Set 4 – due 16 February

"Diffraction problems are amongst the most difficult ones encountered in optics." (From "Principles of Optics," Born and Wolf)

1) [20 points] Jackson 10.9a. Use the Born approximation. The integral peaks strongly in the forward direction, so you can replace $qa \simeq ka\theta$, $d\cos\theta = \theta d\theta$, and take the range of θ from 0 to infinity. You'll get an integral

$$\sigma \simeq 2\pi|\epsilon - 1|^2 k^2 a^4 \int_0^\infty x dx \frac{j_1(x)^2}{x^2}. \tag{1}$$

At that point Bessel function identities near Jackson 9.90 might be useful.

Notice how the Rayleigh k^4 is softened by the extended source to $\sigma \sim k^2$.

2) [20 points] Jackson 10.11. (a,b only). Hint: expand

$$R = [(x - x')^2 + y^{'2} + z^2]^{1/2} \sim z[1 + \frac{(x - x')^2 + y^{'2}}{2z^2} + \ldots] \tag{2}$$

For Fresnel integrals, see Wikipedia "Fresnel Integral," Abramowitz and Stegun, p. 300, or Morse and Feshbach, p. 816. There does not seem to be a standardized notation for these functions.

3) [20 points] Jackson 10.12. In (a), work in the Fraunhofer limit. It is quite similar to the calculation done on pp. 491-492, (it is basically "solve by copy") except that the initial polarization is $\vec{\epsilon}_0 = \hat{y}$. For part (b), use the Dirichlet formula, 10.85. (drop the $+i/(kR)$)