Set 3 – due 9 February

"If you can't be a giant, you want to be the giant killer"–M. Perl

1) [15 points] Jackson 10.4, parts a (5 points) and b (10 points) only. $\delta \gg R$ means that the size of the sphere is much less than the attenuation length, so the scatterer can just be treated as a dielectric sphere. In part b, to find absorption, calculate the power dissipated due to conductivity σ.

2) [15 points] Jackson 10.5. Part a (10 points) only. It might be useful to show that the induced magnetic dipole moment is

$$\vec{m} = \frac{ik\sigma}{2c} \int \vec{x}(\vec{x} \cdot \vec{B}) d^3x. \tag{1}$$

In lieu of part (b), compute [5 points] the differential and total cross sections for scattering in the long wavelength limit including both the induced magnetic moment term from this problem and the electric dipole term from problem 1. Explicitly work through the average over initial polarizations ande sum over final ones. Notice that both the electric and magnetic dipole moments are complex.

The optical theorem is an exact relation when used with the exact scattering amplitude. It is very tricky to apply when you only have an approximate scattering amplitude.

3) [10 points] This is the simple mathematics behind the Weisskopf article. Suppose you model the atom as an electron on a spring of natural frequency ω_0. Include a damping force $F = -\Gamma d\vec{x}/dt$. Put the spring in an external electric field $\vec{E}\exp(-i\omega t)$, solve for the steady state motion, compute the resulting dipole moment, and drop the result into the appropriate formula for the differential cross section. Now do physics: note how you recover Rayleigh scattering for $\omega \ll \omega_0$. At high frequency, the cross section becomes independent of frequency. This is called the Thomson cross section. (It is the same formula for scattering off a free electron.) There is a dimensionful constant (in CGS)

$$r_0 = \frac{e^2}{m_e c^2} \tag{2}$$

where m_e is the electron mass, which characterizes the scale of elastic light scattering on electrons and hence on atoms. Find a number for this in cm. This says that the natural scale for light scattering on the electrons in atoms is $\sigma \sim r_0^2$ (or maybe more correctly, $r_0^2(\omega/\omega_0)^4$, which is even smaller). Notice how all the interesting polarization behavior is frequency-independent. This isn't generally true, but it is true in dipole approximation. There are other ways to derive this result which we'll encounter later on in the course.