Set 8 – due 22 March

"It is quite easy to speak about symmetries, on one side. Everybody has a notion of symmetry, it is a very deeply rooted and widespread concept, ranging from art to science. In some way or another symmetry is perceived by everybody. I think it is worth mentioning that about thirty years ago there was strong interest in experimenting with apes to see how much they were able to learn. One objective was to see how apes would learn to paint. In one of these experiments one dot was made at one side of a piece of paper and the ape would then try to make a dot on the other side to balance it symmetrically. That's exactly what we are doing in physics." – J. Wess

1) [15 points] Jackson 11.13

2) [10 points] Jackson 12.14

3) [10 points] Jackson 11.14, parts a, b only

4) (30 points) The axion is a candidate dark matter field with potential JILA or NIST search modes. The Lagrangian for electromagnetism coupled to an axion field $a(x,t)$ is (in some units)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - J_\mu A^\mu + \frac{1}{4}CaF_{\mu\nu}\tilde{F}^{\mu\nu} \tag{1}$$

where C is a coupling constant and $\tilde{F}^{\mu\nu}$ is the dual field strength tensor defined in Jackson (11.140), $\tilde{F}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$. To set the stage for thinking about experiments, derive the analog Maxwell's equations from the Lagrangian, Eq. 1, expressing the electromagnetic fields in terms of $\vec{E}$ and $\vec{B}$. You'll get source terms involving the a field appearing along with the charge and current density. These new terms are the experimental hooks which different discovery experiments attempt to exploit.