Lattice Simulations of Large N_c QCD

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Outline

- Motivation
- Lattice simulations away from $N_c = 3$
- Mesonic observables
- Baryon spectroscopy and its large-N_c regularities
- Different fermion representations

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Collaborators: Ayyar, Calle Cordon, Goity, Jay, Hackett, Y. Liu, Neil, Svetitsky, Shamir

Motivation

- Large N_c is where QCD is supposed to simplify
- Large N_c counting is about graphs but its consequences are nonperturbative
- Interesting to explore properties of confining, chirally broken theories
- How generic is QCD, anyway? Useful for
  - Qualitative understanding of matrix element regularities
  - Beyond standard model physics (composite Higgs, self interacting dark matter)

However

- If you only care about $SU(3)$, just do $SU(3)$ – everything else is an uncontrolled approximation
- Cost scales as $\propto N_c^{2-3}$
- No single N_c is interesting by itself
- Many potentially interesting tests are hard, even for $N_c = 3$

Technical issues for lattice simulations

I am using an arbitrary-color version of the Milc code written by Svetitsky, Shamir and me

To play the game you need

- Redefine 3 as NCOL – everywhere!
- Some algorithm development needed for smearing, updating beyond $N_c = 3$
- Baryons are made of N_c fermions – need interpolating fields

Large N_c project to-do list

- Simulate on as many N_c's as you can afford
- Tune bare couplings to match lattice spacings
- Use the same volumes, roughly same quark masses
- Compare dimensionless observables

and then you can test large N_c

- See if physics matches at similar (bare) $\lambda = g^2 N_c$ ($\beta = 2N_c/g^2$)
- Compare results against expected regularities

Fortunately – large N_c isn't just about small m_q (or even about the continuum limit)

Specific systems I studied

Related to 't Hooft large N_c

- Quenched $N_c = 3 - 7$: 1205.0235, 1404.2301
- $N_f = 2$, $N_c = 2 - 5$: 1606.01277
- Quenched but u, d, s, $N_c = 3 - 7$: 1308.4114
- Gradient flow $t_0(N_c)$: 1701.00793

Bali et al have much better data (but just for mesons, quenched, $N_c = 2 - 17$) – 1304.4437

Also, Beyond Standard Model inspired systems

- $SU(4)$, $N_f = 2$ AS2's: 1501.05665
- $SU(4)$, $N_f = 2$ AS2's, $N_f = 2$ F's: 1710.00806, 1801.05809

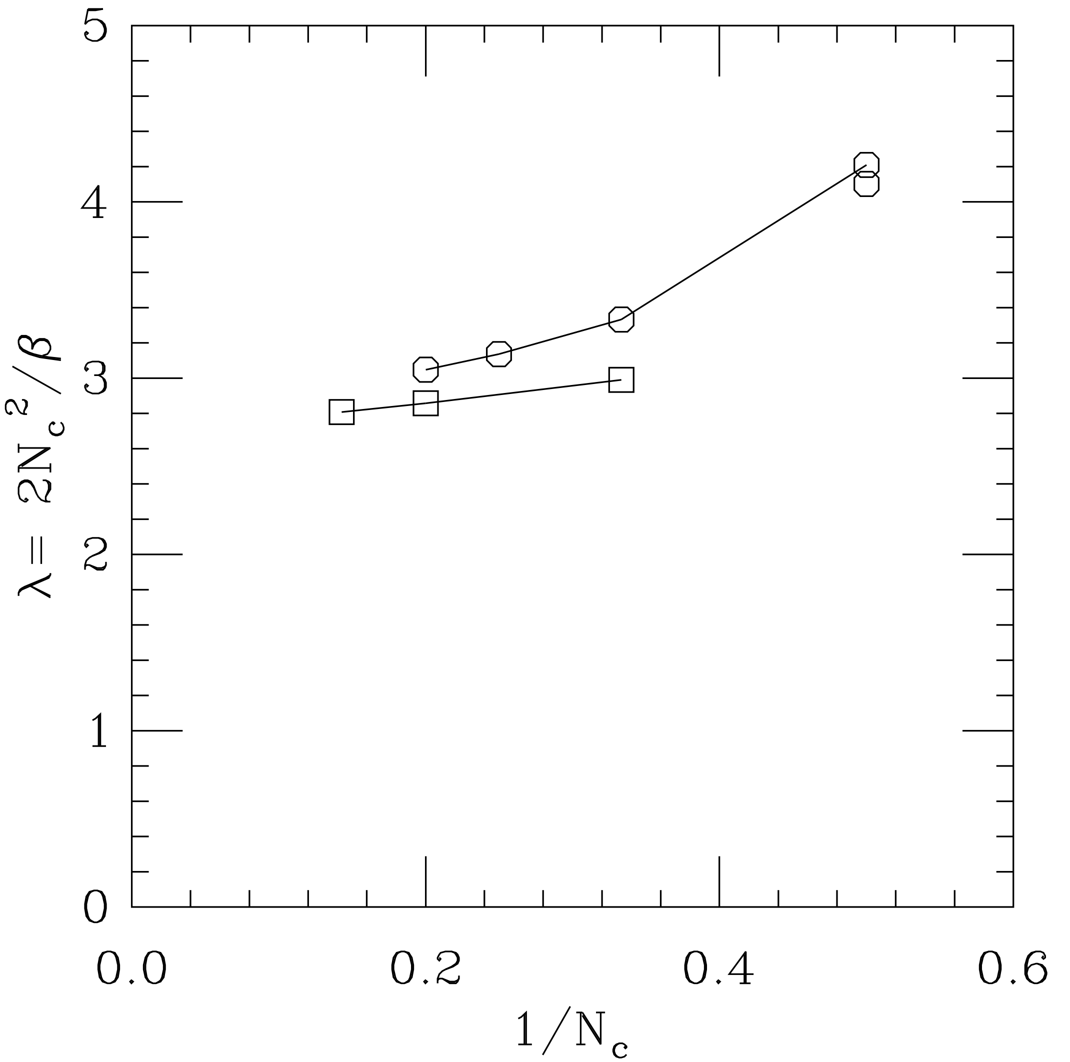
Zeroth order results

- To match gluonic or mesonic masses in lattice spacing a, match bare $g^2 N_c$'s
- As $N_f/N_c \to 0$, fermions affect a less and less
- To match a from one r in $V(r)$ is to match $V(r)$ across r

Lattice comment: I match across N_c with Sommer parameter $r_1 = 0.3$ fm

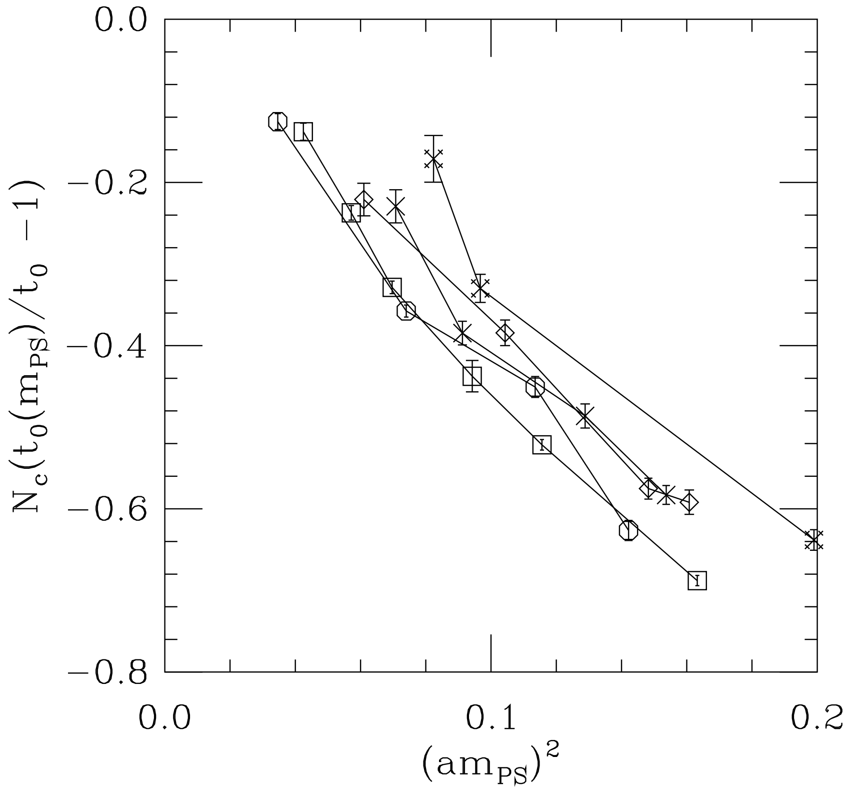
Lattice comment: Sometimes my collaborators use “flow” t_0 but that has its own N_c story

Pictures follow...

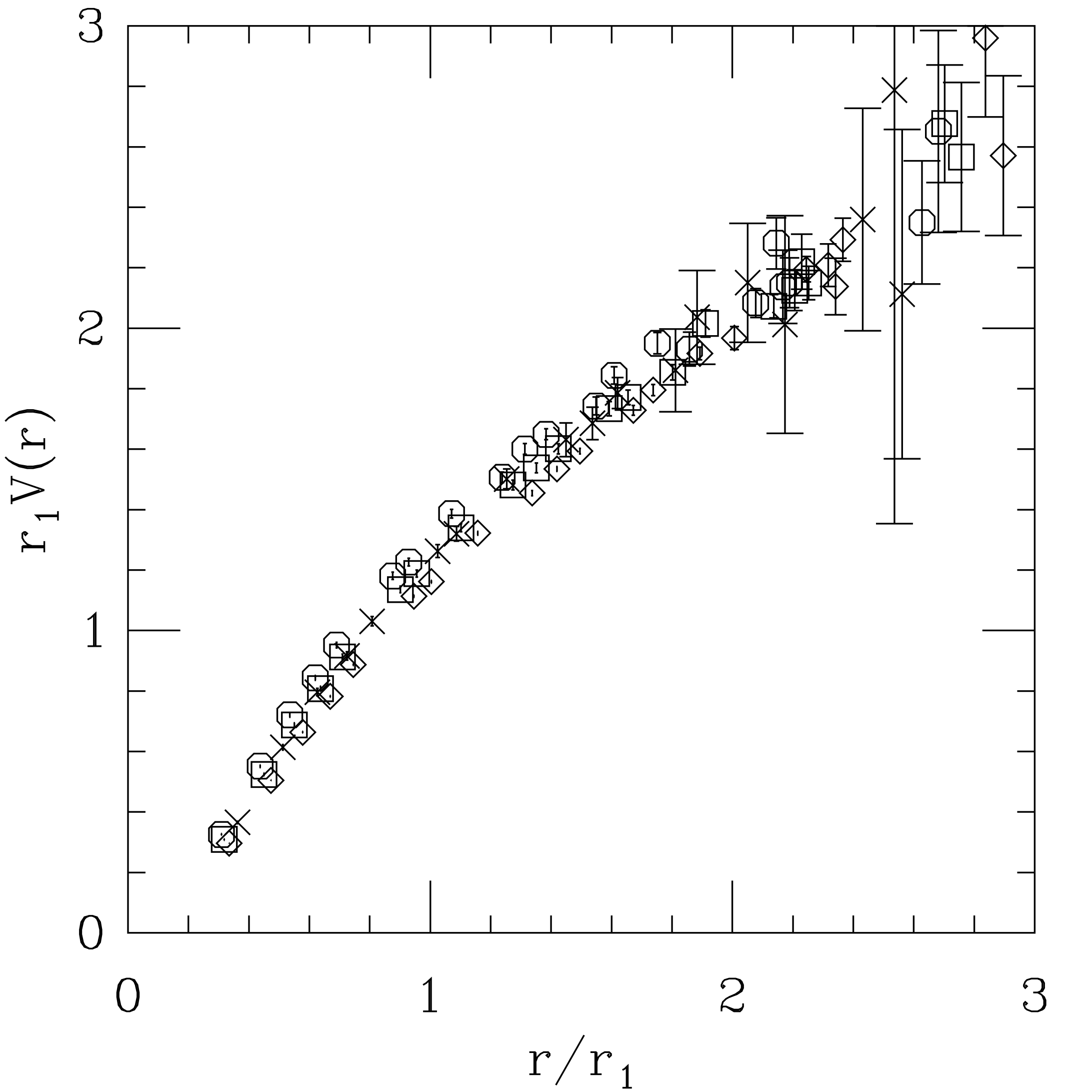


Bare (input) 't Hooft couplings where the lattice spacing matches ($r_1/a \sim 3$) – λ approaches a limit

$$t_0(m_{PS}) = t_0(0)(1 + k_1\frac{m_{PS}^2}{f_{PS}^2} + k_2\frac{m_{PS}^4}{f_{PS}^4}\log(\frac{m_{PS}^2}{\mu^2}) + k_3(\frac{m_{PS}^2}{f_{PS}^2})^2 + \ldots) \quad (1)$$



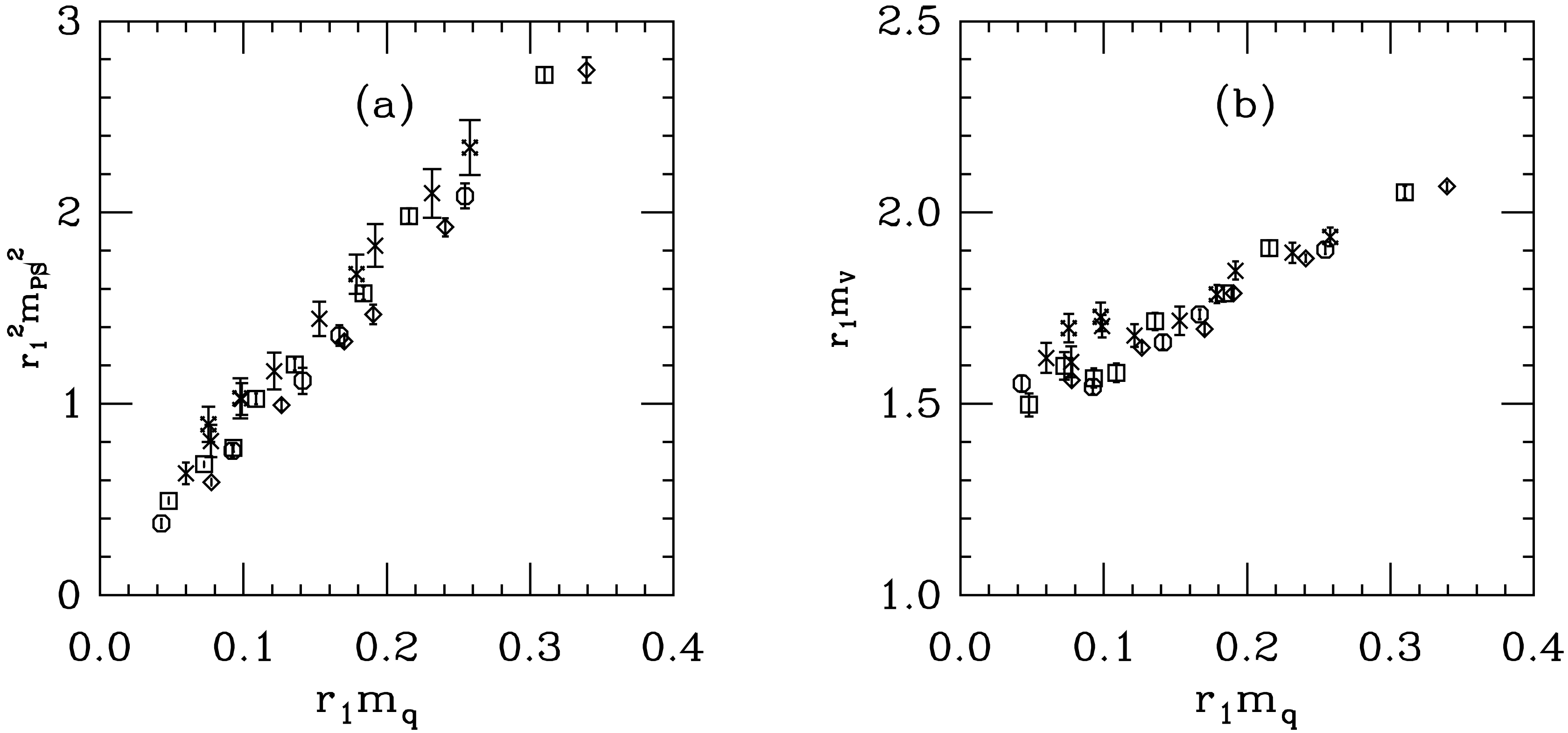
$$N_c(t_0(m)/t_0 - 1) = \frac{N_c}{f_{PS}^2} k_1 m_{PS}^2 \propto O(1) \quad (2)$$



Dimensionless combination $r_1 V(r)$ vs r/r_1 from data sets matched at $(m_{PS}/m_V)^2 = 0.48$. Symbols are crosses for $N_c = 2$, octagons for $N_c = 3$, squares for $N_c = 4$ and diamonds for $N_c = 5$.

Meson spectroscopy

Meson masses should be – and are – N_c-independent



(a) pseudoscalar mass squared (b) vector (Inflection point of potential, $r_1 \sim 0.3$ fm,$1/r_1 \sim 650$ MeV, used to set all scales)

Chiral symmetry breaking

My volumes were too small to do a really good job, can't get to tiny m_q, but...

Decay constants scale as $f \sim \langle 0|V|h\rangle \propto 1/\sqrt{N_c} \times N_c \propto \sqrt{N_c}$

And the condensate – modern methods need smaller m_q

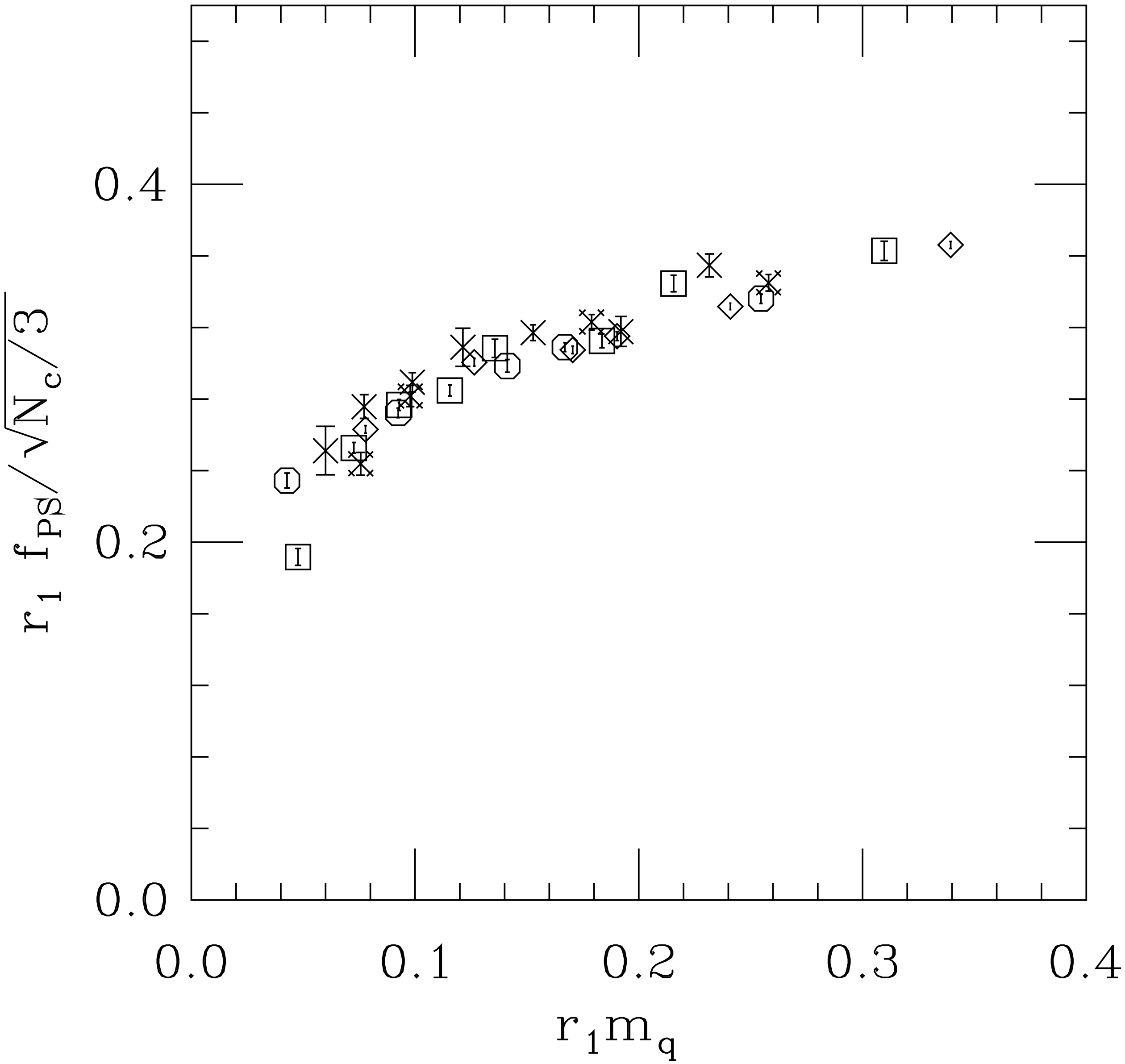
We used the Gell-Mann Oakes Renner relation

$$\Sigma(m) = \frac{m_{PS}^2 f_{PS}^2}{4m_q} \propto N_c? \tag{3}$$

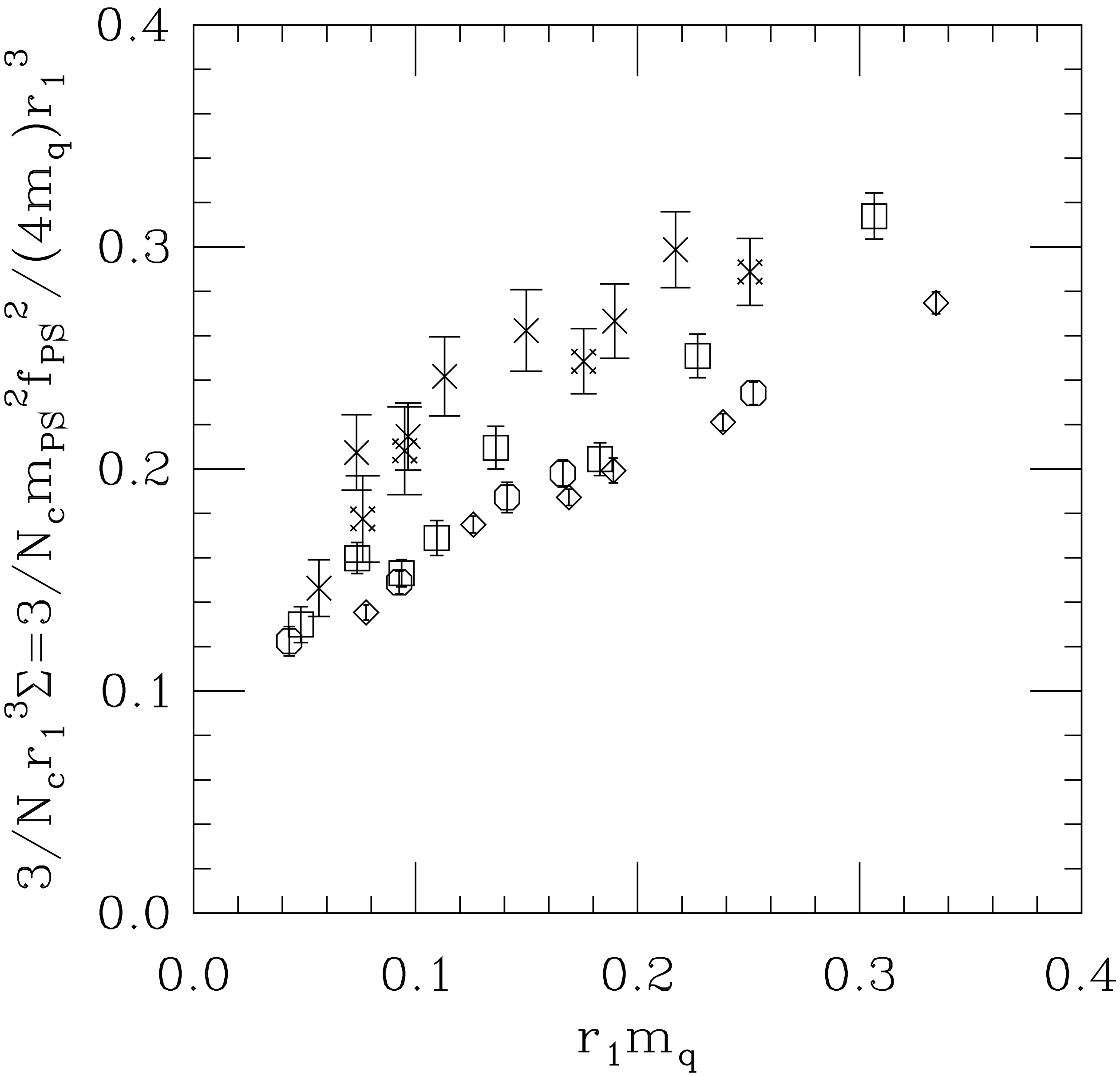
$SU(2)$ is an outlier compared to the rest

- It's the smallest N_c
- different chiral symmetry breaking pattern ($SU(2N_f) \to Sp(2N_f)$)

Decay constants $\langle 0|V|h\rangle \propto 1/\sqrt{N_c} \times N_c \propto \sqrt{N_c}$



Pseudoscalar decay constants with $\sqrt{N_c}$ scaled out, $N_c = 2 - 5$



Rescaled condensate from the GMOR relation. $N_c = 2$ is the outlier compared to the rest

Pause for clashing ideologies

"The scale for chiral symmetry breaking is different from the scale of confinement."

$f_\pi \sim 93$ MeV versus $\Lambda \sim 200$ MeV?

$T_{chiral} \neq T_{deconfinement}$? $r_{chiral} \neq r_{confinement}$?

In

$$\mathcal{L}_2 = \frac{F_0^2}{4}\mathrm{Tr}\,(\partial_\mu U \partial_\mu U^\dagger) + \Sigma_0 \mathrm{ReTr}\,(m_q U). \tag{4}$$

$F_0^2 \propto N_c$, $\Sigma_0 \propto N_c$

As N_c grows, both F_0 and Σ_0 get arbitrarily large, but m_{PS}^2/m_q stays fixed

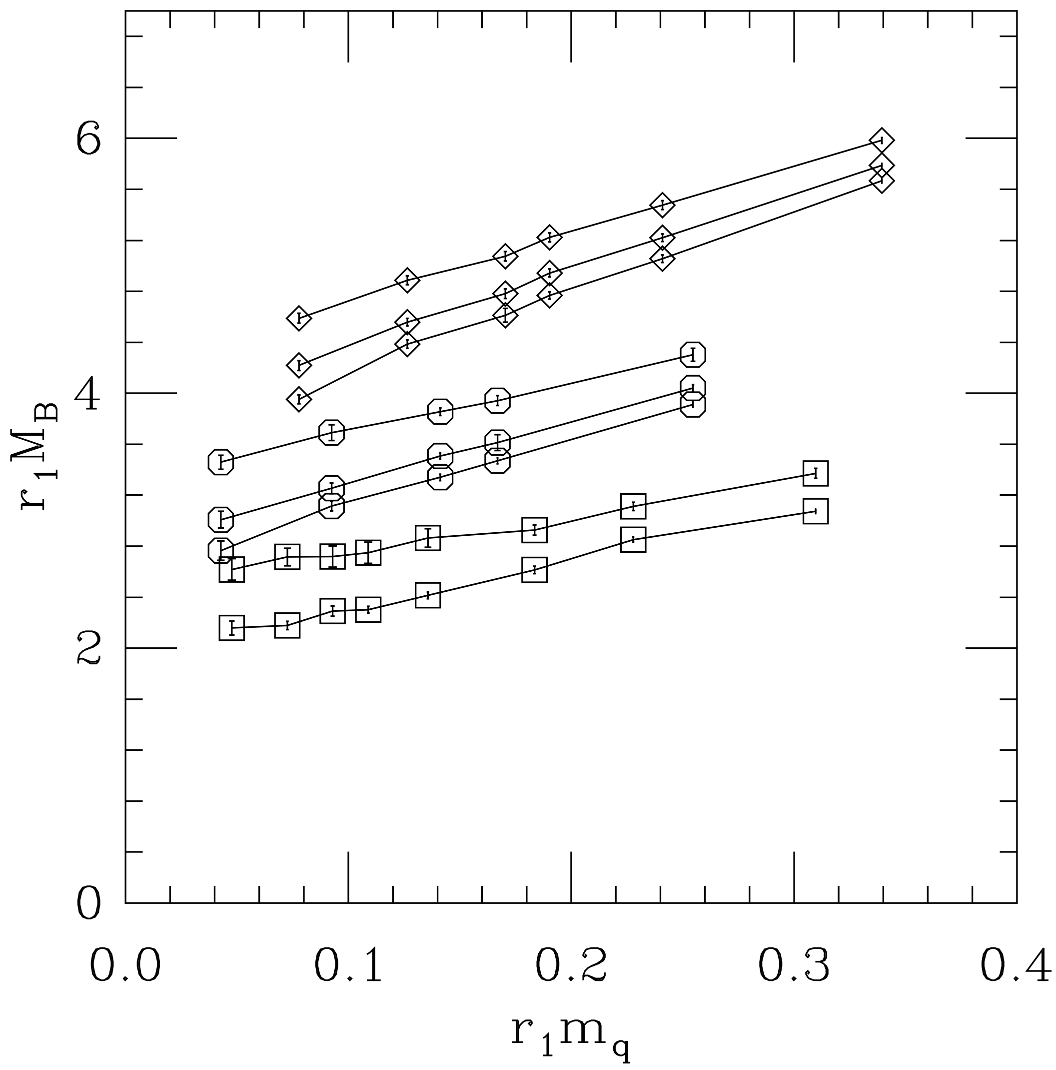
Could work with $B = \Sigma_0/F_0^2$, then $B \sim N_c^0$...

vs

"Hadronic quantities scale as $N_c^p \times$ a typical hadronic scale"

What is a "typical hadronic scale," anyway?

Baryon spectroscopy



Baryons, $N_f = 2$, $N_c = 3$, 4, 5

Baryons – theory

All states are isospin-spin locked, $J = I = N_c/2$, $N_c/2 - 1, \ldots$

A generic large N_c baryon mass formula is a rotor spectrum

$$M(N_c, J) = N_c m_0 + B\frac{J(J+1)}{N_c} \tag{5}$$

m_0 and B are m_q dependent, need to do comparisons versus m_q

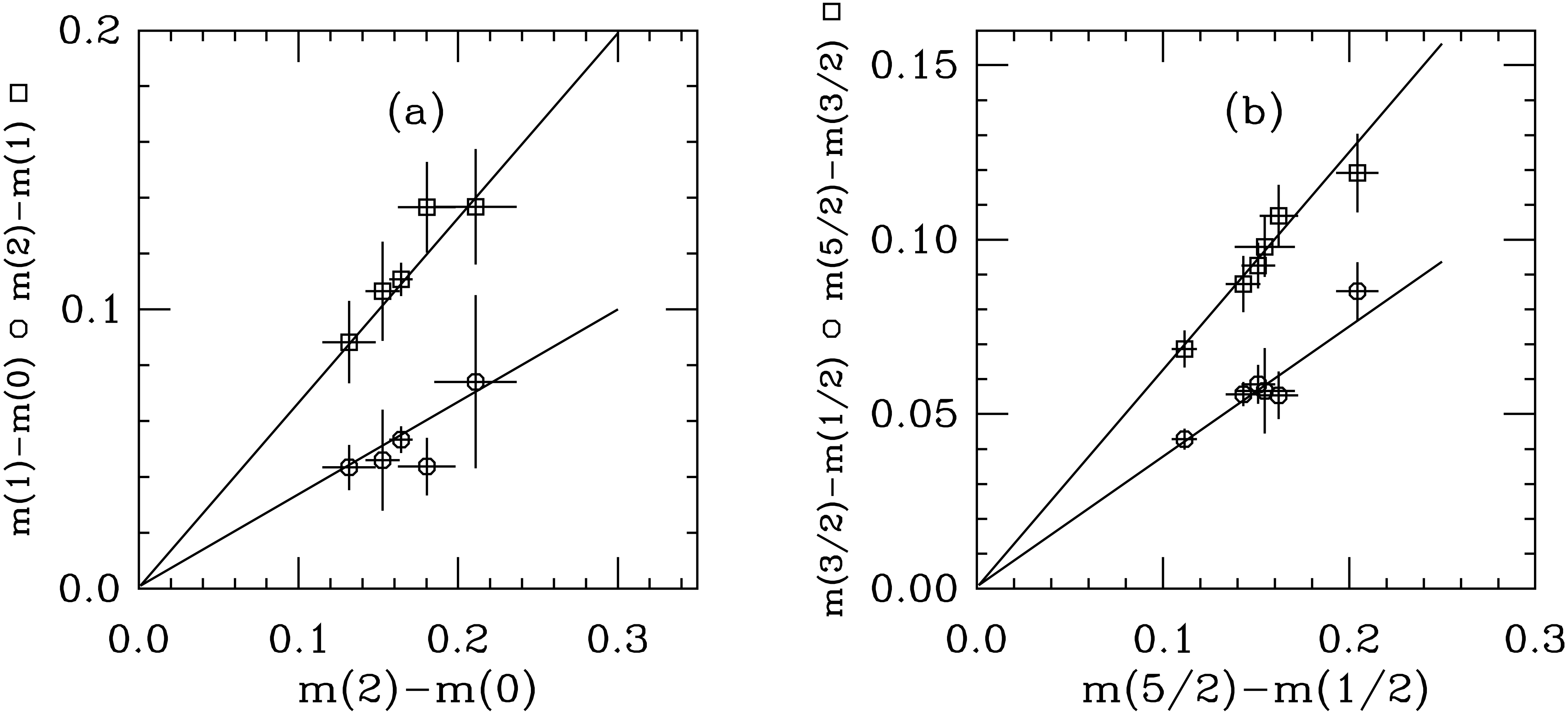
m_0 and B themselves have a $1/N_c$ expansion, $m_0 = m_{00} + m_{01}/N_c + \ldots$

Testing the $J(J+1)$: ratios of mass differences are pure numbers

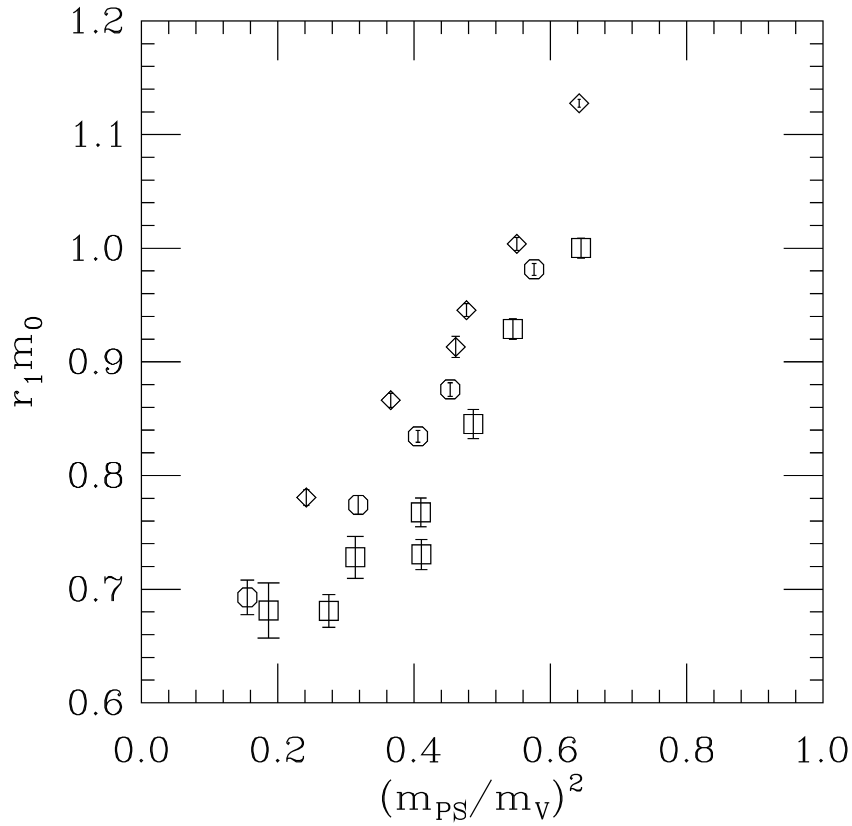
$$\Delta(J_1, J_2, J_3) = \frac{M(N_c, J_2) - M(N_c, J_3)}{M(N_c, J_1) - M(N_c, J_3)}, \tag{6}$$

Plot of one mass difference versus another one has a pure-number slope

It's like the Landé interval rule in atomic spectroscopy



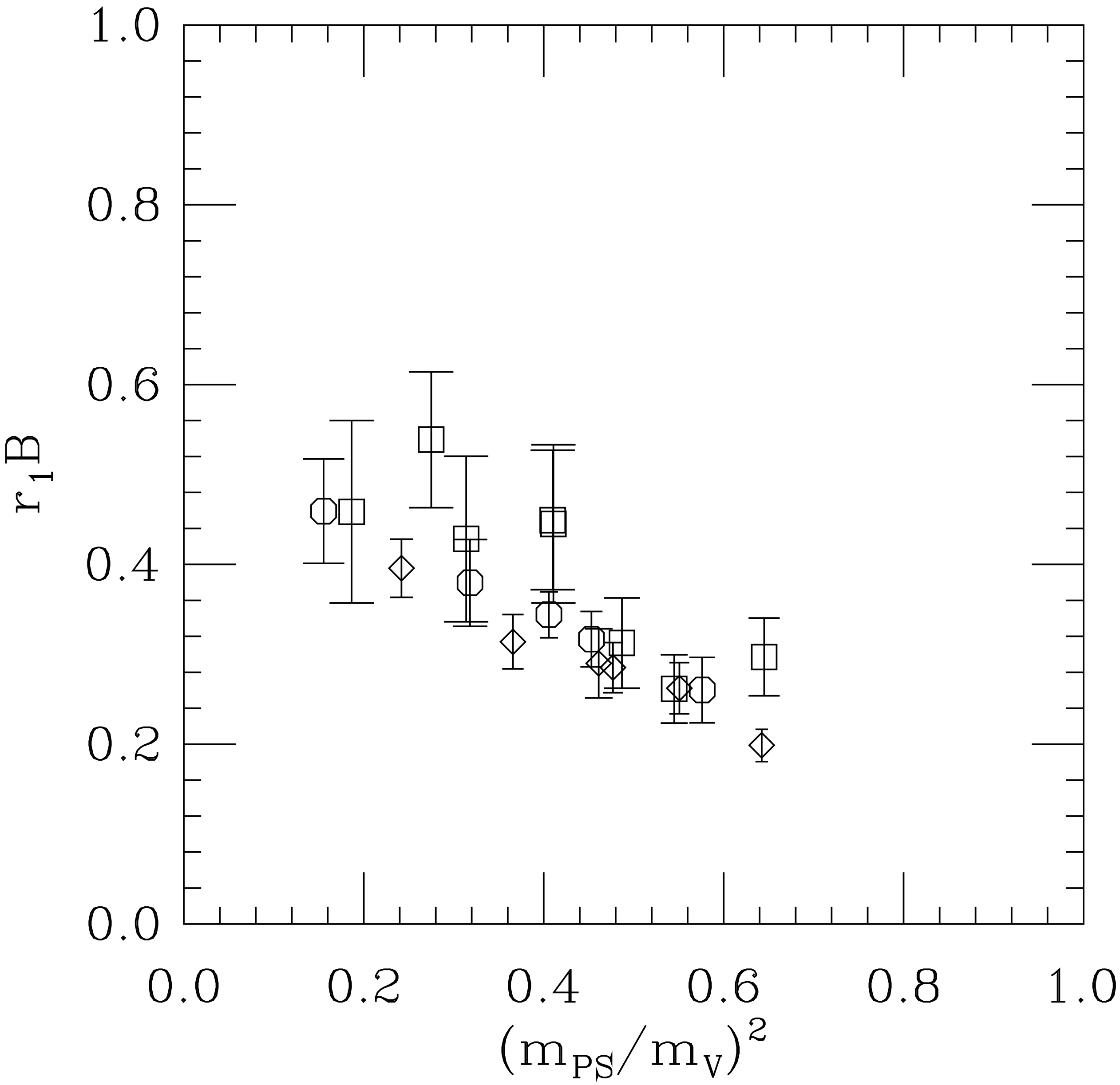
Ratios of mass differences for $N_c = 4$ and $N_c = 5$. Lines are the analytic ratio (NOT a fit)



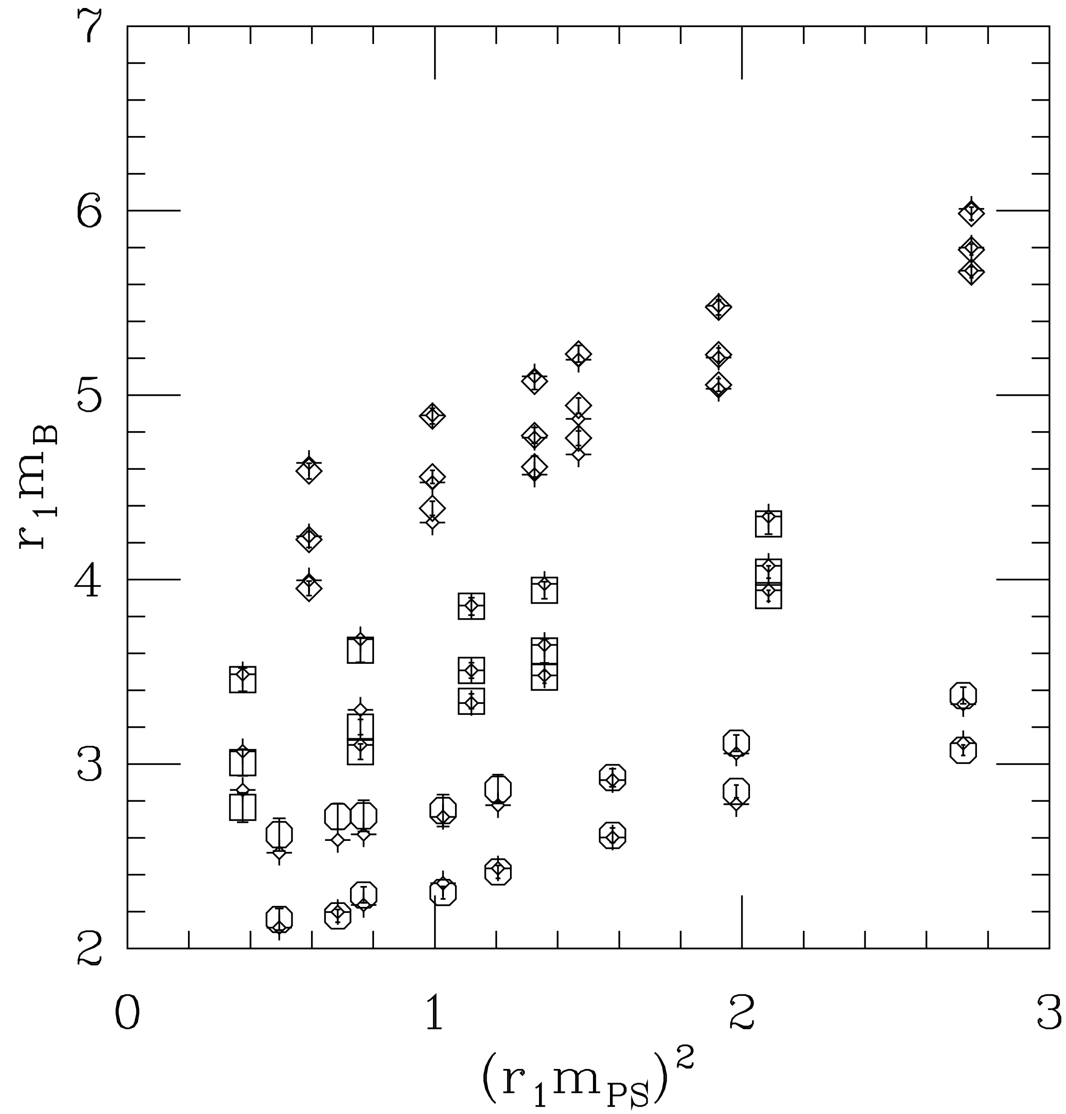
m_0 vs vs $(m_{PS}/m_V)^2$. Data are squares for $SU(3)$, octagons for $SU(4)$, diamonds for $SU(5)$.

m_0 drifts with $1/N_c$: a better rotor formula is

$$M(N_c, J) = N_c(m_{00} + \frac{m_{01}}{N_c}) + B\frac{J(J+1)}{N_c} + \ldots \quad (7)$$



B vs $(m_{PS}/m_V)^2$. Note falling behavior vs $(m_{PS}/m_V)^2$ while m_0 grows.



Doing better! A six-parameter fit to all baryons in $N_c = 3 - 5$

$$m_B = N_c(m_{00} + \mu_1 m_{PS}^2) + (m_{01} + \mu_2 m_{PS}^2) + \frac{J(J+1)}{N_c}(B_0 + b m_{PS}^2) + \dots \quad (8)$$

m_0 and B

$$M(N_c, J) = N_c(m_{00} + \frac{m_{01}}{N_c}) + B\frac{J(J+1)}{N_c} \tag{9}$$

$m_0 = m_{00} + m_{01}/N_c$ is a "constituent quark mass," a smooth rising function of m_q

B is a falling function of m_q. What is it?

Skyrme story: $B/N_c \propto 1/I$, it's an inverse moment of inertia, $B \propto 1/M$

Colorspin (or color HFS) story (de Rujula, Georgi, Glashow or MIT bag model 1975)

$$V_{ij} \propto g^2 t_i^a t_j^a \vec{\sigma}_i \cdot \vec{\sigma}_j \tag{10}$$

$B \propto (m_i m_j)^{-1}$

Data can't decide between these choices (for moderate m_q)

Different fermion representations

Also have $SU(4)$ with two AS2's, $SU(4)$ with two F's and two AS2's

$SU(4)$ with 2 AS2's has

- $SU(4) \to SO(4)$ chiral symmetry breaking (with diquarks)
- Meson spectrum again "universal"
- Pion decay constant for two-index fermions is $F_6 \propto N_c$, not $\sqrt{N_c}$
- Q^6 baryons, also with rotor spectrum

$SU(4)$ with 2 AS2's and 2 F's has (in addition)

- An extra $U(1)$ Goldstone ($\bar{q}q - QQ$), which we've looked for, not seen directly
- $F_6/F_4 \sim 1.5$
- qqQ baryons, like the Σ^*, Σ, Λ, with a more colorful s
- Meson spectra similar to F systems

Colorspin and multiple representations

$$V_{AB} = g^2 t_A^a t_B^a \vec{\sigma}_A \cdot \vec{\sigma}_B \tag{11}$$

Compare meson, F baryon (q^{Nc}), $SU(4)$ AS2 Q^6, q^4 and qqQ baryons

$$0 = \mathrm{Tr}\, \langle H | (\sum_{j=1}^{M} t_j^a)^2 | H \rangle \tag{12}$$

so

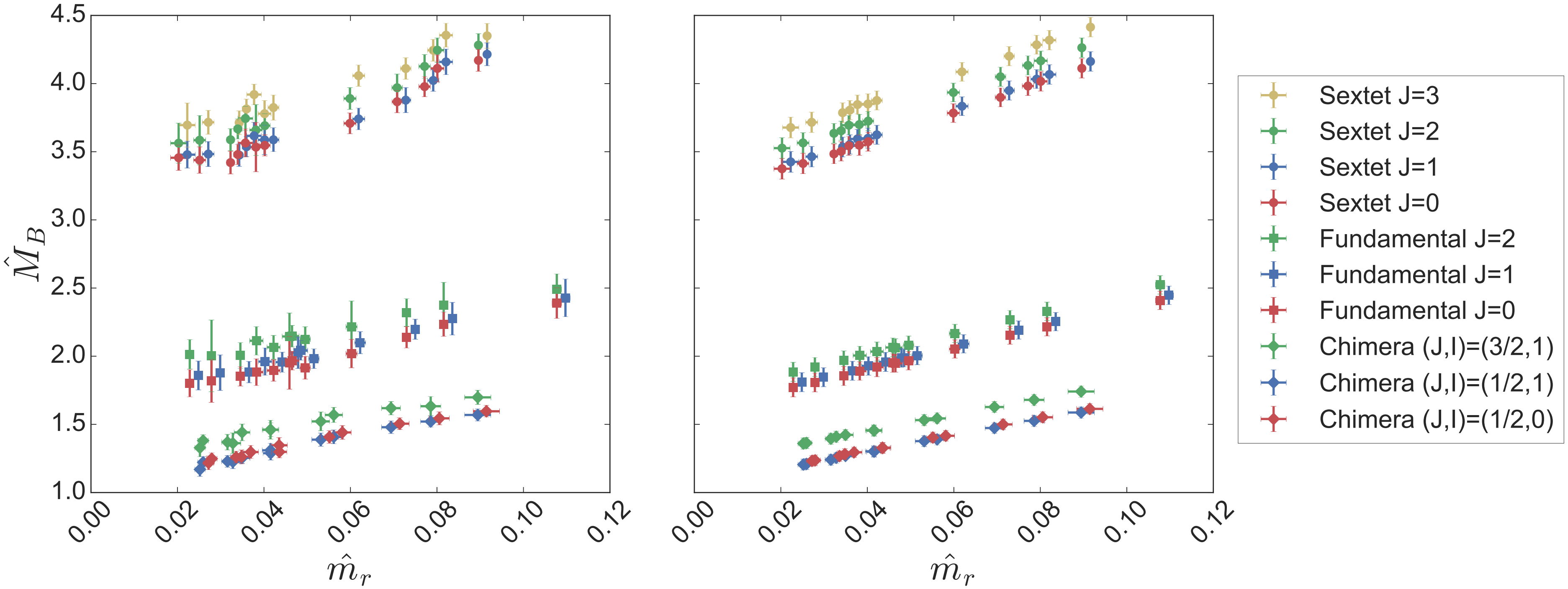
$$0 = M C_2(r) + M(M-1) \langle H | \mathrm{Tr}\, t_A^a t_B^a | H \rangle \tag{13}$$

and

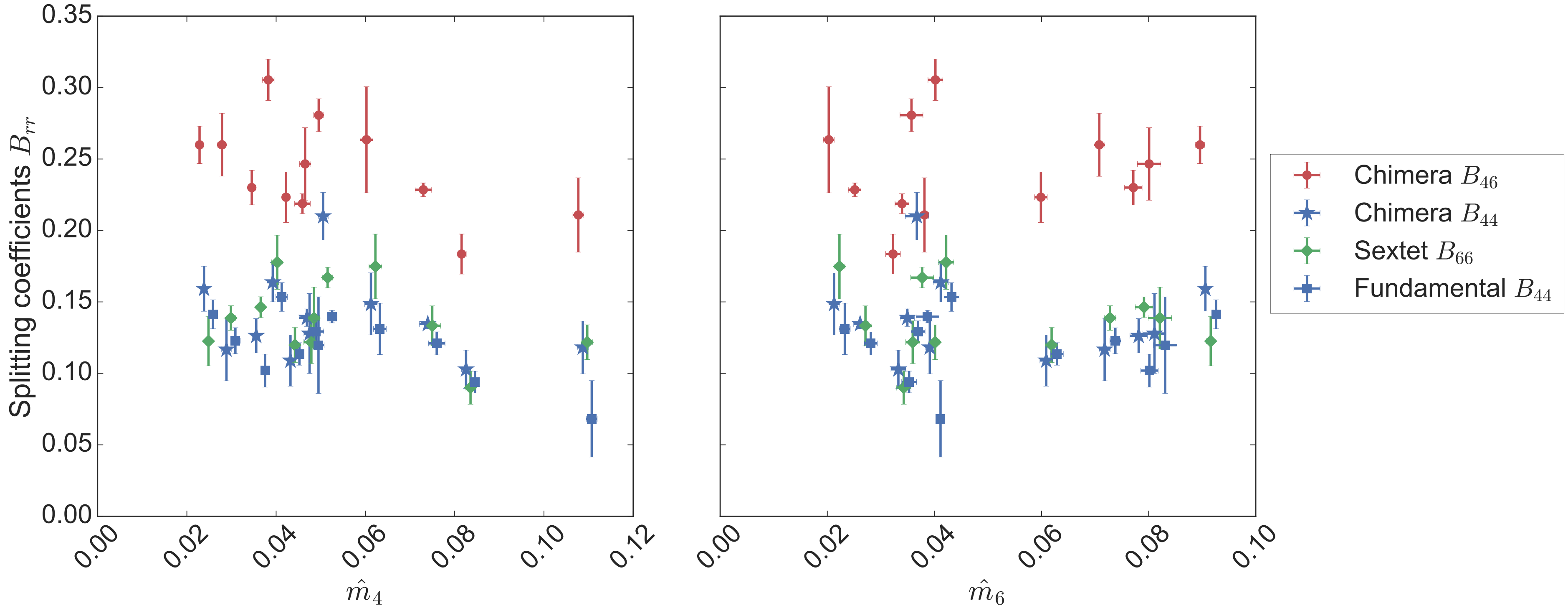
$$\langle \mathrm{Tr}\, t_A^a t_B^a \rangle = -\frac{C_2(r)}{M-1} \tag{14}$$

Cases:

- $q\bar{q}$: $C_2(r) \sim N$, $V \sim g^2 N \sigma_i \cdot \vec{\sigma}_j$
- q^M: $C_2(r)/M \sim 1$, $V \sim g^2 \sigma_i \cdot \vec{\sigma}_j$
- $SU(4)$:
  - q^4: $V \sim \frac{5}{8} \vec{\sigma}_A \cdot \vec{\sigma}_B$
  - Q^6: $V \sim \frac{1}{2} \vec{\sigma}_A \cdot \vec{\sigma}_B$
  - qqQ: $V \sim \frac{5}{8} \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \frac{5}{4} (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{\sigma}_Q$



$SU(4)$ with $N_f = 2$ F's and AS2's. q^4, Q^6 and qqQ baryons (chimeras) data left, fit right, versus average valence quark mass



B terms for $SU(4)$ with $N_f = 2$ F's and AS2's, q^4, Q^6 and qqQ baryons

Colorspin counting $B_{44} = 5/8$, $B_{66} = 1/2$, $B_{46} = 5/4$

Conclusions

Large N_c counting works, lot of aspects are untested but (maybe) accessible

Many possible extensions – E = easy, M =moderate, D =difficult

- B_K and other matrix elements with a large N_c history (E)
- Nonzero temperature (E-M)
- Form factors (N_c independence of wave functions) (M-D)
- Γ vs N_c ($\Gamma(\rho \to \pi\pi)/M \propto 1/N_c \to$ topological expansion) (D)
- Contracted $SU(2N_c)$ algebra (baryon couplings to pions) (D)
- Tiny m_q ($m_{\eta'} \to 0$) (M-D for η')
- Excited states (D)
- Veneziano limit (N_f/N_c fixed, $N_c \to \infty$) (E-M)
- Corrigan-Ramond limit (AS2 fermions) (E if quenched)
- etc

To extrapolate respectably to $N_c \to \infty$ requires better data

Bottom line for BS model pheno: these systems are simple, describe them with a quark model