Linear Sigma EFT for Nearly Conformal Gauge Theories

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Lattice for Beyond the Standard Model Physics
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References:

- Arxiv:1702.00480
- Arxiv:1710.08545
- Appelquist et. al. (LSD). “Linear Sigma EFT for Nearly Conformal Gauge Theories.” In Prep.

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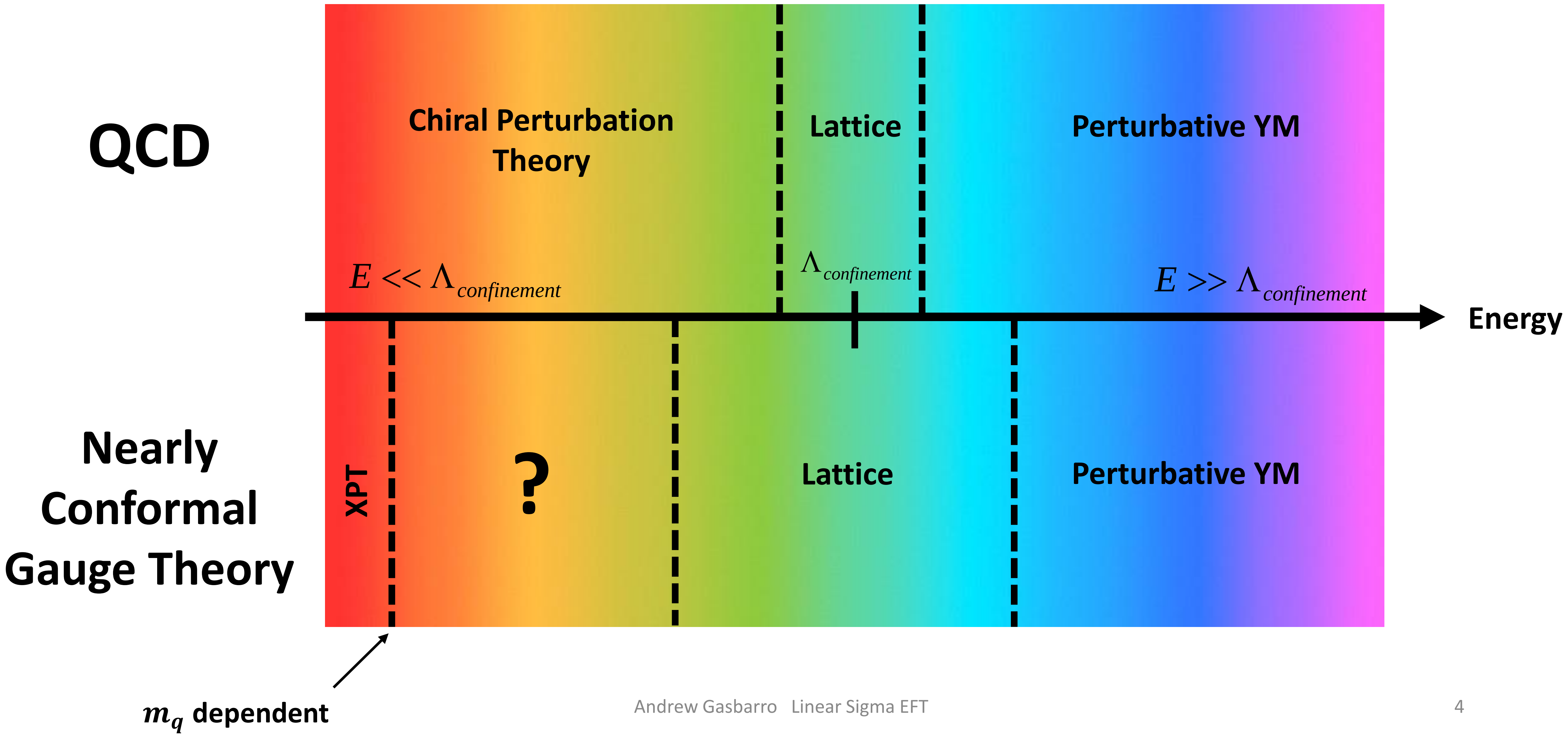
Outline

- EFT Considerations for Nearly Conformal Gauge Theories
  - Light singlet scalars in nearly conformal gauge theories
  - Review properties of eight flavor SU(3) Spectrum
    - Scale Setting in nearly conformal gauge theories
- Linear Sigma EFT
  - Construction for SU(N)xSU(N)
  - Power counting and chiral breaking terms
- Fits of EFT to Lattice Data

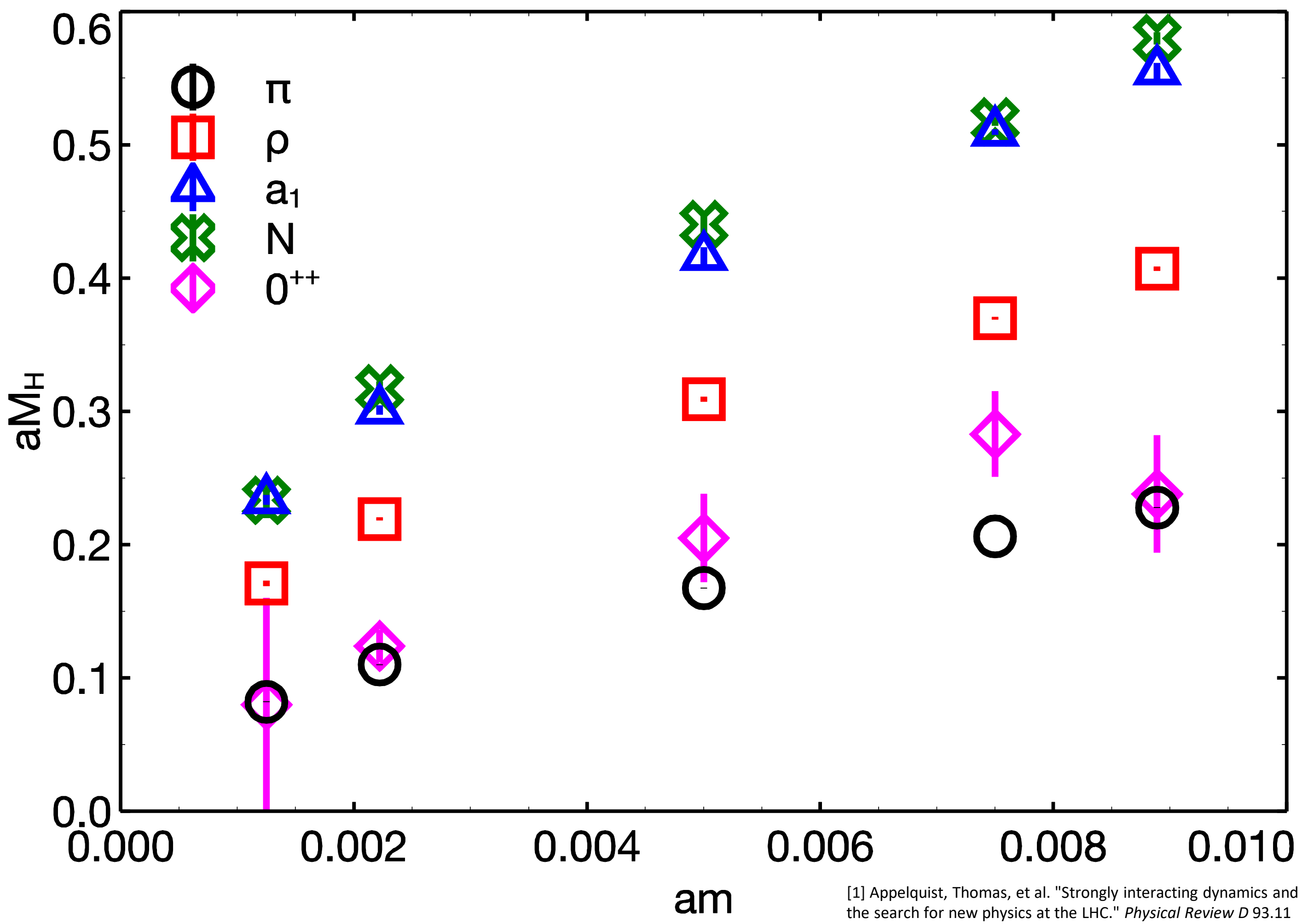
Confining Gauge Theories at High and Low Energies



Light Scalars

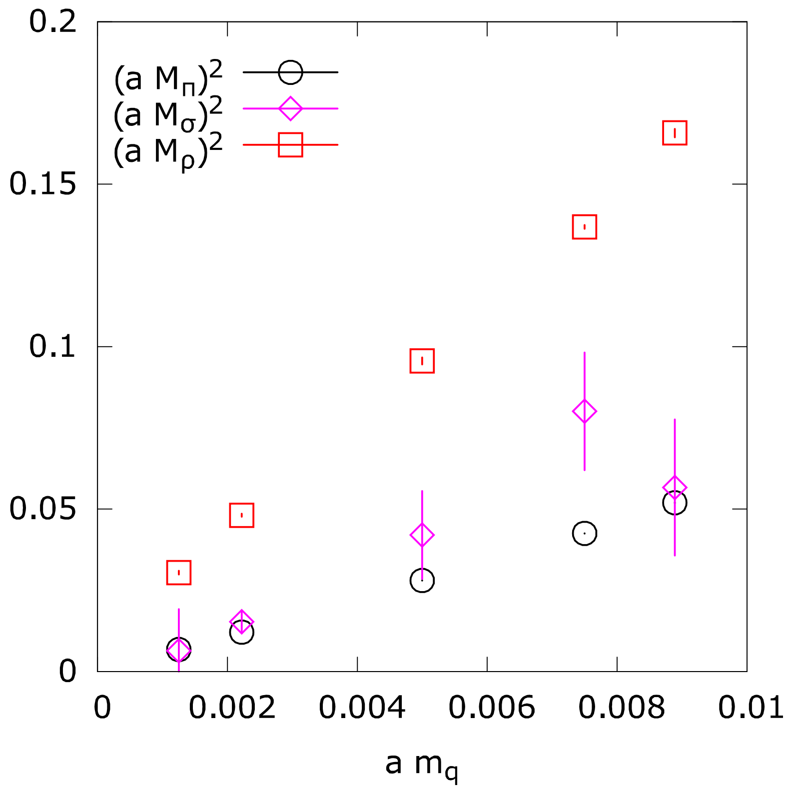
an example: SU(3) with Eight Fundamental Flavors

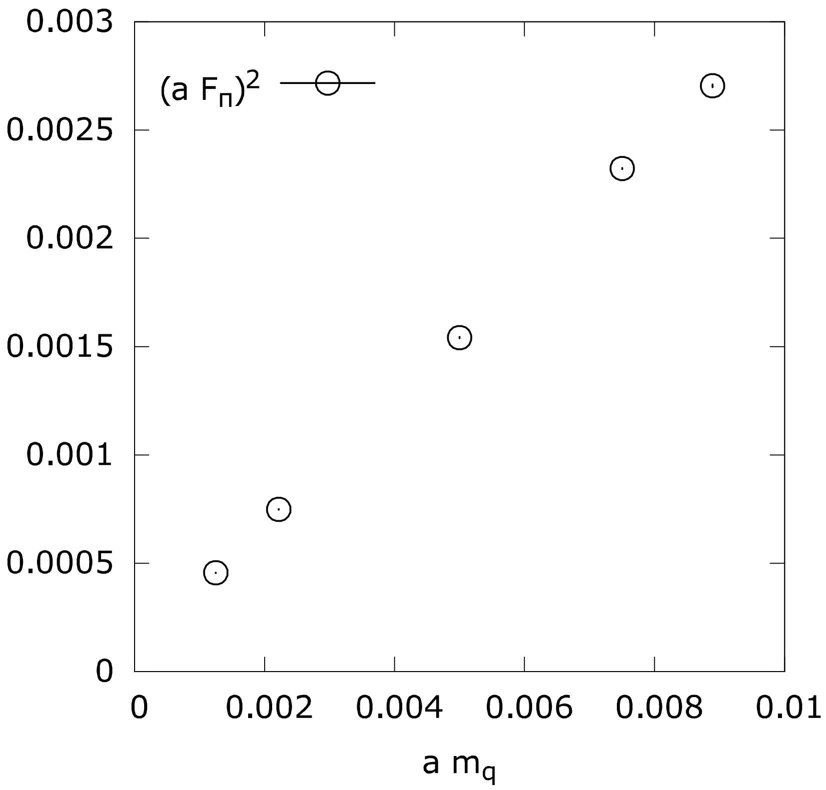
- Sigma similar in mass to the pions
- Separation of Scales
  - sigma and pions separated from rho and heavier resonances
- Many Light Scalars on the Lattice
  - SU(3), Nf=8 Fundamental [1-4]
  - SU(3), Nf=2 Sextet [5,6]
  - SU(3), 4+8 & 12 Fundamental [7,8]
  - SU(2), Nf=2 Adjoint [9]
- Seek EFT descriptions of light states that is good up to cutoff set by next resonance (M_ρ)
- Pions and sigma should "peel apart" at small enough quark mass.
- Update on LSD collaboration study of Nf=8 QCD given at this workshop by George Fleming



[1] Appelquist, Thomas, et al. "Strongly interacting dynamics and the search for new physics at the LHC." *Physical Review D* 93.11 (2016): 114514.
[2] Gasbarro, Andrew D., and George T. Fleming. "Examining the Low Energy Dynamics of Walking Gauge Theory." *arXiv preprint arXiv:1702.00480* (2017).

Interpreting the Nf=8 Spectrum for an EFT





In lattice units, approximate relationships: $(a\,M_x)^2 \sim c_x\,(a\,m_q)$ $(a\,F_x)^2 \sim c_x\,(a\,m_q)$

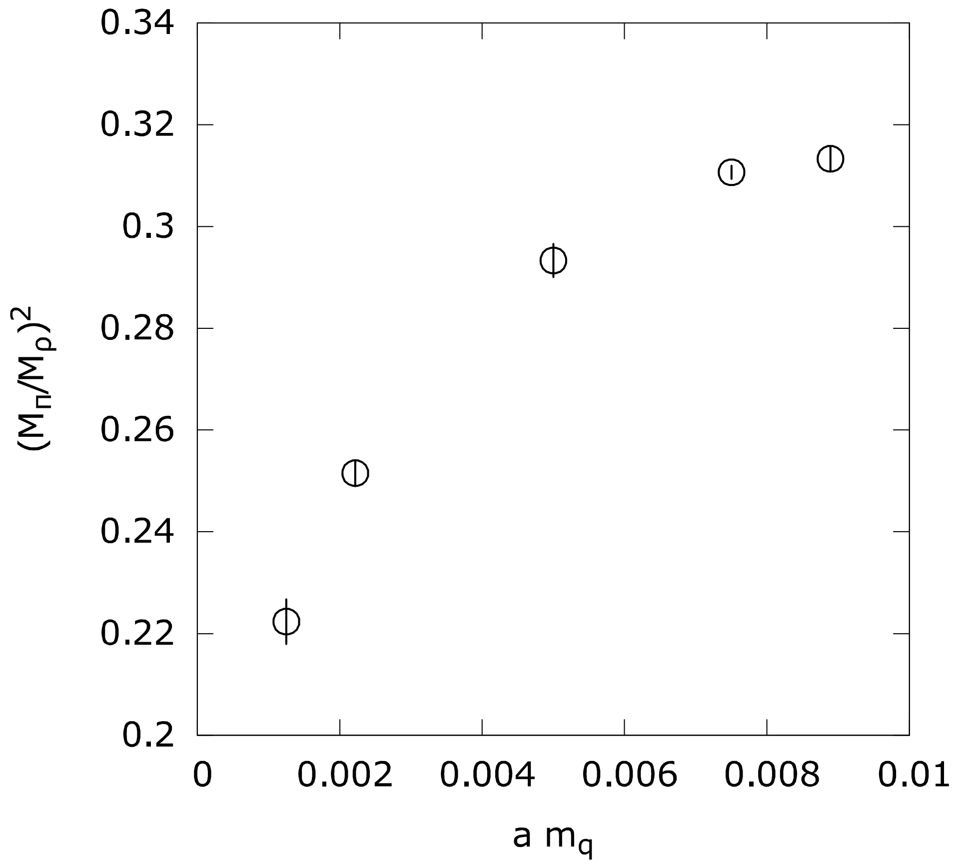
Should we expect EFT to yield these relationships? Not necessarily.
Is pion behaving like GMOR / LOXPT? Not necessarily.

Interpreting the Nf=8 Spectrum for an EFT

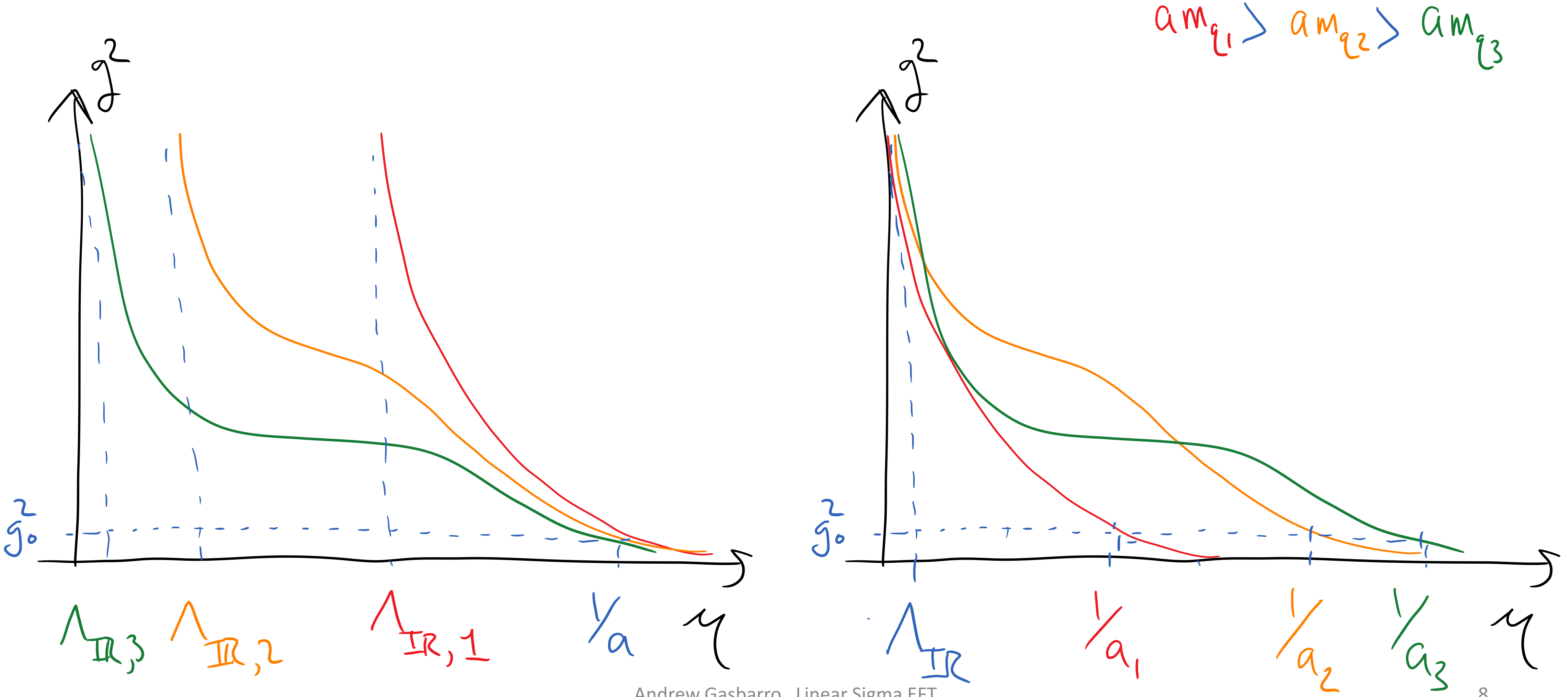
- Not hyperscaling. Ratios do move!
- There is a dominant effect,

$$(a\,\Lambda_{\mathrm{IR}})^2 \sim c\,(a\,m_q)$$

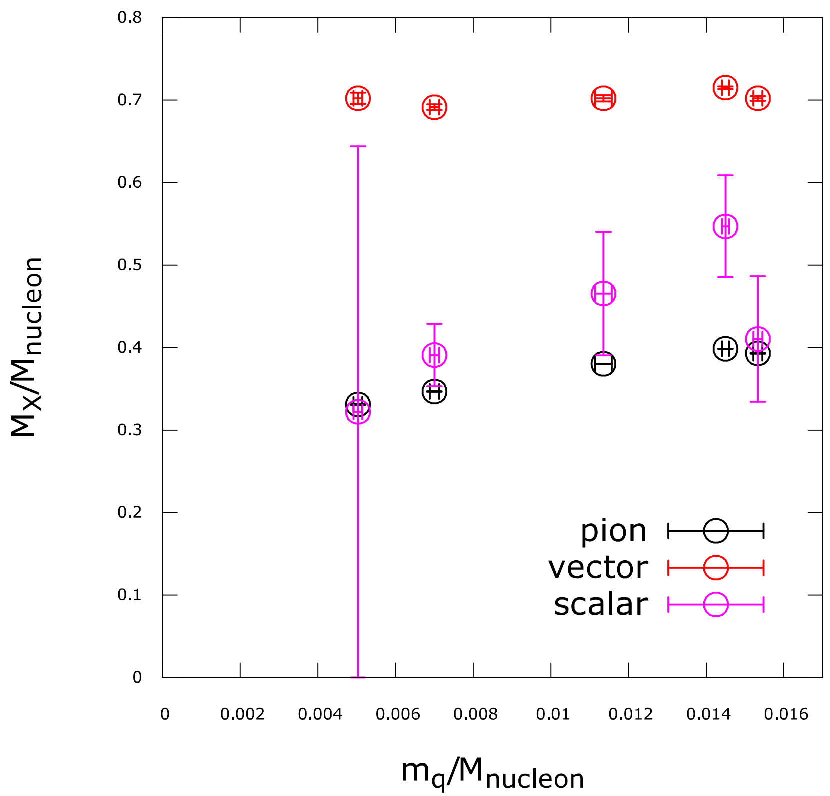
- But also subdominant variation in the ratios
- Is the pion really exhibiting LOXPT / GMOR-like behavior?

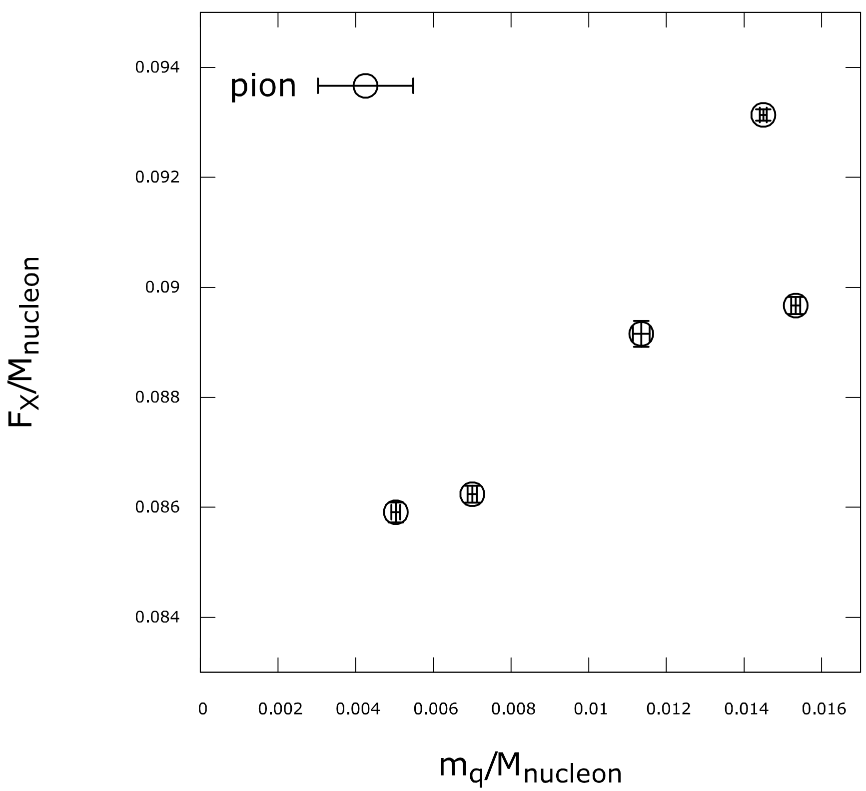


A cartoon of quark mass effect on $a\Lambda$



Interpreting the Nf=8 Spectrum for an EFT





- If we interpret lattice spacing as moving and IR scale as fixed, can use nucleon units
  - EFT Cutoff is relatively constant
  - Finite lattice spacing effects may be substantial.
- Moving forward, we will consider both **lattice units** and **nucleon units**

Field Content of $SU_L(N_f)$x $SU_R(N_f)$ Linear Sigma Model

- Fields live in bifundamental, $(N_f, \overline{N_f})$, representation: $M_a^{\bar{b}}(x)$
  - Transform Linearly under $U_L(N_f)$x $U_R(N_f)$

$$M(x) \to L M(x) R^\dagger$$

- N_f = 2 is special. Admits a real linear representation due to isometry with O(4)
  - Otherwise, matrix is complex. $2N_f^2$ Real degrees of freedom
- Express degrees of freedom via a polar decomposition:
  - Nonlinear realization of chiral symmetry

$$M = \Sigma(x)S(x) \qquad \Sigma(\mathrm{x}) = \exp\left[\frac{i\sqrt{2}}{F_\pi}\left(\frac{\eta'(x)}{\sqrt{N_f}} + \pi_i(x)T_i\right)\right] \qquad S(x) = \frac{\sigma(x)}{\sqrt{N_f}} + a_i(x)T_i$$

- Without flavored scalars, fields do not form a complete linear representation of $SU_L(N_f) \times SU_R(N_f) \times U_V(1)$
- Without eta prime, fields do not form a complete representation of $U_A(1)$ but do form a complete representation of $SU_L(N_f) \times SU_R(N_f) \times U_V(1)$
  - Set eta prime field to zero because it is heavy. Does not break the chiral symmetry

Leading Order $SU_L(N_f)$x $SU_R(N_f)$ Linear Sigma Model

- Most general Lagrangian,
  - symmetric under flavor group and parity
  - includes all relevant and marginal operators

$$\mathcal{L} = \frac{1}{2}\langle \partial_\mu M^\dagger \partial^\mu M \rangle - V_0(M)$$

$$V_0 = \frac{\mu^2}{2}\langle M^\dagger M \rangle + \frac{\lambda_1}{4}\langle M^\dagger M \rangle^2 + \frac{N_f \lambda_2}{4}\left\langle \left(M^\dagger M\right)^2 \right\rangle$$

- $\mu^2 < 0$, symmetry spontaneously breaks: $SU_L(N_f) \times SU_R(N_f) \rightarrow SU_V(N_f)$

$$\frac{\partial}{\partial M^\dagger} V_0(M) = \frac{1}{2}\left(\mu^2 + \lambda_1 \langle M^\dagger M \rangle + N_f \lambda_2 M^\dagger M\right) M = 0 \qquad \langle 0|M_{ab}|0\rangle = \frac{f}{\sqrt{N_f}}\delta_{ab} \qquad f = \frac{-\mu^2}{\lambda_1 + \lambda_2}$$

- Can show: $F_\pi = f\sqrt{2/N_f}$

- Convenient to rewrite: $V_0 = \frac{-m_\sigma^2}{4}\langle M^\dagger M \rangle + \frac{m_\sigma^2 - m_a^2}{8f^2}\langle M^\dagger M \rangle^2 + \frac{N_f m_a^2}{8f^2}\left\langle \left(M^\dagger M\right)^2 \right\rangle$

Explicit Breaking in $SU_L(N_f)$x $SU_R(N_f)$ Linear Sigma Model

- Introduce operators to explicitly break symmetry to $SU_V(N_f)$
- Can organize breaking terms by looking to underlying gauge theory for guidance.
- Only source of explicit breaking in

  Yang Mills model is quark mass. $\boldsymbol{\mathcal{L}}_{YM} \supset \psi_L^\dagger \boldsymbol{\mathcal{M}} \psi_R + \psi_R^\dagger \boldsymbol{\mathcal{M}} \psi_L$
- Spurion analysis: treat chiral breaking as an external source which transforms as $\chi(x) \to L\chi(x)R^\dagger$
- Symmetry broken when spurion field set to a constant matrix: $\chi(x) = B\boldsymbol{\mathcal{M}}$
- V.e.v. (and other quantities) depend on breaking at tree level

$$\frac{F^3}{f^2} - F + \frac{2}{m_\sigma^2}\frac{\partial V_{SB}}{\partial \sigma}\bigg|_{\sigma=F} = 0$$

Symbol	Operator
O_1	$\langle \chi^\dagger M + M^\dagger \chi \rangle$
O_2	$\langle M^\dagger M \rangle \langle \chi^\dagger M + M^\dagger \chi \rangle$
O_3	$\langle (M^\dagger M)(\chi^\dagger M + M^\dagger \chi) \rangle$
O_4	$\langle \chi^\dagger M + M^\dagger \chi \rangle^2$
O_5	$\langle \chi^\dagger \chi M^\dagger M \rangle$
O_6	$\langle \chi^\dagger \chi \rangle \langle M^\dagger M \rangle$
O_7	$\langle \chi^\dagger M \chi^\dagger M + M^\dagger \chi M^\dagger \chi \rangle$
O_8	$\langle \chi^\dagger \chi \rangle \langle \chi^\dagger M + M^\dagger \chi \rangle$
O_9	$\langle (\chi^\dagger \chi)(\chi^\dagger M + M^\dagger \chi) \rangle$

Power Counting in $SU_L(N_f)$x $SU_R(N_f)$ Linear Sigma EFT

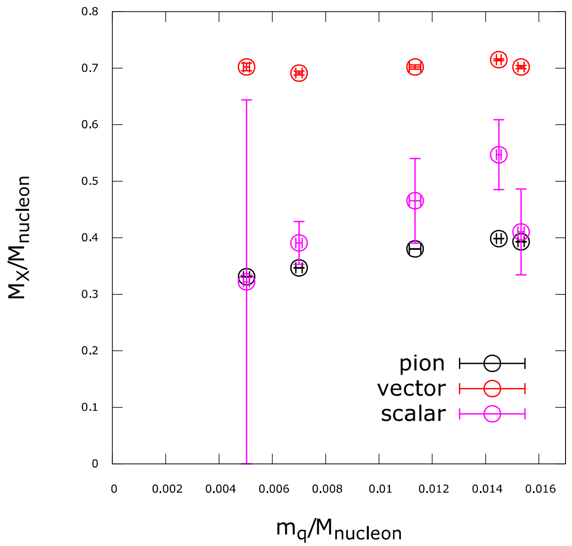
- Expansion in chiral limit:

$$\frac{M(x)}{\Lambda}, \frac{\partial}{\Lambda} \sim \frac{F_\pi}{\Lambda}, \frac{M_\sigma}{\Lambda} < 1$$

- Chiral Breaking:
  - Power counting dimension may differ from 2
  - Also, $\chi(x)$ has no natural engineering dimension

$$\chi(x) \quad \text{Corresponds to } \; m_q \frac{\langle 0|\bar{\psi}\psi|0\rangle\big|_{m_q=0}}{f_\pi^2} \overset{?}{\leftrightarrow} M_\pi^2$$

$$\frac{\chi(x)}{\Lambda^\alpha} < 1$$



- Additional operators in breaking potential and affect tree level observables

$$\mathcal{L} \supset \Lambda^4 \left(\frac{\partial}{\Lambda}\right)^{N_p} \left(\frac{M(x)}{\Lambda}\right)^{N_M} \left(\frac{\chi(x)}{\Lambda^\alpha}\right)^{N_\chi}$$

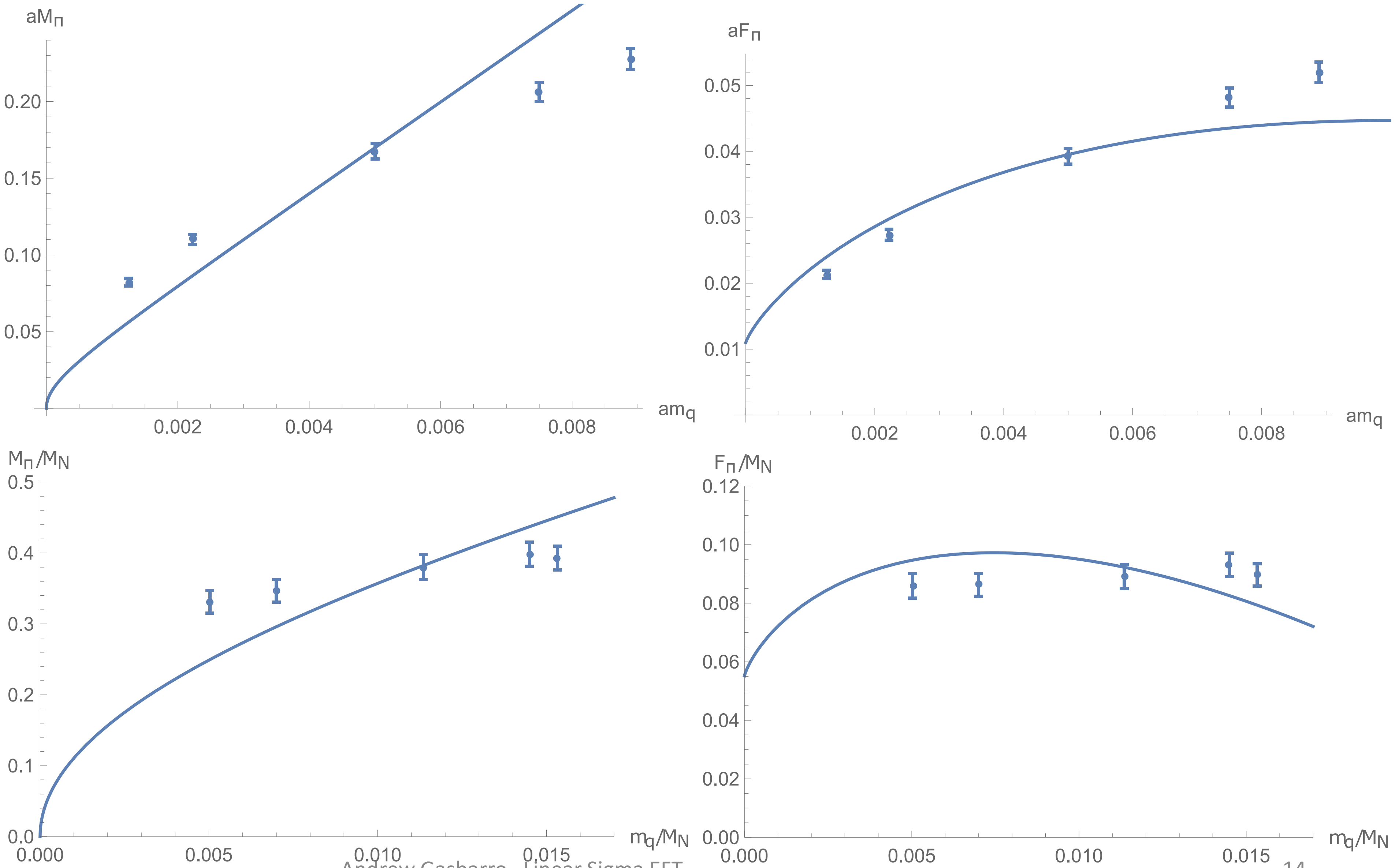
Symbol	Operator	$3/5 < \alpha \leq 1$	$1 < \alpha \leq 3$
O_1	$\langle \chi^\dagger M + M^\dagger \chi \rangle$	✓	✓
O_2	$\langle M^\dagger M \rangle \langle \chi^\dagger M + M^\dagger \chi \rangle$	✓	X
O_3	$\langle (M^\dagger M)(\chi^\dagger M + M^\dagger \chi) \rangle$	✓	X
O_4	$\langle \chi^\dagger M + M^\dagger \chi \rangle^2$	✓	X
O_5	$\langle \chi^\dagger \chi M^\dagger M \rangle$	✓	X
O_6	$\langle \chi^\dagger \chi \rangle \langle M^\dagger M \rangle$	✓	X
O_7	$\langle \chi^\dagger M \chi^\dagger M + M^\dagger \chi M^\dagger \chi \rangle$	✓	X
O_8	$\langle \chi^\dagger \chi \rangle \langle \chi^\dagger M + M^\dagger \chi \rangle$	✓	X
O_9	$\langle (\chi^\dagger \chi)(\chi^\dagger M + M^\dagger \chi) \rangle$	✓	X

$$M_\sigma^2 - 3M_\pi^2 = m_\sigma^2 - 4B^2 m_q^2 (2c_4 - c_5 - c_6 + 4c_7)$$

Nf=8 QCD and NLO XPT

χ^2/d.o.f. = 29.77

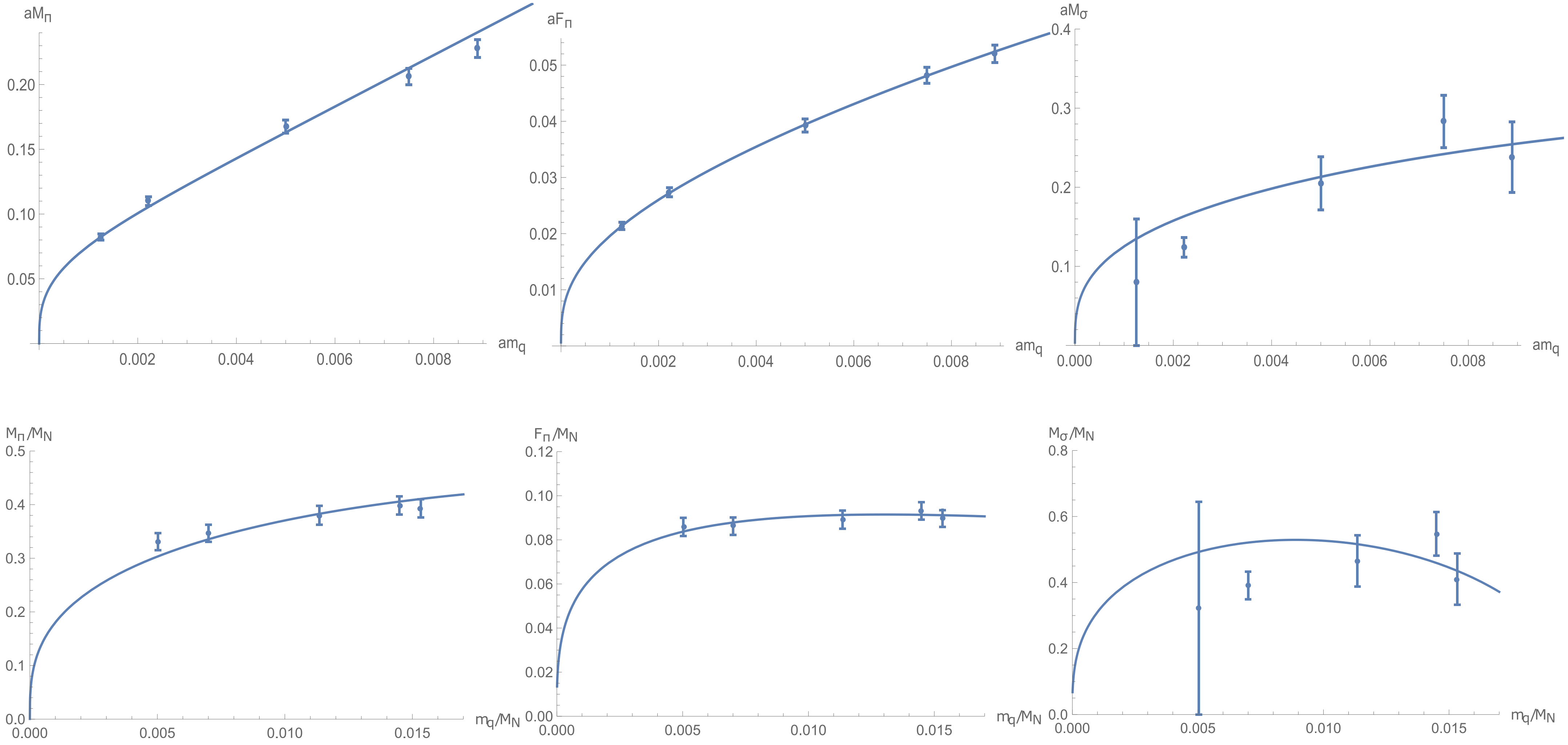
χ^2/d.o.f. = 7.65



Nf=8 QCD and LSM9

χ^2/d.o.f. = 1.30

χ^2/d.o.f. = 1.39



Conclusions

- Reviewed Nearly Conformal Gauge Theory Results from the Lattice
- Motivated New EFT framework
- Introduced EFT Framework Based on Linear Multiplet
- Developed an appropriate Power counting
- Fits to data show marked improvement over XPT

Ongoing & Future Work

- Better analysis of fitting parameter space
- Incorporate finite lattice spacing effects into EFT
- Include additional observables
  - Chiral Condensate
  - Pi-Pi Scattering Length
  - Form Factors

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