Set 1– due 8 September

"You won't become a good pianist by listening to good concerts." – J. Wess

For this week, we have some short problems, while we build tools: First some (rather formal) Green's theorem manipulations...

1) [5 points] Jackson 1.10: note that this is only true if there is no charge inside the sphere!

2) [5 points] Jackson 1.12: this is needed for

3) [10 points] Jackson 1.13

4) [20 points] A sphere of radius R has a uniform charge density ρ and total charge Q. Find the electrostatic potential energy of the sphere, that is, the energy required from some non-electric interaction to hold the sphere together. On dimensional grounds, the answer will be

$$W = C\frac{1}{4\pi\epsilon_0}\frac{Q^2}{R} \tag{1}$$

and the interesting quantity is the constant C. Do this problem "like an undergraduate," that is, use Gauss's law to find $\vec{E}$, then integrate it to find $\Phi(r)$. Do the problem two ways: compute (a) [10 points]

$$W = \frac{1}{2}\int d^3x\rho(x)\Phi(x) \tag{2}$$

(b) [10 points]

$$W = \frac{\epsilon_0}{2}\int d^3x|\vec{E}(x)|^2. \tag{3}$$

We'll visit the third alternative

$$W = \frac{1}{2}\int \rho(r)\rho(r')\frac{1}{4\pi\epsilon_0}\frac{1}{|\vec{r}-\vec{r'}|}d^3rd^3r' \tag{4}$$

later in the course–this looks like a lot but it is actually the easiest way to do the problem.