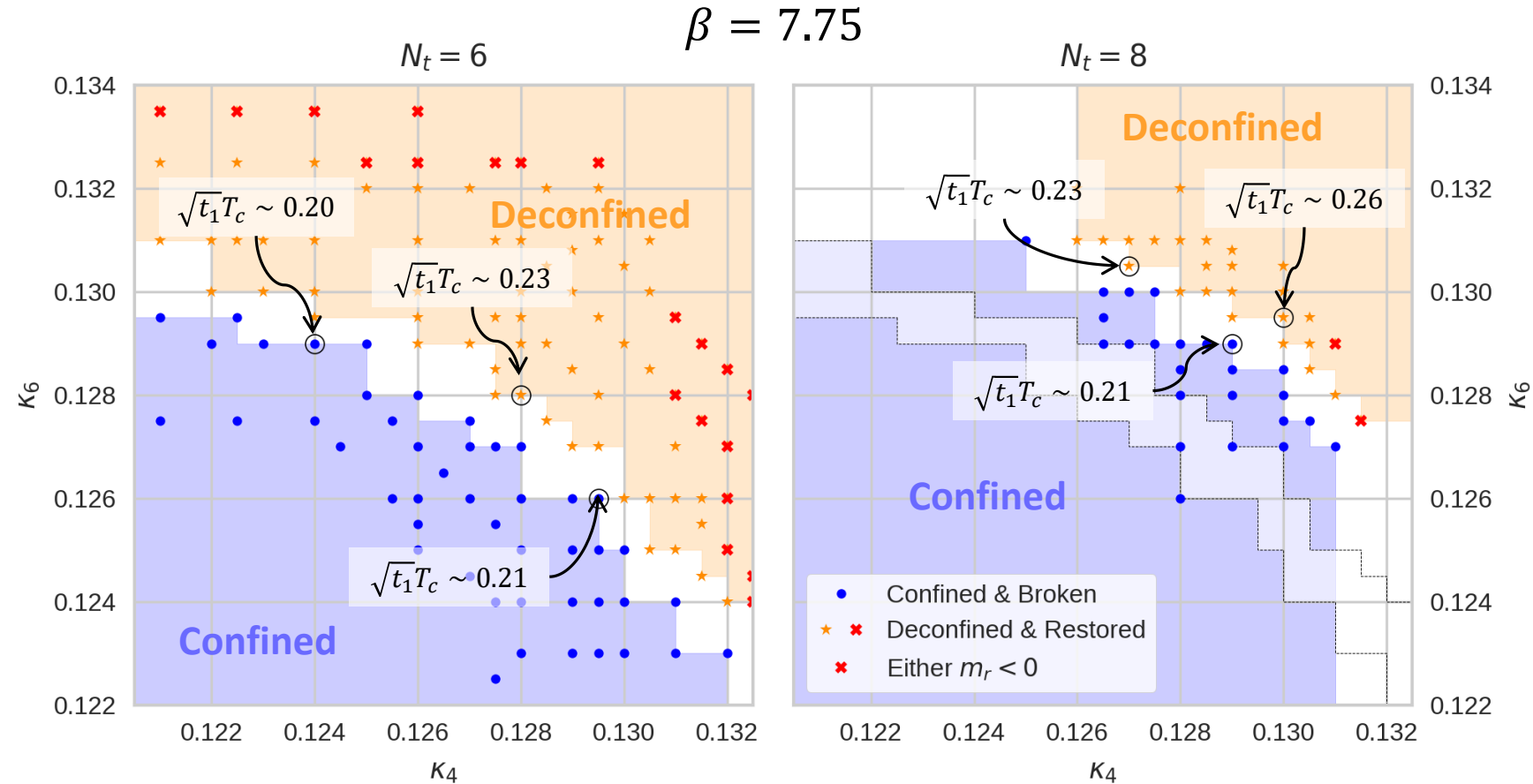
$\Rightarrow T_c \sim 150$  MeV

Phenomenology:

Experimental bound:  $F_6 \gtrsim 1.1$  TeV

$\Rightarrow 1/\sqrt{t_1} \gtrsim 3.7$  TeV

$\Rightarrow T_c \gtrsim 720$  GeV



# Conclusions

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Theory “acts like QCD”

- Pseudoscalar spectrum described by  $\chi$ PT

- Vector resonances probably broad [but narrower than QCD]

- Baryon spectrum described by quark model

- Phase structure like QCD's [except transition is first-order]

- Transition temperature QCD-like

Fitting to models (physically-motivated and empirical) has been a very fruitful, efficient approach

Many predictions to make contact with phenomenology