Set 4 – due 29 September

"You have to think until it hurts, and then keep going" – Heisenberg

1) Jackson 3.7. [10 points] (a)–5, (b)–5.

Two multipole problems, both involving quadrupoles. When I saw them the first time, I didn't understand the question, so here is a translation:

In these problems, the Cartesian quadrupole moments are defined through

$$Q_{ij} = \int d^3x \rho(x)[3x_i x_j - \delta_{ij} r^2], \tag{1}$$

There could be five of them (Q_{ij} is real – 9 entries, symmetric – 6 entries, traceless – one constraint). But with azimuthal symmetry, there can be only two nonzero ones. You can find an axis where Q_{ij} is diagonal, and the phrase "Nucleus with a quadrupole moment Q" means $Q_{33} = eQ = -2Q_{11} = -2Q_{22}$.

In Prob. 4.6, the surface of the nucleus is an ellipsoid. Its semimajor axis (of radius a) is oriented along the z axis, and its semiminor axis (of radius b) lies in the $x - y$ plane. The surface is given by

$$1 = \frac{z^2}{a^2} + \frac{\rho^2}{b^2} \tag{2}$$

where $\rho^2 = x^2 + y^2$. Also, read carefully the remarks (and footnotes!) on pp. 150-151.

In Prob. 4.7, you will have to re-insert units to get a number.

2) Jackson 4.6 [20 points] (a)–5, (b)–5, (c)–10.

3) Jackson 4.7 [20 points]. (a)–7 (b)–7 (c)–6

4) [10 points] Show that the Green's function for the sphere, eq. 2.17, is equivalent to the "shell expression", eq. 3.125, in the limit that the radius of the inner shell is taken to zero. In the former expression, x and x' live inside a sphere of radius a, so in the latter expression, remember this, and relabel "b" as "a" for consistent notation to 2.17.