Set 9 – due 3 November

"The nation that controls magnetism will control the universe" – Dick Tracy (1935)

1) Jackson 5.27 [10 points]

2) Jackson 5.33 [10 points] (a)–5, (b)–5.

3) Jackson 5.34 [20 points] (a)–3: Use the formula given in Problem 5.10b as the start. (b)–7; (c)–7; (d)–3: No discussion of Prob. 5.18 is needed.

4) Jackson 6.8 [20 points]

The hard part of this problem is the start. $\vec{P}$ always follows $\vec{E}$, so $\vec{P}$ points along $\hat{x}$. You need the surface magnetic pole density $\sigma_M = \vec{M} \cdot \hat{n}$ to source Φ_M. Once you have it, the problem comes apart in your hands.

There are (at least) three ways to begin. First, you could use the surface current density $\vec{K}_M$ and surface magnetization $\vec{M}$, $\vec{K}_M = \vec{M} \times \hat{n}$ where $\hat{n}$ is an outward normal to the surface. The surface current density comes from the surface polarization density $\vec{K} = \sigma_P \vec{v}$ where σ_P is the surface polarization charge density, and $\vec{v} = \vec{\omega} \times \vec{r}$. $\vec{K} = \vec{M} \times \hat{n}$ so $\vec{M} = \hat{k}\omega P_0 x$ where P_0 is the magnitude of the polarization vector.

Second, you could look at the volume magnetization M and find the volume current $\vec{J}_M = \vec{\nabla} \times \vec{M}$. You imagine a little dipole whose head and tail are separated by a small difference, so $\vec{J} = Nq(\vec{v}_+ - \vec{v}_-)$. This is nice, but wrong by a sign – the dipole remains oriented along $\hat{x}$, so the charge hops from dipole to dipole in the *opposite* direction to what you have found. You can find $\vec{M}$ from $\vec{J}_M = \vec{\nabla} \times \vec{M}$, you discover $\vec{\nabla} \cdot \vec{M} = 0$ and construct σ_M.

The third way is to look arouhd Jackson Eq. 6.100: a material in bulk motion acquires an effective magnetization $\vec{M}_{eff} = \vec{P} \times \vec{v}$. The derivation is awful, it is fiddling along the lines of Eqs. 6.93-6.96.