SET 6–DUE 25 FEBRUARY

"What a pity that I have to die in the age of relativity's development." – H. Minkowski (1909)

1) [20 points] "Why, these monsters had star travel!" In Heinlein's classic novel, "Have Space Suit, Will Travel," the earth is menaced by aliens from Proxima Centauri (distance 4.3 light years), who travel in ships which accelerate for half the trip at $A = 8$ g, and decelerate for the other half. What is the elapsed time of a one-way trip from there to here, as seen by the crew, and as measured by observers on either planet?

It is very useful to first show that

$$\frac{du}{dt} = \frac{1}{\gamma}\frac{du'}{dt'}\frac{1 - v^2/c^2}{(1 + vu'/c^2)^3}$$

In the limit $u' \to 0$, this reduces to A/γ^3 for this problem, and (using $u = v$ in this limit), you can show $At = \gamma v$, which you can use to find $v(t)$ or $\gamma(t)$, to integrate $dt' = dt/\gamma$.

2) [20 points] Two equal mass particles (of mass m) scatter elastically with a scattering angle α in the center of mass frame. Show that the scattering angle in a frame where one particle is at rest and the other has energy E is given by

$$\cos^2\theta = \frac{\cos^2\alpha/2}{1 - \frac{E - mc^2}{E + mc^2}\sin^2\frac{\alpha}{2}}$$

Comment on the nonrelativistic and extreme relativistic limits.

3) [10 points] A set of invariants which describe the scattering of unequal mass relativistic particles ($1 + 2 \to 3 + 4$) are $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, $u = (p_1 - p_4)^2$. (a) [5 points] Show $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$. (b) [5 points] Now assume all the particles have equal mass m. From (a) only two of s, t, u are independent. Draw a two dimensional plot of s vs t showing the kinematically allowed region(s)– i.e., ANY scattering experiment will populate only your allowed region. ($c = 1$ here.) The reason why this is interesting:

scattering amplitudes are most usefully computed in terms of invariants, because then it is easy to evaluate them in different frames. It is useful to know the ranges of s, t, etc, when you examine the amplitudes to look for “interesting” behavior.