SET 12–DUE 15 APRIL

Above the front door of his country cottage in Tisvelde Bohr nailed a horseshoe, which is proverbially instrumental in bringing luck. Seeing it, a visitor exclaimed: “Being as a great scientist as you are, do you really believe that a horseshoe above the entrance to a home brings luck?” “No,” answered Bohr, “I certainly do not believe in this superstition. But you know,” he added with a smile, “they say that it does bring luck even if you don’t believe in it!” from G. Gamow, “Thirty years that shook physics”

1) [20 points] Jackson 14.12. Hint: (but you must do the integral!)

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{dx\ \cos^2 x}{(1+b\sin x)^5}=\frac{1}{8}\frac{b^2+4}{(1-b^2)^{7/2}}.$$

I found the integral a challenging puzzle. After differentiating repeatedly under the integral sign, the difficult piece of it reduces to

$$J(\alpha)=\frac{1}{2\pi}\int_0^{2\pi}\frac{dx}{\alpha+\sin x}$$

which can be done using the residue theorem.

2) [20 points] Jackson 14.13. The Poisson summation formula is

$$\sum_{m=-\infty}^{\infty} e^{2\pi i m\omega/\omega_0}=\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$$

As a hint to get going on this, begin with (or derive) Eq. 14.67, and then notice that you can write

$$\int_{-\infty}^{\infty} f(t)dt=\sum_{m=-\infty}^{\infty}\int_0^T dt f(t+mT)$$

3) [20 points] Jackson 14.14. Use the result of 14.13, plus the definition for the cylindrical Bessel function

$$J_m(x)=\frac{1}{2\pi i^m}\int_0^{2\pi} d\phi\exp(i(x\cos\phi-m\phi))$$

plus some identities for sums or differences of the Bessels.