Set 12 – due 28 April

The final is Sunday May 7, 1:30-4 PM in our classroom.

1) [30 points] For a flavor of how a typical perturbative quantum field theory calculation is done, find the differential cross section for Thomson scattering: a photon of momentum $\vec{q}_1$ and an electron of momentum $\vec{p}_1$ collide to produce a photon of momentum $\vec{q}_2$ and an electron of momentum $\vec{p}_2$. At very low energy, this goes entirely through the $(e^2/(2mc^2)\vec{A}\cdot\vec{A}$ term (the pA terms vanish when $\vec{p}$ goes to zero) and so this becomes a first order Golden Rule calculation. The photon operator is (as usual)

$$\vec{A}(x,t) = \sum_\sigma \sum_k (\frac{2\pi\hbar c^2}{\omega V})^{1/2}[\vec{\epsilon}_{k\sigma} e^{i\vec{k}\cdot\vec{x}/\hbar} a_{k\sigma} + h.c.]. \tag{1}$$

Take the electron's wave functions as

$$\psi_i(x) = \frac{1}{\sqrt{V}} e^{(i\vec{p}_1\cdot\vec{x})/\hbar} \qquad \psi_f(x) = \frac{1}{\sqrt{V}} e^{(i\vec{p}_2\cdot\vec{x})/\hbar}. \tag{2}$$

Equivalently, if you want, you could work in second quantized formalism for the electrons, and the last formula would pick up creation operators. The initial and final photon states are $a^\dagger_{q_1\sigma_1}|0\rangle$ and $a^\dagger_{q_2\sigma_2}|0\rangle$. There is one very useful magic result (basically around Eq. 13.79 - 13.81 of the notes): if

$$\langle f|H_I|i\rangle = T\frac{1}{V^2}\int d^3x e^{(i\vec{K}\cdot\vec{x})/\hbar} \tag{3}$$

where T is the part of the matrix element not in the total momentum conserving integral, then

$$d\sigma = \frac{d^3q_2}{(2\pi\hbar)^3}\frac{d^3p_2}{(2\pi\hbar)^3}(2\pi\hbar)^3\delta^3(\vec{K})\frac{2\pi}{\hbar}\delta(E_f - E_i)\frac{1}{v_{rel}}|T|^2. \tag{4}$$

This is the analog of "squaring the delta function" for energy in the Golden Rule,

$$\lim_{T\to\infty} |\int_0^T dt \exp(-i\omega t)|^2 \to 2\pi T\delta(\omega) \tag{5}$$

so

$$\lim_{L\to\infty} |\int_0^L dx \exp(ikx)|^2 \to 2\pi L\delta(k). \tag{6}$$

So many hints: $v_{rel} = c$, neglect the electrons' kinetic energies compared to the photon energies. Stop when you get the known (see Jackson) formula,

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2 |\vec{\epsilon}_1\cdot\vec{\epsilon}_2|^2 \tag{7}$$

Every part of this calculation has an interesting (longish) story for you to discover on your own. Happy hunting!