Set 12 – due 1 December

"The task is not to see what no one else has seen, but to think what no one else has thought, about that which everyone else has seen." – Schrödinger

1) [20 points] Jackson 7.6 (a)–6, (b)–6, (c)–8.

2) [20 points] More about Jackson 7.2. You have already solved this problem to find the transmission and reflection coefficients. Now consider the case of an infinite planar dielectric medium of thickness d in the vacuum (i.e. $n_3 = 1$ in 7.2). Assume that there is an incident electromagnetic wave of energy density u_0 and frequency ω directed normal to the surface of the plane, and compute the radiation pressure of the wave on the plane in two ways: (a) [7 points] Calculate the field momentum of the incident wave, p_i, the transmitted wave, p_t, and the reflected wave p_r. Then use momentum conservation to write $\vec{p}_i = \vec{p}_t + \vec{p}_r +$ momentum of plate. (b) [7 points] Evaluate the stress tensor to the left and to the right of the slab. (c) [6 points] For what values of thickness d is the pressure a maximum?

3) [10 points] Suppose that there is a magnetic field $\vec{H}_0$ parallel to, and at the surface of, a good conductor. Beginning with the expressions for $\vec{E}$ and $\vec{H}$ in conductors (in Jackson Sec. 8.1), (a) [5 points] show that the time-averaged power loss into the conductor per unit area is

$$\frac{dP}{dA} = \frac{\mu_c \omega \delta}{4} |H_0|^2 = \frac{1}{2\sigma\delta} |H_0|^2. \tag{1}$$

(b) [5 points] What is the time-averaged magnetic energy density (energy per unit area) stored in the conductor, in terms of $\vec{H}_0$?

The point of this problem is that this is how loss occurs for electromagnetic waves confined in a region with metallic boundaries: it is used to compute attenuation in a waveguide or the Q of a cavity.