Set 5 – due 23 February

"The ideas of space and time which I wish to develop before you grew from the soil of experimental physics. Therein lies their strength. Their tendency is radical. From now on, space by itself and time by itself must sink into the shadow, while only a union of the two preserves independence." – H. Minkowski (1908)

1) [5 points] Jackson 11.4 part (a). (b) [5 points] Instead of Jackson's part (b), do the following: "Length contraction" occurs when an observer infers a length from an elapsed time. Suppose a rod moves parallel to its length at velocity v, past an observer. The (unprimed) observer sees one end of the rod pass the origin at time $t_1 = 0$. The other end passes the origin at time t_2. The observer infers a length $l = vt_2$. A (primed) observer on the rod also has $(x'_1, t'_1) = (0, 0)$. The other end of the rod is at l_0 and the moving first observer passes this end at time $t'_2 = l_0/v$. Show $l = l_0/\gamma$.

2) [10 points] Consider a stick with a mechanism which, in its proper rest frame, can simultaneously release a drop of ink from each end. The stick moves parallel to its length with a velocity v along the floor. The stick has a length (measured in its rest frame) of l. The floor has lines spaced a distance l apart, again as measured in the floor's frame, aligned perpendicular to the direction of motion of the stick. When the mechanism is set off, how far apart (in the rest frame of the floor) are the marks on the floor? Justify your result from the point of view of an observer in the rest frame of the stick and an observer in the rest frame of the floor.

3) [15 points] Consider a stick with a mirror on the right end. At a given moment a photon and a particle moving with velocity $u < c$ leave the left end, moving to the right along the stick. The photon reaches the mirror first, is reflected, and, moving back to the left, encounters the particle still moving to the right, a fraction x of the way along the stick. The situation at three typical moments is shown below.

Using the fact that x is an invariant quantity (think about it!) show using only the constancy of the speed of light (i. e. you don't have to assume anything about the shrinking factor for the moving stick) that for an observer moving to the left with velocity v with respect to the stick, the velocity of the particle must be

$$w = \frac{u+v}{1+\frac{uv}{c^2}} \tag{1}$$