Set 1– due 26 January

"The world of our imagination is narrower and more special in its logical structure than the world of physical things." (Max Born)

1) [10 points] Jackson 9.3. In CGS, the discontinuity in $E \cdot \hat{n}$ across a conductor is $4\pi\sigma$ where σ is the surface charge density. Or, just do the problems in MKS and let us deal with the units...

2) [20 points] Consider an array of $2N + 1$ dipoles. Each has a time-dependent dipole moment $\vec{p}(t) = \hat{z} p e^{-i\omega t}$, and they are spaced a distance D apart along the $\hat{z}$ axis, centered on the origin. What is the resulting antenna pattern? Explicitly work out what happens at θ (the polar angle measured with respect to the $\hat{z}$ axis) near $\pi/2$.

This is an iconic problem: many similar radiators distributed in space, so the antenna pattern tells you things about the spatial distribution of radiators.

3) (a) [15 points] The surface of a charge distribution of uniform density is almost a sphere: the radius as a function of the polar angle is $R(\theta) = R_0(1 + \gamma \cos\theta)$. The quantity γ oscillates in time with frequency ω, so we are describing something like surface waves on a water balloon. Working to lowest nontrivial order in the small parameter γ, and in the long wavelength limit, find an expression for the antenna pattern of emitted radiation and the total power radiated.
(b) [5 points] Part (a) is a variation of Jackson 9.12, except in that problem $R(\theta) = R_0(1 + \beta P_2(\cos\theta))$. What is the leading multipole behavior of the radiation in that case? Why?