Set 5 – due 24 February

1) [15 points] Consider an idealized laser pulse as a time dependent electric field $\vec{E}(t) = \hat{z}E_0 \exp(-t^2/\tau^2)$. Suppose that the pulse is incident on a hydrogen atom which is in the 1S state at $t = -\infty$. Neglecting spin, compute the probability as $t \to \infty$ that the hydrogen atom has been driven into each of the three $2p$ states ($m = 1$, 0, -1) by the pulse. The point of this problem is to introduce selection rules in electromagnetic transitions.

2) [15 points] Derive a Golden Rule expression for the transition probability per unit time through first order, for $V(t) = V_0 \cos(\omega t)$. Be very careful to show all details for the cross term. Answer:

$$W_{\beta\alpha} = \frac{2\pi}{\hbar}|\langle\beta|V_0|\alpha\rangle|^2 \frac{1}{4}\{\delta(E_\beta - E_\alpha - \hbar\omega) + \delta(E_\beta - E_\alpha + \hbar\omega)\} \tag{1}$$

Getting rid of the cross term is sufficiently tricky that you wouldn't believe it if I did it for you.

3) [10 points] We have a one dimensional harmonic oscillator of natural frequency ω. We probe it with a spatially uniform but time dependent force

$$F(t) = -\frac{F_0\tau/\omega}{\tau^2 + t^2}. \tag{2}$$

Far in the past, the system is in its ground state. Using first-order perturbation theory, calculate the probability that the system is found in the first excited state at times far in the future.

4) [10 points] We have another odd Sakurai problem: we have a one-dimensional particle in a time-independent potential, and we know all the eigenfunctions. We probe the system with a traveling pulse corresponding to a time-dependent potential

$$V(t) = A\delta(x - ct). \tag{3}$$

Suppose that the system is in its ground state far in the past, so $\langle x|i\rangle = u_i(x)$. Find the probability that the system is in some excited state with energy eigenfunction $\langle x|f\rangle = u_f(x)$ at late times. Then interpret your result: you can regard the delta function as a superposition of harmonic perturbations, since

$$\delta(x - ct) = \frac{1}{2\pi c}\int_{-\infty}^{\infty} d\omega \exp(i\omega(x/c - t)). \tag{4}$$

Discuss the connection between what you found and the energy conservation associated with harmonic perturbations.