SET 9–DUE 18 MARCH

"Never be the brightest person in the room" (J. Watson)

1) [15 points] The Lagrange density for a two component self-interacting scalar field is

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu\phi_1)(\partial^\mu\phi_1) + (\partial_\mu\phi_2)(\partial^\mu\phi_2)] + \frac{1}{2}\mu_0^2[\phi_1^2 + \phi_2^2] - \frac{\lambda}{4!}[\phi_1^2 + \phi_2^2]^2.$$

For $\mu_0^2 > 0$ with the sign conventions as shown, the vacuum has broken symmetry and there is a Goldstone boson.

(a) [10] To see this more clearly, consider the change of variables to two new fields $\rho(x,t)$ and $\chi(x,t)$ with $\phi_1(x,t) = \rho(x,t)\cos\chi(x,t)$ and $\phi_2(x,t) = \rho(x,t)\sin\chi(x,t)$ or $\phi(x,t) = \phi_1(x,t) + i\phi_2(x,t) = \rho(x,t)e^{i\chi(x,t)}$. Show that ρ is the massive field and χ is the Goldstone boson.

(b) [5] Next, suppose that the last symmetry of the Lagrangian is broken by a term $V' = -\epsilon\phi_1$. Show that the Goldstone bosons also get a mass, and that $m_\chi^2 \simeq \epsilon/\rho_0$ where ρ_0 is the vacuum expectation value of the ρ field. This behavior is seen in ferromagnets, in the presence of an external magnetic field, and in QCD, where the pion is the Goldstone boson and ϵ represents the quark mass.

I do not do much index-pushing for a living, so we have a couple of Jackson examples instead of something original.

2) [10 points] Jackson 12.14

3) [10 points] Jackson 11.14, parts a, b only

4) A plane wave is represented by a four-potential

$$A^\mu = a^\mu \exp ik^\alpha x_\alpha$$

where a^μ is a constant four-vector and $k^\mu k_\mu = 0$.

(a) [3 points] If the electromagnetic field is in Lorentz gauge, how are the a^μ's related?

(b) [3 points] Find an expression for $F^{\mu\nu}$ and (c) [3 points] show that $F^{\mu\nu}k_\nu = 0$.

(d) [6 points] Show that $\vec{B} \times \vec{k} = \omega \vec{E}/c$ follows from (c).