

Set 6 – due 3 March

The midterm will be Wednesday evening, March 8, in G-125 (our classroom) from 7 to 8:30 PM.

1) [10 points] Show that if the vector potential has its “photon normalization,”

$$\vec{A}(x, t) = \left(\frac{2\pi\hbar c^2}{\omega V}\right)^{1/2} \vec{\epsilon} [e^{i(k \cdot r - \omega t)} + e^{-i(k \cdot r - \omega t)}] \quad (1)$$

then the field momentum density,

$$\frac{\vec{P}}{V} = \frac{1}{4\pi c} \langle \vec{E} \times \vec{B} \rangle \quad (2)$$

has a sensible value. The bracket means “time average.” (Blue Jackson calls the field momentum density $\vec{g}$: see Eq. 6.118 (in MKS) or Θ^{0i}/c in CGS in Eq. 12.114.)

2) Consider a hydrogen atom in the $F^2 = 0$ state ($\vec{F} = \vec{S}_{proton} + \vec{S}_{electron}$), in a magnetic field $\vec{B} = \vec{B}_0 \cos(\omega t)$. The B-field induces a $V = (e/(m_e c)) \vec{B} \cdot \vec{S}_{electron}$ perturbation; the $\vec{B} \cdot \vec{S}_{proton}$ perturbation is too small to matter. (a) [20 points] Compute the transition probability per unit time W using the Golden Rule to the $F = 1, F_z = 0$ state, if $\vec{B}_0 = \hat{z} B_0$. (b) [10 points] The B-field is part of an electromagnetic field with intensity per frequency interval $I(\omega)$. Integrate the W you got in (a) over the frequency spectrum of radiation to find the rate at which atoms are excited to the $F = 1$ state. (c) [10 points] If $\vec{B}_0 = \hat{x} B_0$, what transitions are allowed? How do the rates for these transitions compare to the one found in (b)?

3) [10 points] In class, I derived the Golden Rule expression for the spontaneous decay of an atom in initial state i to a final state f plus emission of a photon:

$$\frac{d\Gamma}{d\Omega} = \frac{2\pi}{\hbar} |\langle f | H_I | i \rangle|^2 \{ \delta(E_1 - E_f - E_\gamma) \frac{p_\gamma^2 dp_\gamma}{(2\pi\hbar)^3} \} \quad (3)$$

and we completed the calculation by setting $|p_\gamma| = E_\gamma/c$, and integrating over the delta function. The object in the curly brackets is called the “phase space factor” (in this case, the “one body phase space factor”). How would the phase space factor change if we replaced the photon by a particle G with a nonzero

mass and an energy - momentum relation $E_G = [p_G^2 c^2 + (m_G c^2)^2]^{1/2}$? What I am looking for is basically how

$$\frac{dI}{d\Omega} = \int \delta(E_i - E_f - E_G) p_G^2 dp_G \quad (4)$$

varies as a function of $\Delta E = E_i - E_f$, with a discussion which includes the low energy (ΔE very close to $m_G c^2$) and high energy ($\Delta E \gg m_G c^2$) limits.