SET 8–DUE 11 MARCH

"It is quite easy to speak about symmetries, on one side. Everybody has a notion of symmetry, it is a very deeply rooted and widespread concept, ranging from art to science. In some way or another symmetry is perceived by everybody. I think it is worth mentioning that about thirty years ago there was strong interest in experimenting with apes to see how much they were able to learn. One objective was to see how apes would learn to paint. In one of these experiments one dot was made at one side of a piece of paper and the ape would then try to make a dot on the other side to balance it symmetrically. That's exactly what we are doing in physics." – J. Wess

1) [15 points] Jackson 11.13

2) One could think of the Schrodinger equation as the classical equation of motion for a classical complex field $\psi$, which extremizes the action

$$S = \int dt d^3x [i\hbar\psi^* \frac{\partial\psi}{\partial t} - \frac{\hbar^2}{2m}(\vec{\nabla}\psi)^* \cdot (\vec{\nabla}\psi) - V(r,t)\psi^*\psi].$$

(a) [10 points] Varying $\psi^*$ and $\psi$ separately, and integrating by parts if you have to, verify this assertion.

(b) [10 points] What are the conserved current and associated charge density associated with the invariance of the action (and Lagrange density) under global phase rotations $\psi(x,t) \to e^{i\alpha}\psi(x,t)$, $\psi(x,t)^* \to e^{-i\alpha}\psi(x,t)^*$.

(c) [10 points] In quantum mechanics we usually write the desired local gauge transformation as

$$\psi(x)' = \exp(-i\frac{q}{\hbar c}\theta(x))\psi(x)$$

$$A_\mu(x)' = A_\mu + \partial_\mu\theta(x).$$

Show that the appropriate covariant derivative is

$$D_\mu = \partial_\mu + i\frac{q}{\hbar c}A_\mu.$$

Hint: show $D'_\mu\psi' = \exp(-i\frac{q}{\hbar c}\theta(x))D_\mu\psi$

(d) [10 points] Next, make the replacement of the covariant derivatives in the action and derive the appropriate Schrodinger equation for a nonrelativistic quantum mechanical particle in an electromagnetic field.

For background material (why would you want to do this?) see Schiff, "Quantum Mechanics," Ch. 14.