Set 4 – due 17 February

Most of a comment by Fermi: "There are two ways of doing theoretical physics. One way, and this is the way I prefer, is to have a clear physical picture of the process that you are calculating. The other is to have a precise and self-consistent mathematical formalism."

1) [20 points] This is an old Merzbacher problem. We have our variational helium wave function

$$\psi(\vec{r}_1, \vec{r}_2) = \left(\frac{Z_{eff}^3}{\pi a_0^3} \right) \exp(-Z_{eff}(r_1 + r_2)/a_0) \tag{1}$$

with $Z_{eff} = 2 - 5/16$. You are asked to use this wave function to compute the diamagnetic susceptibility of helium. The game is to use the second order Hamiltonian $H_1 = -e^2 A^2/(2mc^2)$ with $\vec{A} = -(1/2)(\vec{r} \times \vec{B})$ for constant B, and to compute the first order energy shift, $\Delta E = -(1/2)\chi B^2$. The experimental number is $\chi = 1.88 \times 10^{-6}$ cm^3 per mole. How well did this calculation do?

2) Consider charmonium, a bound state of a heavy quark and an antiquark, with reduced mass $\mu = m/2$. The confining potential is $V(r) = -g^2/r + \kappa r$. (a) [7 points] For big m the wave function peaks at small r. Taking $H_0 = p^2/(2\mu) - g^2/r$, compute the ground state energy treating the κr term as a perturbation. (b) [10 points] Now take a variational wave function $\psi \simeq \exp(-Cr)$ (with variational parameter C) and compute the ground state energy of charmonium. Carry your calculation far enough to get the nonlinear equation whose solution will determine C. (Setting $C = \lambda/a_0$ where λ is the variational parameter and a_0 is the Bohr radius, but in terms of μ and g, will allow you to quickly use results from the Helium calculation.) (c) [3 points] When m is very large solve the equation and show that your answer coincides with the answer you got in (a).

3) [15 points] Consider a two-state system $H = H_0 + H_1$ where

$$H_0 = \begin{pmatrix} \epsilon_1 & 0 \\ 0 & \epsilon_2 \end{pmatrix} \tag{2}$$

and

$$H_1 = \begin{pmatrix} 0 & \gamma e^{i\omega t} \\ \gamma e^{-i\omega t} & 0 \end{pmatrix}. \tag{3}$$

At $t = 0$, the system is in the state whose unperturbed energy is ϵ_1. Find the exact time dependent probability that the system is in the ϵ_2 state (recall last semester), then do the problem with first order perturbation theory, and discuss when and how well the perturbative answer reproduces the exact result.