Set 8 – due 27 October

"As usual, mathematical calculations could not win over unexpected conditions." – D. Eisenhower

1) Jackson 5.3 [10 points] The classic problem of the solenoid...

2) (a) [7 points] Continuing this problem, show that near the axis and near the center of the solenoid (length L and radius R) the magnetic induction is mainly parallel to the axis, but has a small radial component

$$B_\rho = 24\mu_0 NI \frac{R^2 z\rho}{L^4} \tag{1}$$

correct to order R^2/L^2 and for $z << L$, $\rho << R$. The coordinate z is measured from the center point of the solenoid, with the ends of the solenoid at $z = \pm L/2$.
(b) [3 points] Show that at the end of a long solenoid the magnetic induction near the axis has components

$$B_z \simeq \frac{\mu_0 NI}{2}, B_\rho \simeq \frac{\mu_0 NI}{4}\frac{\rho}{R}. \tag{2}$$

Hint: there is a short way to do this, or a very long way.

3) Jackson 5.13 [20 points]

4) [20 points] The Helmholz coil is a pair of circular current loops of radius a with a common axis, separated by a distance $2b$ which is chosen to make the second derivative of $\vec{B}$ vanish at a distance at a point on the axis halfway between the two coils. Show that this distance is $2b = a$ and also find the coefficient C in the Taylor expansion

$$B_z(b+\delta) = B_z(b)(1 + C(\frac{\delta}{a})^4 + \ldots]. \tag{3}$$

You'll probably have to start by finding B for a single coil along its axis; this is simple Biot-Savart cranking.

This problem is a bit messy but if you are a table top experimentalist you might actually use Helmholz coils someday.