Set 11 – due 21 April

1) [15 points] Evaluate the cross section for a spherical square well ($V(r) = -V_0$ for $r < R$, $V(r) = 0$ otherwise) at low energy when the potential is very weak. Your hierarchy of scales is $1 >> 2mV_0R^2/\hbar^2 >> R^2k^2$ where the CM energy is $E = \hbar^2k^2/(2m)$. It's useful to find the s-wave phase shift first, and then take the limit. You can compare your answer to the $q \to 0$ limit of the Born approximation from Set 8.

2) [10 points] Often a resonance is narrower than the resolution of one's instruments. Show how measuring

$$\Sigma_{ij} = \int_{E_0-\Delta E}^{E_0+\Delta E} dE \sigma_{i\to j}(E) \tag{1}$$

around a resonance at E_0, where $\Delta E >> \Gamma$ the total width of the state, can give information on branching ratios.

3) [15 points] Calculate and graph the total unpolarized (spin-averaged) neutron-proton cross section for laboratory kinetic energies from threshold to 20 MeV, given the following low energy parameters: Singlet: $a^{S=0} = -24.5 \times 10^{-13}$ cm, $r_0^{S=0} = 2.7 \times 10^{-13}$ cm. Triplet state: deuteron binding energy is 2.23 MeV, $r_0^{S=1} = 1.7 \times 10^{-13}$ cm.

4) [15 points] Starting from the boundary condition for S-wave scattering from a square well of radius r_0, depth $-V_0$,

$$k \cot(kr_0 + \delta) = K \cot(Kr_0) \tag{2}$$

(k is the wave number for $r > r_0$, K is the internal wave number) show that

$$S(k) = e^{2i\delta} = e^{-2ikr_0} \frac{K \cot(Kr_0) + ik}{K \cot(Kr_0) - ik} \tag{3}$$

has poles at the known bound state energies of the square well. Hint: recall

$$e^{2i\delta} = \frac{\cot\delta + i}{\cot\delta - i} \tag{4}$$