Evidence in the Literature for a Cosinusoidal Gravitational Potential with a Universal Wavelength

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ABSTRACT

A replacement for Newtonian gravity is proposed: $m\phi(r) = -(GmM/r)\cos(2\pi r/\lambda_o)$. (This replacement is motivated by the recent observation that only a very few central point potentials have an associated uniqueness theorem.) The spacing of external shells around the elliptical galaxy NGC 3923 gives a tentative value of 1800 light-years for the universal constant λ_o. This value of λ_o also is accommodated by the observed distribution in the diameters of lenses associated with bright barred disk galaxies and by the spacing of gravitationally lensed images in the Einstein Cross (2237+0305).

The potential is consistent with the flat rotation curves and the Tully-Fisher law for disk galaxies. It also explains several features of our Galaxy. A previously catalogued asymmetry in the azimuthal velocity distribution of stars near the sun is interpreted as evidence for the hypothesis and against a smoothly-varying spherical halo of galactic dark matter. The observed broad distribution in radial space velocities of nearby stars is only understood if the sun is near an *inner* turning point. This point is confirmed directly. Circular features near the Galactic center are consistent with the potential as is a central bar. The bar is related dynamically to the spiral arms and, surprisingly, to the dwarf spheroidals. The dispersion in the radial velocity within each of the nearby spheroidal dwarf galaxies is seen as

a consequence of the Galactic potential, rather than internal dark matter. Catalogued radial velocities of the globular clusters test both the proposed and the Newtonian potentials.

The cosinusoidal potential is written in a form appropriate for general relativity. This is done by adding a term $-\eta_{\mu\nu}\Lambda$ to the usual term $g_{\mu\nu}\Lambda$ which expresses the cosmological constant. Λ is identified with λ_o: $\Lambda = -(1/2)(2\pi/\lambda_o)^2$. A consequence of the relativistic formulation is that the bending of light by gravitational lenses, even those with no apparent lens, can probably be explained without dark matter. Alternatively, the phase and rapid oscillation of the cosinusoidal potential reduces to nearly zero its contribution to the time delay between images. The remaining, geometric, time delay is predicted to be 25 days between the two semicircular rings in the radio lens MG 1634+1346. This prediction is yet to be tested.

The cosmological implications are investigated. Surprisingly, the self-interaction of a large homogeneous sphere is explosive, a feature which assures that the age of the universe $\simeq H_o^{-1}$ (not $(2/3)H_o^{-1}$). The proposed value of λ_o together with the present density of baryons point to an inflationary period at a red shift $z \simeq 100,000$. This is a lookback time which is consistent with that expected from observed periodicities in present galactic densities. The potential also provides a natural explanation for the stability of the recently discovered chain galaxies.

Finally, this proposal requires that gravitational radiation have a dispersive group velocity, $v_g = c(1 + \nu^{-2}\lambda_o^{-2}c^2)^{1/2} > c$. The consequences of this tachyonic behavior are discussed. The appendix addresses the complementary issue of whether the photon has a small mass, $m_\gamma = h/\lambda_o$, and thus a velocity, $v_\gamma = c(1 - \nu^{-2}\lambda_o^{-2}c^2)^{1/2}$. Evidence for such a massive photon is found in the magnetic fields within the Galaxy, M31, and the Coma cluster.

SUBJECT HEADINGS: Gravitation, Galaxy: NGC 3923, Relativity, Cosmology: Dark Matter, Galaxies: Structure, Stars: Statistics

1. INTRODUCTION

Recently Bartlett & Su (1994) have shown that of all possible central point potentials, $\phi(r)$, only two permit a uniqueness theorem in all bounded geometries. These potentials are well-known: the Newtonian, $\phi(r) = A/r$ and the Yukawa, $\phi(r) = Ae^{-kr}/r$. Each has the property that knowledge of the potential on an arbitrary boundary gives uniquely the potential within. The extension of the Yukawa potential to imaginary k yields the cosinusoidal potential,

$$\phi(r) = A\cos(kr)/r. \tag{1}$$

We have shown that this third potential generally permits uniqueness, failing only when the boundary forms a resonant cavity for the given k.

The Newtonian and Yukawa potentials have been applied in situations well removed from electrostatics, the study for which the uniqueness theorem is typically used. The question naturally arises as to whether there is any use for the cosinusoidal potential. Has nature found a roll for a potential of the form $\cos(2\pi r/\lambda_o)/r$?

2. HYPOTHESIS

As the least well understood of the fundamental forces, gravitation is a possible place for the new force. Begin by considering the usual limit of small velocities and not too dense local matter. In distinction to the Newtonian potential which requires but a single universal constant, the cosinusoidal potential needs two. The coefficient A is fixed by the need to recover Newtonian dynamics at short distances. The range parameter λ_o is less than or of the order of galactic dimensions, since this is the distance over which Newtonian mechanics first shows any evidence of failure. Thus

$$m\phi(r) = -(GmM/r)(\cos(2\pi r/\lambda_o)), \tag{2}$$

where $\lambda_o >> 1$ AU is a universal constant.

In the limit $r/\lambda_o \to 0$, $\cos(2\pi r/\lambda_o)/r \to (1/r)[1 - (1/2)(2\pi r/\lambda_o)^2]$ and we recover Newtonian gravity as we must for the solar system, but not necessarily for galactic systems.

Our goal, then, is to alleviate the dependence of astrophysical theories on dark matter by modifying the law of gravity. In order for such an hypothesis to succeed it must overcome widely held beliefs: 1) that whether identical units attract (as is the case with gravity) or repel (as with electricity) is dependent on the evenness or oddness of the spin of the mediating field and 2) that no scheme of modified Newtonian mechanics that is based solely on changing the dependence of the gravitational law on distance can satisfy the Tully-Fisher relation for disk galaxies, ($M_{lum}\ \alpha\ v_{rot}^4$). Since (1) is crucial to the whole thesis, it is discussed now. We will return to the Tully-Fisher relation later.

Jagannathan & Singh (1986) have carefully examined the question of attraction vs. repulsion. They conclude that the connection between even spin and attraction is upheld, particularly when the fundamental potential is of the Yukawa form, $e^{-kr}/r,\ k \geq 0$. The cosinusoidal potential considered here can formally be considered to be the real part of a Yukawa potential with an imaginary k. The consequent distinction between negative and positive k^2, however, spoils the positive definiteness of a critical integral [eq. (5)] in Jagannathan and Singh's proof. Thus the cosinusoidal potential may profitably be considered further.

We begin by examining the astronomical evidence for a particular value of λ_o. I then show the compatibility of eq. (2) with the form of general relativity. In the following paper general relativity itself is generalized in a way that allows the interaction between matter and antimatter to be different from that of matter and matter, a difference which could be responsible for the decay $K_L(2\pi)$.

3. COSINUSOIDAL POTENTIAL

The conjunction of luminous matter and the usual Newtonian mechanics is inadequate to explain several astronomical phenomena. These include the average speed of galaxies within clusters, gravitational lensing, x-ray halos of elliptical galaxies, and the flat rotation curve of disk galaxies. Reviews have been given recently of the roll that either dark matter (Trimble 1987) or non-Newtonian gravity (Sanders 1990) could have in these effects. Subsequently, Gerhard and Spergel (1992) showed that even the most successful of the non-Newtonian theories, that of Milgrom (1984, 1995), may require dark matter to explain the motions of the dwarf spheroidal galaxies near the galaxy. They also emphasized a problem for Newtonian theories: the mass to light ratio must vary by at least a factor of ten among the eight dwarf spheroidals. Alternatively, recent observations of gravitational microlensing by ordinary, non-luminous matter (MACHOS) seems to account for a fair fraction of the flat rotation curve of the Galaxy, at least if Newtonian dynamics is assumed (Alcock et al, 1995).

We shall return to these important problems later. First it is necessary to develop the general dynamics to be expected from a cosinusoidal potential. Let us begin with elliptical galaxies. Their shape often is approximately spherical and one particular case (NGC 3923) offers a good opportunity for determining λ_o.

3.1. Elliptical galaxies

According to the hypothesis, the motion of galactic matter m in the presence of a point source M is governed by eq. (2). The associated gravitational field,

$$g_r = -\partial\phi/\partial r = -(GM/r^2)\cos(2\pi r/\lambda_o) - (2\pi GM/r\lambda_o)\sin(2\pi r/\lambda_o), \quad (3)$$

reduces to the familiar Newtonian one in the limit $r << \lambda_o$. In case $r >> \lambda_o$, the second term dominates and

$$g_r = -(2\pi GM/r\lambda_o)\sin(2\pi r/\lambda_o). \quad (4)$$

This field differs in two essential ways from the usual one. It is periodically attractive and repulsive; it diminishes as r^{-1} rather than r^{-2}.

An extended source has the same features: periodicity and $1/r$ behavior of both potential and field. Significantly, not just the monopole, but *all* multipole moments (which have reflection symmetry) contribute coherently to a potential which falls off as $\cos(2\pi r/\lambda_o)/r$.

The multipole expansion for the source is readily developed in terms of the spherical harmonics $Y_{nm}(\theta', \phi')$ and the spherical Bessel functions $j_n(2\pi r'/\lambda_o)$ and $y_n(2\pi r'/\lambda_o)$. The point potential, eq. (2), is expressed in terms of the source coordinate $\mathbf{r}'(r', \theta', \phi')$ and field coordinate $\mathbf{r}(r, \theta, \phi)$ through the identity (Antosiewicz 1964)

$$-\cos(2\pi s/\lambda_o)/s = (2\pi/\lambda_o)\Sigma(2n+1)j_n(2\pi r'/\lambda_o)y_n(2\pi r/\lambda_o)P_n(\cos\gamma), \quad (5)$$

where γ is the angle between $\mathbf{r}$ and $\mathbf{r}'$, $s = |\mathbf{r} - \mathbf{r}'|$ and $r > r'$.

Using the addition theorem to express $P_n(\cos\gamma)$ in terms of the source and field angles we find for the potential away from an extended source,

$$\phi(r,\theta,\phi) = (2\pi G/\lambda_o)\Sigma_{nm}(4\pi)^{1/2}A_{nm}y_n(2\pi r/\lambda_o)Y_{nm}(\theta,\phi), \tag{6}$$

where the multipole

$$A_{nm} = (4\pi)^{1/2}\int \rho(r',\theta',\phi')Y^*_{nm}j_n(2\pi r'/\lambda_o)dv'. \tag{7}$$

For elliptical galaxies of low ellipticity we are most interested in the monopole,

$$A_o = \int (\lambda_o/2\pi r')\rho\sin(2\pi r'/\lambda_o)dv'. \tag{8}$$

Note that, in distinction to the Newtonian case, the monopole A_o is not simply the mass, but rather is dependent on the radial distribution of matter.[1] The resulting potential outside is given by eq. (6),

$$\phi(r) = -GA_o\cos(2\pi r/\lambda_o)/r. \tag{9}$$

This potential leads to radially confined orbits; in the extreme cases, either circular or purely radial. In the former, the periodicity of the potential yields shells having a radius of $r_N = (N+1/4)\lambda_o$, where N is an integer. In the latter case, the maximum radius of each shell is $(N+1/2)\lambda_o$.

Although weak shell-like structures are evident in some of the galaxies illustrated in Arp's catalogue (1960), shells were not studied systematically until Malin

(1977) developed special techniques to enhance photographic contrast. Using these techniques, Malin & Carter (1980, 1983) catalogued 137 examples of shells around isolated (field) southern elliptical galaxies and discovered the picture-book example, NGC 3923, which has more than 20 identifiable shells. Recently Seitzer & Schweizer (1990) have used CCD's (Charge Coupled Devices) to find that more than 50% of their surveyed field ellipticals possess partial shells.

The galaxy NGC 3923 has been particularly well studied (Quinn 1984; Pence 1986; & Hernquist & Quinn 1987). In the latest publication, and the only one to quote errors for each shell, Prieur (1988) combines photographic and CCD techniques to measure the radii of 22 shells: the innermost being only 19 arc-seconds and the outer 1170".

The origin of the shells is controversial. Originally they were thought to be the residue of shock waves from a galactic merger, a direct hit. This view has recently been criticized by Thomson and Wright (1991) who believe a near miss is the more likely origin. They further assume that the observed galaxy must, before the encounter, have its exterior stars in nearly circular orbits. Ironically, this assumption comes automatically if one assumes a cosinusoidal potential.

We now use the measurements of Prieur (1988) to obtain a tentative value for the length λ_o. The shells, of slight ellipticity, cover about 60 degrees, extending 30 degrees to either side of the galaxy's major axis. Prieur tabulated the maximum radius of each shell $R_n, \quad n = 1, 22$ including the shell's thickness. Fortunately the facts that (1) all measurements are made close to the same axis and (2) The spherical Bessel function $y_2(x) \simeq -y_o(x)$ for all $x > 2\pi$ ensures that the R_n should be quantized even if the galaxy has substantial quadrupole moments. In particular, the observed R_n should occur near $(N + 1/2)\lambda_o$ for *some* integer N. Accordingly we form the chi-square

$$\chi^2(r_o, \phi_o) = \Sigma_{n=1}^{22}(r - R_n)^2. \tag{10}$$

Here the predicted value r is given by $r = (N + \phi_o)r_o$, and the integer N is chosen to minimize the discrepancy between r and R_n. Trials were made for eight values of the putative offset phase $\phi_o = 2\pi m/8,\ m = 1, 8$ and for a continuous range of the putative step length r_o. For all eight $\phi_o' s$ some 3" $< r_o <$ 4" will give an acceptable χ^2. Clearly our method has no power for such small $r_o' s$. However, for larger $r_o' s$, only one choice of ϕ_o ($\phi_o = 5/8$ cycles) gives an acceptable fit, ($\chi^2 = 15$ when $r_o = 5.34$")[2]. Figure 1 compares Prieur's measurements with this best fit.

Figure 2 shows how the underlying χ^2 changes as r_o is varied. Note that the background χ^2 increases quadratically with r_o, as expected. This is why the data are not accurate enough to rule out small $r_o' s$. However, the same quadratic variation in the background makes the minimum at 3.5", say, much more likely to be a chance occurrence than the one at 5.34". For this reason, the latter value is claimed as a tentative measure of the fundamental length λ_o.

We may use the value of the step length in NGC 3923 together with the distance to this galaxy and an assumed Hubble constant to determine λ_o. The proximity of this galaxy to the Virgo cluster makes the extraction of its distance controversial even when the galaxy's red shift is well known. Both Prieur (1988) and Tully (1988) assume a Hubble constant of 75 km s^{-1}Mpc^{-1}. The former finds a distance of 17.5 Mpc; the latter of 25.8 Mpc. We assume an average and thus adopt a tentative value for $\lambda_o = 5.34" \times 21.7$ Mpc = 560 pc = 1800 light- years.

It is clearly desirable to have this length confirmed in other shell galaxies. NGC 3923 is the prototype of a type 1 shell galaxy whose shells cluster about a common axis. NGC 474 (Arp 227) is a type 2 galaxy whose partial shells are at position angles all around the galaxy (Prieur 1990). As Prieur observes, type 2 galaxies offer a problem for Newtonian physics, “No study has been done about this, but I doubt that a family of stars formed a Hubble time ago with *approximately* the same initial conditions will remain orbiting with the accuracy needed to make the edges of the shells very sharp, sometimes not even resolved”. By contrast, the cosinusoidal potential can have a minimum in free space. The shells do not have to be rotating. Stability is not a problem.

To my knowledge, NGC 474 is the only other galaxy to have shells of published radii (Schombert and Wallin 1987). The angular radii of the four measured shells are given in their table I as 100”, 135”, 195” and 204”. The differences of these radii (35”, 60”, and 9”) share a largest common divisor (d=8.64”) such that all 3 differences are within 0.5” of an integer times d. As can happen with only 3 data points, the distance corresponding to d is close to $2\lambda_o$, rather than λ_o itself. To see this, we use our analysis of NGC 3923 and the ratio of the distances to NGC 474 and NGC 3923 as listed by Tully (1988) to find that the predicted d is (2)(5.34”)(25.8 Mpc/32.5 Mpc)=8.48”. We can also find the distance corresponding to d directly using values for the linear radii also given in Schombert and Wallin’s table I. This also gives a distance of almost exactly twice λ_o, but for an assumed value of the Hubble constant of 100 km s^{-1}/Mpc, rather than the 75 assumed here. Thus we may conclude that NGC 474 confirms, with limited statistics, the tentative value of λ_o found in NGC 3923.

3.2. Disk Galaxies

Of all objects having a scale greater than a kiloparsec, the disk galaxies are the only ones whose internal velocities are well specified in all 3 directions.[3] Thus these galaxies offer a good test for the proposed gravitational theory. Some might say that this test is bound to fault any periodic potential. After all, the dominant feature of disk galaxies is their spiral arms. These are not observed to be striated by spherical channels. We shall return to this problem in sections 3.2.5 and 3.3. In the meantime the reader is urged to temporarily suspend disbelief.

I believe that a proper test of the cosinusoidal potential is that it predicts the observed *circular* features in the disk galaxies. These are (i) The distribution of diameters of lenses and rings in barred galaxies, (ii) The flat rotation curves, and (iii) The Tully-Fisher relation.

3.2.1 Lenses

The 121 barred galaxies listed in the *Second Reference Catalogue of Bright Galaxies* (de Vaucoulers, de Vaucouleurs and Corwin 1976) have been surveyed by Kormendy (1976). He finds that the majority of these galaxies have circular or nearly circular lenses or inner rings. In his fig. 5 Kormendy shows the distribution of the diameters of 38 rings, 26 lenses, and 4 ambiguous cases vs the absolute magnitude of the individual galaxies. The figure shows that rings and lenses have similar distributions. However, a distinction appears when the distributions are projected on the ordinate. The lenses, but not the rings, appear clumped around evenly spaced diameters.[4]

A measure of this clumping is given by the weighted sum

$$N_{wt}(\lambda_o, \phi_o) = \Sigma_{i=1}^{26} \cos(2\pi r_i/\lambda_o - \phi_o), \tag{11}$$

where r_i is the radius of the *ith* lens, λ_o is the putative gravitational wavelength, and ϕ_o is a constant offset phase. Unfortunately ϕ_o is not known *a priori*, so I evaluated $N_{wt}(\lambda_o, \phi_o)$ for many ϕ_o and show the positive branch $N_{wt}(\lambda_o)$ of the envelope of such curves in Fig. 3.

This envelope shows a peak of $N_{wt} = 10.8$ at $\lambda_o = 475$ pc. A perfectly periodic spacing in radii would give $N_{wt} = 26$. Alternatively, the projection of a two-dimensional random walk gives a noise level of $N_{wt} = (26/2)^{1/2} = 3.6$. Particularly significant is that the putative λ_o from the 26 barred galaxies is within the range of that determined by the single elliptical galaxy, NGC 3923.

Whereas there are other peaks near submultiples of 475 pc, there is none near 950 pc. This is consistent with the identification of 475 pc as the fundamental period. Finally, note that the spread of observed diameters in Kormendy's original plot corresponds to $r = (8 \pm 3) \times 475$ pc. This moderate multiple makes our method somewhat sensitive to peculiar velocities which may explain why the peak of N_{wt} is only 40 % of the possible 26.

The question of why the *rings* fail to show any periodicity is an open one. Kormendy observed that the rings and lenses are strongly correlated with the type of galaxy. The rings are associated with late-type barred galaxies; the lenses with early-type. Additionally the specification of the diameters of a ring is a two parameter one; whereas only one is required for the sharp outer edge of a lens.

3.2.2 Dynamics

The central dynamical problem of disk galaxies is how to reconcile two apparently conflicting observations: the flat rotation curves and the Tully-Fisher relation. The galactic rotational velocity v tends to a constant v_∞ as $r \rightarrow \infty$ rather than falling off as $r^{-1/2}$ as expected for a central $1/r$ potential. This rotational velocity is related to the total *luminous* mass by the relation $v_\infty^4 \; \alpha \; M_{lum}$ (Aaronson et. al. 1982). Evidently, M_{lum} is inadequate to explain the flat rotation curves, but adequate for the Tully-Fisher relation.

The Newtonian solution is to posit dark matter in an approximately spherical halo to account for the flat rotation curves. Since the amount of dark matter is not otherwise constrained, the solution must work, but it begs the question of how the spheroidal dark matter "conspires" with the flat luminous matter to preserve both the rotation curves and Tully-Fisher relation. Alternatively, Milgrom (1984) has dispensed with dark matter by modifying Newton's second law so that for the small accelerations at the periphery of a galaxy, $F \simeq m(aa_o)^{1/2}$, where a_o is a constant $\simeq 10^{-8}$cm s^{-2}.

The present proposal also does not require dark matter to explain either (i) the flat rotation curves or (ii) the Tully-Fisher relation. The former follows because $g_r \; \alpha \; r^{-1}$ and the latter because the cosinusoidal potential weights matter in the source by r^{-1} thus favoring central over peripheral matter. The following two sections fill in the details.

3.2.3. flat rotation curves

We rewrite the potential eq. (6) in a form appropriate for an axially and equatorially symmetric galaxy:

$$\phi(r,\theta) = (2\pi/\lambda_o)G\Sigma_{n-even}A_n y_n(2\pi r/\lambda_o)p_n(\cos\theta), \tag{12}$$

where the multipole

$$A_n = \int \rho(r',\theta')p_n(\cos\theta')j_n(2\pi r'/\lambda_o)2\pi r'^2 d(\cos\theta')dr' \tag{13}$$

and the p_n are normalized Legendre polynomials, $p_n(\cos\theta) = (2n+1)^{1/2}P_n(\cos\theta)$.

For even n, the spherical Bessel functions $y_n(x) \rightarrow -(-1)^{n/2}\cos(x)/x$ and $j_n(x) \rightarrow (-1)^{n/2}\sin(x)/x$ as $x \rightarrow \infty$. At the equator the normalized Legendre polynomials reduce to $p_n(0) \simeq (-1)^{n/2}$ for all even n. Thus for large r, *all* multipoles give contributions to g_r which are in phase and fall off as r^{-1},

$$g_r(r,\pi/2) \simeq -(2\pi G/\lambda_o r)\sin(2\pi r/\lambda_o)\Sigma_{even\ n}A_n. \tag{14}$$

Observation shows that the curves of v vs r are approximately flat and consequently that $g_r \,\alpha\, r^{-1}$. This observation follows follows from our theory if we assume that the galactic matter is disposed so that the weighted average of $\sin(2\pi r/\lambda_o)$ over a cycle, ζ is substantially independent of r.

3.2.4. Tully-Fisher relation

It is important to contrast the $1/r$ fall-off of g_r [eq. (14)] for multipoles of arbitrary order n with that generated by multipoles with the Newtonian potential, $g_r \simeq 1/r^{2+n}$. Thus to investigate the source strength for the Tully-Fisher relation, it is now necessary to look at every multipole moment A_n, not just the monopole.

To check that the cosinusoidal potential is consistent with the Tully-Fisher relation we must make some assumptions. At first sight, these assumptions may appear very coarse, but then the Tully-Fisher relation is only satisfied to $\pm$ an astronomical magnitude. Begin by evaluating the A_n [eq. (13)] subject to the following assumptions:

Universal ζ: Assume that galactic matter is positioned so that for all galaxies, and all radii $\zeta \equiv < \sin(2\pi r/\lambda_o)) >_{cycle} = const. \simeq 0.4$

Complete Disk: Neglect any contribution from a spherical core. Assume all galactic matter is in a disk of surface density $\sigma = \sigma_o e^{-r/l}$, where σ_o is a *universal* constant and the size parameter l varies from galaxy to galaxy (Kent 1987).

Galactic Scaling: Assume all galaxies have similar profiles of thickness $2z$ vs radius, $z = l f_o(r/l)$, where f_o is a universal function. In particular, let the ratio between z and r at $r = l$ be $1/n_o = f_o(1)$. (Data from our galaxy indicates $n_o \simeq 3.5$ kpc/750 pc $= 5$.)

Minimum Thickness: Assume $l > (n_o^2/20)\lambda_o$. The multipole source strength $A_n \to 0$ for large n for either of two reasons. The galaxy is so thick for the critical $r < l$ that the integral of the $p_n(\cos\theta)$ over angles averages to zero. Since the first zero near the equator of the even $p_n(\cos\theta)$ occurs at $|\pi/2 - \theta| \simeq 1/n$, the assumption of geometrical scaling then sets a limit $n = n_o$ on the order of the multipole. Alternatively, n could be so large that the phase of $j_n(2\pi l/\lambda_o)$ is substantially different from its asymptotic phase. A study of the zeros of the spherical Bessel functions shows that $j_n(2\pi l/\lambda_o)$ has slipped by a quarter of a cycle or more whenever $n^2 > 20 l/\lambda_o$. The criterion of minimum thickness ensures that the Legendre polynomials, not the Bessel functions limit the order of A_n.

With these assumptions we have simply:

$$A_n = \int 2\pi r' dr' j_n(2\pi r'/\lambda_o)\sigma(r') = \int \zeta\lambda_o\sigma_o e^{-r'/l} dr' = \zeta\sigma_o l\lambda_o, \tag{15}$$

for $n < n_o$ and $A_n = 0$ for $n > n_o$. By contrast the mass, in the absence of dark matter, is the integral,

$$M = M_{lum} = \int 2\pi r'\sigma_o e^{-r'/l} dr' = 2\pi\sigma_o l^2 \tag{16}$$

Thus $l = (M/(2\pi\sigma_o))^{1/2}$ and $A_n = \zeta(M\sigma_o/2\pi)^{1/2}\lambda_o$, for n even and $\leq n_o$. Finally we substitute for A_n in eq. (14) above and find for the average value of g_r weighted by the matter in one cycle:

$$< g_r >= -(n_o/2+1)\zeta^2 G(2\pi M\sigma_o)^{1/2}/r, \tag{17}$$

a result which does not depend on λ_o and which is consistent with the TF relation, $v_\infty^4 \ = \ (< g_r > r)^2 \ \alpha \ M_{lum}$.

We can go another step and ask how much dark matter is required to generate the $< g_r >$ of eq. (17) assuming Newtonian mechanics. The dynamical Newtonian mass is

$$M_N \equiv - < g_r > r^2/G$$

$$= (n_o/2+1)\zeta^2 r(2\pi M\sigma_o)^{1/2} = (n_o/2+1)\zeta^2(r/l)M \simeq (4r/l)\zeta^2 M. \tag{18}$$

Evidently the ratio M_N/M increases linearly with the *ratio* of the maximum *observable* radius of a particular galaxy to its own size parameter. This behavior agrees with the observation of Sanders (1990) that M_N/M does not increase with the absolute size of a galaxy, but is evident for small disk galaxies as well as large.[5] At present, the luminosity of a galaxy can be followed through 6 magnitudes, from the central maximum σ_o of about 20 Mag/arc-sec^2 down to 26 Mag/arc-sec^2 (Kent 1987). Thus $r/l \simeq 6$ and $M_N/M \simeq 24\zeta^2 \simeq 4$, in rough agreement with observation.

The argument above may be a little too simple for real galaxies. In particular the assumptions of universal ζ and a complete disk are too draconian for a potential which weights sources inversely with their distance from the center. Allowing a central bulge of $M_{eff} \propto \sigma_o l^2$ does not compromise either the flat rotation curves or the Tully-Fisher relation. This bulge could account for the observation that the motion of stars near the sun may require a local $\zeta < 0.4$. (See section 3.3.1).

3.2.5 Stability

For the Newtonian potential, an isolated spherical shell of matter will always produce an attractive force at all external points. The same is not true for the cosinusoidal potential. Here the potential and field both alternate in sign. For a shell of mass M_a and radius a, the potential and field are

$$\phi(b) = -GM_a(\lambda_o/(2\pi ab))\sin(2\pi a/\lambda_o)\cos(2\pi b/\lambda_o) \qquad a < b. \tag{19}$$

$$g_r(b, M_a) \simeq -(GM_a/ab)\sin(2\pi a/\lambda_o)\sin(2\pi b/\lambda_o). \quad r > \lambda_o \tag{20}$$

The presence of two sine functions in g_r has an important consequence for stability. Matter which is orbiting in an inner shell ($(N-1/2)\lambda_o < r < (N+1/2)\lambda_o$) will always increase the amplitude of the potential at an outer shell. To see this, we have only to apply the virial theorem to the shells in succession. Start with central matter M_o at a radius $a < \lambda_o/2\pi$. This matter produces a field $g_r \simeq -(2\pi GM_o/b\lambda_o)\mathrm{sin}(2\pi b/\lambda_o)$ at the location of the first and successive shells ($b \simeq N\lambda_o, N = 1, 2...$). But by the virial theorem $0 < \ < T > = < rd\phi/dr > =$ $+2\pi GM_o < \mathrm{sin}(2\pi b/\lambda_o) >$ for each star in a given shell. The fact that kinetic energies must be positive thus ensures that each shell contributes coherently with the central mass to the monopole moment A_o in eq. (8).

There also is a distinction between the cosinusoidal and the Newtonian potentials for the fields inside a shell. For the inverse square law there can be no gravitational field inside an isolated spherical shell of matter. The same is not true for the cosinusoidal potential. For it, the reciprocity theorem given an expression identical to eq. (19)

$$\phi(a) = -GM_b(\lambda_o/(2\pi ab))\mathrm{sin}(2\pi a/\lambda_o)\mathrm{cos}(2\pi b/\lambda_o) \qquad a < b. \tag{21}$$

This equation gives the the potential at $r = a$ due to a uniformly dense spherical shell at a larger radius b. Thus the gravitational field inside the shell is

$$g_r(a, M_b) \simeq +(GM_b\lambda_o/ab)\mathrm{cos}(2\pi a/\lambda_o)\mathrm{cos}(2\pi b/\lambda_o). \quad r > \lambda_o \tag{22}$$

Unlike the Newtonian case, this field can be as large as the reciprocal external field, $g_r(b, M_a)$. Assume that the galaxy is formed from the inside out. Inorder for the succeeding layers not to disturb the interior is is necessary that

$$< \rho\mathrm{cos}(2\pi b/\lambda_o)/b > = 0 \tag{23}$$

over each successive layer.

This equation is a "galaxy building principle" which galaxies have evidently met. To confirm this fact observationally, however, is difficult. The equation constrains *circular* orbits to be at discrete radii, $r/\lambda_o = N + 1/4$. The phase space available for such a circular orbit is vanishingly small, however. It is much more likely that a galaxy will form with $\zeta \simeq < v_\phi/v_\phi(max) >^2$ being substantially less than 1. In that case, the galaxy building principle can be satisfied, particularly if, in addition to azimuthal velocity, there are large radial motions that bring the orbits out to nearly the potential *maxima* at $r/\lambda_o \simeq N + 1/2$.

These necessary radial motions help explain why galactic shells are so difficult to detect that they have only been studied in the last 15 years. The shells are not a series of delta functions; they are broad (and thick) bands (which are generally viewed obliquely).[6]

Finally, note that the galaxy building principle can be directly extended to higher multipole moments. The internal field caused by matter at radius b having an angular distribution $P_{2n}^m(\cos\theta)\cos m\phi$ is proportional to $y_{2n}(2\pi b/\lambda_o)$ which behaves like $(\lambda_o/2\pi b)\cos(2\pi b/\lambda_o)$for $b > (n^2/5)\lambda_o$. Thus for large b, the same radial distribution of matter used to kill the internal monopole field will kill that from even the high multipoles needed to make a disk. Similarly, the azimuthal density fluctuations needed to make spiral arms (Lin & Shu 1964) can occur without disturbing the interior of the galaxy. Additionally, the internal potential (as well as the field) is killed by the galaxy building principle.

For the Newtonian potential the situation is dramatically different. Here there is no option for killing any internal field except for the monopole field. Further each additional layer adds coherently to the monopole *potential* near the center.

These differences will be important in Sect. 3.3.3 where bars and spiral arms are discussed.

3.3. The Galaxy

In this section we study the Milky Way as an example of a disk galaxy. The velocity distributions of stars near the sun give an estimate of the local phase of $cos(2\pi r/\lambda_o)$ and of the magnitude of ζ. The dominantly circular motion of gas near the centers of the both the Milky Way and Andromeda show the oscillations in the cosinusoidal potential. Perhaps surprisingly, the cosinusoidal potential is found to be compatible with a bar which extends over several cycles. Then the observed bar is related to spiral density waves and the dwarf spheroidal galaxies which orbit the Galaxy at large radii. Finally, the motion of the globular clusters test both the cosinusoidal and Newtonian potentials.

3.3.1 Stars near the Sun

Over 60 years ago, Oort (1928) realized that the 3-D velocity distributions of local stars offers a unique possibility for exploring the dynamics of the galaxy. We are interested in these distributions now because they provide detailed tests of the proposed potential as well as a local measurement of ζ. Our study is helped by the recent release of the 3rd edition of the Catalogue of Nearby Stars (CNS3) in electronic form (Gleise & Jahreiss 1991). The catalogue list all known stars within 25 pc of the sun (and a few beyond). About half of these or 1946 stars have sufficiently well determined parallaxes that CNS3 lists their three velocity coordinates (relative to the sun): u,v, and w. These form a right handed coordinate system and respectively are the space velocity in the galactic plane and directed to the galactic center; the velocity in the direction of galactic rotation; and the velocity perpendicular to the plane and in the direction of the north galactic pole.

The analysis which follows is based on these 1946 stars. Call this restricted set CNS3R. I have made no attempt to separate the stars by age or even to remove the second member of a doublet. The first decision was to avoid the interjection of the possible bias of the author. The second decision has in effect already been made by Gleise & Jahreiss. Although 30% of the local stars are doublets, only 149 of the l946 selected stars are the second (or third) member of a multiplet. The space velocities of light companion stars (e.g. that of Sirius) have been omitted, probably because they change so quickly.

Histograms in the space velocities u, v, and w are shown in Fig. 4. A striking feature of these graphs is the asymmetry in the plot for the azimuthal velocity v. There is a sharp, high-velocity cut-off in the distribution for v. Oort interpreted this feature as evidence for the escape velocity from the galaxy. The modern explanation is more tentative. The cut-off at v=+65 km/s could reflect an edge to the galaxy at 25 kpc or the absence of a mechanism which could bring in stars from, say, 40 kpc on a highly elliptical orbit (Mihalas & Binney 1981).

The modern explanation is not without problems, however. The fall-off from the peak is distinctly asymmetric. Even within just $\pm$ 30 km/s of the peak, the decline on the positive side is 40% greater than the decline on the negative. It is hard for Newtonian mechanics to account for such asymmetries over a short interval about velocities near the circular velocity of the sun about the center of the galaxy, currently accepted to be $v_{sun} = 231$ km/s (Jones, Klemola, and Lin 1994). The scale for such changes is set by the difference between the escape and circular velocities for a galaxy compressed to its core: $\Delta v_{Newton} = v_{sun}(2^{1/2}-1) = 90$ km/s.

Dark matter could, help, but is not likely to do so if, as in current models,(Primack, Seckel, & Sadoulet 1988), it is spherical and diffuse (to provide galactic stability).

The problem in the azimuthal velocity v could be just be an environmental one. Nevertheless, it is worthwhile to try to find a dynamical origin. Ironically, the proposed cosinusoidal potential makes this quest easier than it is with Newtonian mechanics. This is because the cosinusoidal potential weights the contribution of central matter to $g(r)$ more heavily than that of distant matter. Further, as we have seen in section 3.2.4, even a disk galaxy is unlikely to have many higher order multipoles. Those that it does have oscillate in phase with the monopole (at least near the galactic equator).

Thus it is fruitful to consider the *effective* one-dimensional potential, of an isolated mass M at the origin, $\phi_e(r) = -(GM/r)\cos(2\pi r/\lambda_o) + L^2/(2r^2)$, where $L = v_\phi r$ is the angular momentum per unit mass of the test particle (Goldstein 1951). This potential may be given a universal form by setting $GM = 1/2\pi$ and measuring distances in "wavelengths": $x = r/\lambda_o$. Then

$$\phi_u = -(1/2\pi x)\cos(2\pi x) + (1/2)(L_u/x)^2, \tag{24}$$

where the universal potential and universal angular momenta are given by $\phi_u = \phi_e(\lambda_o/2\pi GM)$ and $L_u = L/(2\pi GM\lambda_o)^{1/2}$. The universal form is particularly useful here where we do not know the effective source strength GM but do know the distances r and λ_o and wish to use the universal form ϕ_u to predict the behavior of the radial velocities of nearby stars $-u$, given their azimuthal velocity v.

To set the scale for x, note that the spacing of shells in NGC 3923 has given the tentative value $\lambda_o = 560$ pc. Using the adopted value $r_{sun} = 8.5$ kpc, we find that $x_{sun} \simeq 15$. Fig. 5 shows the universal potential ϕ_u for $x < 18$ and several values of L_u.

The most striking feature of the plots is the series of plateaus at $\phi_u = 0.5$. These occur whenever $x = L_u = N + 1/4;\ N = 0, 1, 2....$Here $\sin(2\pi x) = 1$ and as expected from eq. (24) the stable situation is one of a circular orbit with maximum possible (and x-independent) azimuthal velocity:

$$v_\phi(max) = L_u/x = 1. \tag{25}$$

This $v_\phi(max)$ is the global escape velocity, $(2\pi GM/\lambda_o)^{1/2}$ in dimensional units. It is the maximum velocity which a star can have and still be bound by the galaxy. Note, however, that to be bound the star must be close to $x = N + 1/4$ cycle. The local escape velocity for a star not near 1/4 cycle is much smaller. Note, further, that as a concept, (but not a magnitude), $v_\phi(max)$ replaces the usual v_ϕ(LSR). The Local Standard of Rest is inherently a Newtonian concept which assumes that circular orbits of velocity only weakly dependent on radius can exist anywhere throughout the disk. This is hardly the case with the cosinusoidal potential depicted in Fig. 5.

The opposite extreme, radially oscillating orbits, occur when $L_u = v_\phi x = 0$ Here the effective potential reduces to $\phi_u = -(1/2\pi x)\cos(2\pi x)$. This potential leads to radial oscillations about the minima at $x = N$. The maximum excursion is only from $x = N - 1/2$ to $x = N + 1/2$. (This is in marked contrast to the Newtonian limit where radially plunging orbits go right through the origin). The maximum velocity amplitude slowly decreases with N:

$$v_r(max) \simeq (4/2\pi N)^{1/2} = (4\lambda_o/2\pi r)^{1/2} v_\phi(max). \tag{26}$$

In the general case of fixed total energy E_u,

$$v_r = \pm v_\phi(max)(2(E_u - \phi_u(x, L_u))^{1/2}. \tag{27}$$

The central question is whether one can find a place for the sun on Fig. 5. Neither of the two extremes above is possible. If $L_u = 0$, the sun would have to move on a radial orbit. Alternatively if L_u takes on its maximum value x, there is no potential well and no radial motions are possible. This is in contrast to the observed broad distribution in radial velocities for neighboring stars (Fig. 4). For any given x, stable orbits will occur only over a restricted range of v_ϕ and v_r. This range can readily be found using eq. (27). Here the limits on the range are determined by the condition that the star not have so large an E that it goes past the *maxima* that occur at $x \simeq N + 3/8$. The results for $14\ 5/8 \leq x \leq 15\ 3/8$ are shown in Fig. 6 where the permissible range is under the bell-shaped curves.

These limiting curves show a curious asymmetry about $x = 15\ 1/4$. The ones for $x < 15\ 1/4$ are broad maxima; the one for 15 3/8 is a cusp. This asymmetry is a direct consequence of the *maximum* in ϕ_u which occurs near the outer turning point, but not the inner. If a star of energy $E = \phi_u(max)$ is a small distance Δr away from an outer turning point, both its maximum radial velocity v_r and the departure of its azimuthal velocity Δv_ϕ from that appropriate for a circular orbit vary as Δr. Conversely, near an inner turning point, Δv_ϕ still $\alpha\ \Delta r$, but $v_r\ \alpha\ (\Delta r)^{1/2}$.

It is instructive to relate these theoretical curves to the observed scatter plot of the azimuthal vs the radial velocity for the stars in CNS3R. This comparison is shown in Fig. 7, for various assumptions about v_{sun} and the global escape velocity, $v_{\phi}(max)$. Unfortunately, neither of these velocities is very well known. The most comfortable fit is for v_{sun} given by the dwarf spheroidals (Sect 3.3.4) and $v_{\phi}(max)$ near the upper end the range ($450 km/s < v_{esc} < 650 km/s$) found in a recent study of high velocity stars (Leonard and Tremaine 1990).[7]

That the fit (by eye) is made with only a 5 km/s offset in the radial velocity u indicates that the sun is close to a turning point. Many nearby stars have azimuthal velocities comparable to the sun's, but have appreciable radial velocities. This feature can only be accommodated if the sun is near an *inner* turning point, where the maxima in Fig. 6 are broad. (Parenthetically, a radial phase between 5/8 cycle and 7/8 cycle, as suggested by the data, implies that the sun is presently being repelled by the galactic center)!

Near an inner turning point there is a positive correlation between the apex of the dynamic limit in v_{ϕ} and the radius x of a given star. (Compare the curves for 14 3/4 and 14 7/8 in Fig. 6) It is important to check whether the stars in CNS3R show this correlation. (Although the spatial position of stars is not given directly in CNS3, the position can be readily calculated from the tabulated parallax, declination and right ascension.) The leverage given by the 25 pc range of CNS3 is too small for this correlation to be evident in a u-v scatter plot, but it can be seen in the histograms for $v = v_{\phi\ star} - v_{\phi\ sun}$.

Of the 1946 stars in CNS3R, 579 satisfy $r_{star} - r_{sun} < 6pc$ and 572 satisfy $r_{star} - r_{sun} > 6pc$. The former group consists of stars which are at least 6 pc closer to the center of the galaxy than the sun. Call this group '$-$' stars; the latter '$+$'. Histograms of N_+ and N_- vs v are given in Fig. 8. Separately each resembles a cusp, peaking near v=0, as expected since the sun ($v \equiv 0$) is close to the dynamic limit. But the cusp for + stars is shifted to higher v. This is revealed in a histogram of the difference $\Delta N = (N_+ - N_-)$. (ΔN is a particularly significant variable since local idiosyncrasies such as streaming motions, which can appear in the separate plots of N_- and N_+ should cancel in the difference plot). The distribution in ΔN shows the expected double cusp near $v = 0$.

Significantly, this double cusp is evident in both the subsets $6pc < |r_{star} - r_{sun}| < 12pc$ and $12pc < |r_{star} - r_{sun}|$. Also the cusp does not appear in the remaining sample of 795 stars which have radii very close to that of the sun ($|r_{star} - r_{sun}| < 6pc$). See Fig. 9.

3.3.2. Galactic Center

The cosinusoidal potential of a an isolated *point* mass separates space into three qualitatively different parts. Stable circular orbits are permitted only over about 1/4 of the space ($N\lambda_o < r < N\lambda_o + 0.25$). More room is available for non-circular orbits which are merely radially confined ($r(max) - r(min) < \lambda_o$). Finally there are periodic "forbidden" regions which no stable orbits can enter. When $r >> \lambda_o$, these regions are small (At R = 8.5 kpc, the forbidden region covers only 5% of a cycle). However, their size increases markedly as one approaches the origin. The first forbidden region (r between 0.45 and 0.68 λ_o) covers nearly a quarter of a cycle. (See Fig. 10).

We may look for evidence for or against forbidden regions near the center of the Milky Way. A tracer for the central potential is the 21-cm emission from a thin disk of hydrogen gas first observed systematically by Rougoor and Oort (1960). Mihalas and Binney (1981) summarize these observations, "[assuming $R_o = 10$ kpc] the data indicate that there is an inner disk of radius about 300 pc and rotation speed of about 200 km/s surrounded by a ring having a radius of about 750 pc and a rotation speed of about 265 km/s. There is no sign of expansional motion in either disk or ring." After converting to the currently accepted value $R_o = 8.5$ kpc, these radii correspond to 0.46 λ_o and 1.14 λ_o, in agreement with allowed regions for circular motion. Confirmation of the first forbidden region centered at 0.56 λ is given by the modern ^{13}CO emission data of Bally et al (1988). Their data shown in their fig. 3 as plots of integrated emissivity vs latitude and longitude shows conspicuous voids for longitudes $1.7^o < |l| < 2.6^o$ corresponding to the projected radii (250 pc and 380 pc) which limit the first forbidden region.

A similar thin sheet of gas is evident near the center of M31. The kinematics of this sheet were thoroughly explored by Rubin and Ford (1971). Although the gas does show substantial non-circular motions, Rubin and Ford were able to use Balmer alpha and nitrogen emissions to establish a mean rotation curve shown here as Fig. 11. The plot of v_{rot} vs r shows conspicuous breaks at the beginning and end of the first forbidden region. Additionally v_{rot} declines as 1/r in the forbidden region as expected from angular momentum conservation. That the gas in the forbidden region is tenuous can be seen directly from Rubin and Ford's spectrographic plate, (their fig. 1).

Finally, we note that regions are absolutely forbidden only for a central *point* potential. The next section examines the consequences of an extended source.

3.3.3. Bulges, Bars and Spiral Arms

For Newtonian gravity there is no question that the center of the galaxy is at a deep potential minimum. The potential rises monotonically as r increases. The rate of rise $\partial\phi/\partial r$ may be a function of r and angles, but the slope is always positive. This dull behavior comes from Newton's third theorem, that a mass experiences no force when inside a thin homogeneous shell whose boundaries are two similar, concentric ellipsoids. Each shell, however, contributes its own negative potential to the space within. As r increases, the number of shells simply diminishes. Realistic density distributions (Binney & Tremaine 1987) lead only to small departures from this naive model.

An extended source has a radically different effect on the cosinusoidal potential. Here the galaxy building principle (Sect 3.2.5) is needed to ensure that the internal field is unaltered by the addition of successive cycles of matter. But, in distinction to the Newtonian case, the same distribution of matter within a cycle that makes no contribution to an internal *field*, also makes no contribution to an internal *potential*. The potential at the center will still be negative (due to the inner (Newtonian) half-cycle), but the potential will not be at a deep minimum.

As r increases from the center in the plane of the disk all the multipole moments (of order less than $(20r/\lambda_o)^{1/2}$) act coherently. This makes it likely that the envelope of the potential will go through a *maximum* before decaying as $1/r$. This maximum is a novel feature, not present with the Newtonian potential, nor even with a cosinusoidal potential surrounding a point source. The maximum separates the internal region where radial motion can extend over several cycles from the external region where it cannot. I identify the internal region with the bulge; the external with the disk.

The potential is sketched in Fig 12. Here the bulge extends to the radius $r = r_B$. Interior stars of large radial velocity (but small azimuthal velocity) go past the center and are reflected at r_B and at lesser radii of similar phase ($r \simeq N\lambda_o + 3/8$). The consequent accumulation of stars at a phase where $\sin(2\pi r/\lambda_o) > 0$ deepens the oscillations of the exterior potential thus enhancing the stability of the entire disk. The stars of large radial velocity and excursion coexist with the ones of dominantly azimuthal velocity and limited radial excursion. This feature is compatible with the observations of both motions near the galactic centers of the Milky Way and Andromeda (Rougoor and Oort 1960; Rubin and Ford 1971; Bally et al 1988).

Our discussion thus far has assumed axial symmetry. The presence of interior stars of large radial velocity should make it possible for disk galaxies to destroy this symmetry by forming bars. The alternation of sign in the appropriate ordinary Bessel function, Y_o, gives stability even to thick bars. An essential requirement is that the ends of the bar rotate much more slowly than the corotation velocity. This is a consequence of the virial theorem (for the dominantly central potential), $< T >=< rd\phi/dr >$. The radially unconfined stars of the bar spend less time at maximum radii (where $d\phi/dr > 0$) than do the radially confined stars. Thus the kinetic energy of stars in the bar must be lower that those of the disk generally. (For the bar to be linked dynamically to the dwarf spheroidals, as discussed in the next section, the bar's angular velocity will have to be very small: $\Omega < 300\mathrm{km-s}^{-1}/200\mathrm{kpc}$.)

The ends of the bar ($r = r_B$) are a secondary source of potential that may be important in generating spiral arms. Typically the arms originate at the ends of a bar and begin their outward spiral in approximately the azimuthal direction. It is dynamically impossible for a single star to make the 90^o bend from bar to arm. However, it is quite possible for the accumulation of stars at r_B to provide the potential needed to focus the exterior (and radially confined) disk stars which move in orbits having $r > r_B$).

The end of the bar may resemble more a plane than a point. The potential near a plane varies as $\sin(2\pi\Delta r/\lambda_o)$ and thus has a nearest minimum at $r - r_B = 3\lambda/4$. The potential near a point varies as $-\cos(2\pi\Delta r/\lambda_o)$ and has a minimum at $r - r_B = \lambda$. Since the end of the bar is at $r \simeq (N + 3/8)\lambda$, the minimum in the near potential produced by the bar will be on the axis of the bar at a radius of r/λ_o between $N + 1 + 1/8$ and $N + 1 + 3/8$. The effect of the bar will then be to focus stars around the radius $r/\lambda_o = N + 1 + 1/4$ which is approximately the central radius for circular orbits.

There is a possibility for feedback if the focal point is at the opposite end of the bar ($\Delta\phi = 1/2$ cycle) or indeed if the focus is at any integral number of half-cycles. The accumulation of matter at the ends of the bar needed to focus the nearby disk stars after 1/2 a cycle is not large. A simple application of the impulse approximation shows the needed mass to be $(1/2\pi)^2(r/\lambda_o)M_{eff}$, where M_{eff} is the effective central mass of the galaxy. Accumulation of disk stars at the focal points will tend to depress the potential at the ends of the bar, $r = r_B$. Ultimately, there is negative feedback as the peak at $r = r_B$ becomes shielded by the preceding peak. For small depressions, however, the feedback can be positive. This happens when the spectrum of energies of the stars in bar is a rapidly decreasing function of energy. Then the loss in the number of stars being reflected is more than compensated by an increase in dwell time of those that remain. That the feedback can be positive initially and negative ultimately would seem to be an ideal circumstance for the formation of bars.

The formation of spiral arms that trail from the ends of the bar can also be understood. The arms begin as density enhancements at the focal points in the largely circular orbits that pass close to the ends of the bar. Because of feedback, these focal points are also near the ends of the bar. At radii more that a few λ_o beyond the bar, the bar's ends appear as point sources of an oscillating force which decreases as $1/|\mathbf{r} - \mathbf{r_B}|$. But the length along the orbit over which the phase of the force is coherent increases as $r - r_B$. In the impulse approximation, the two factors cancel, leaving a focal point which is a constant distance as measured along the arc of the orbit. This constant distance corresponds to an angle which diminishes as r increases.

3.3.4 Central Bar & Dwarf Spheroidals

The dwarf spheroidal galaxies are the few small ellipticals that orbit the Milky Way at radii between 60 and 220 kpc. Kunkel and Demers (1976) and Lynden-Bell (1976) observed that these galaxies are roughly in a plane perpendicular to the galactic disk and that the same plane includes the small and large Magellanic clouds. The origin of this "Magellenic Plane" is still not well understood (Majewski and Cudworth 1993; Mateo et al 1991).

The cosinusoidal potential may furnish a dynamical explanation. The Magellenic Plane is perpendicular not just to the galactic disk; it is perpendicular to the central bar as well. When viewed from the north galactic pole, the axis of the bar is about 16^o clockwise of the radius vector from the galactic center to the sun (Binney et al 1991). All eight spheroidals and the two Magellanic clouds are within 29^o of the plane perpendicular to this bar. If we allow the ten satellites to specify the best fitting plane, they are all within 24^o of a plane which is perpendicular to a bar that is 9^o clockwise. (See Fig. 13 and Table I, where the angle between the satellite and best fitting plane is called A).

A dynamical link between central bar and distant satellites is ridiculous for the Newtonian potential. But the link is sensible for the cosinusoidal potential. Here the weighting factor $j_n(r/2\pi\lambda_o)$ favors central matter over distant as a source for a multipole moment n. Additionally. the distant effect of all multipoles is in phase and falls off simply as $1/r$. The total potential (bar plus disk) factors into the product of radial and angular parts,

$$\phi(\vec{r}) = \Psi(b, l)(-\cos(2\pi r/\lambda_o)/r). \tag{28}$$

Earlier we saw that coherent radial enhancements in the density distribution within the disk would maximize Ψ in the plane of the disk. Similarly, matter within the bar maximizes Ψ in the plane perpendicular to the bar's midpoint. This latter maximum provides a channel in which the loosely bound dwarf spheroidals can move without being ripped apart by tidal forces. Because of the disk, Ψ is not a constant around the channel, but rather increases from pole to equator by a factor equal to the number of Legendre polynomials needed to describe the disk. In section 3.2.4 we estimated that number to be ≈ 4.

The angular dispersion of the dwarf spheroidals is consistent with the bar as the source of the potential "channel". Reasoning familiar from physical optics shows that all matter within a bar of length $2a$ acts coherently on distant objects within an angle of $\arcsin(3\lambda_0/4a)$ of the median plane. Taking 24^o for this angle, we find that the effective (not total) radius of the bar is 1.8 λ_o or 1000 pc. (See Fig. 12).

Qualitative confirmation of the potential channel is also given by the position angles of the major axes of the spheroidals. These generally are elongated in a direction parallel to the Magellanic Plane. Specifically, the position angles of the five faint spheroidals, Ursa Minor, Draco, Carina, Sextans, and Leo II, are within 26^o of the direction of the Magellanic plane. The three bright spheroidals, Sculptor, Fornax, and Leo I, have position angles at larger angles as is evident in Table I.

In the absence of dark matter, the faint spheroidals have smaller mass and self-gravity than the bright. The faint are thus the better tracers of the Galactic potential. This fact will be important as we now consider the orthogonal sides of the potential channel, namely those formed by the radial variation, $-\cos(2\pi r/\lambda_o)/r$.

It is this *Galactic* radial factor which enables all the faint dwarf spheroidals to have an observed radial velocity dispersion near 7 km/s. A simple scaling argument lets us infer the expected dispersion at the mean radius of the faint dwarfs from the observed dispersion of stars near the sun,

$$\sigma_{dwarfs} = (1/2)\sigma_{stars}(r_{dwarfs}/r_{stars})^{-1/2} = (1/2)(40)(110/8.5)^{-1/2} = 6\text{km/s}. \tag{29}$$

(The factor 1/2 reflects the decrease of Ψ by a factor of 4 from the galactic pole to the disk.)

The Galactic Newtonian potential lacks the oscillating radial factor. Here the dwarf spheroidal is bound with enough internal dark matter to balance the kinetic energy associated with the dispersion in the velocities. The difficulty is that an uncomfortable amount of dark matter is required, particularly for the faintest dwarfs, Ursa Minor, Draco, and Sextans. Estimates of the required M/L vary, but most give values above 40 times that of the sun. To my knowledge, no one has heeded the admonition of Gerhard and Spergel (1992) and given an explanation for the source of so much localized dark matter.

The observed velocities of the spheroidals offers another test of the cosinusoidal potential. Here, in distinction to the Newtonian case, the dwarf spheroidals must be in essentially circular orbits, each confined within $\Delta r \simeq \lambda_o$. Both the spatial radial and azimuthal velocities of the spheroidals are limited. They can contribute estimated maximal amounts to the observed heliocentric radial velocities RV_{obs} of

$$\Delta v_r = (600/2)[(4/2\pi)(\lambda_o/r_{dwarf})]^{1/2} \tag{30}$$

and

$$\Delta v_\theta = (600/2)\cos(60^{\mathrm{o}})(r_o/r_{dwarf}). \tag{31}$$

Here 600 km/s is taken as the escape velocity from the galaxy, which we have conservatively derated by a factor of 2 to account for the fragility and the polar orbits of the dwarf spheroidals; 60^o is the minimum angle between v_θ and r_o. A final contribution to RV_{obs} is RV_{sun} as computed from the dwarf's position and the generally assumed solar motion with respect to the galactic center, $(u, v, w) = (9, 231, 6\mathrm{km/s})$ (Jones, Klemola, and Lin 1994).

The test is whether, particularly for the faint spheroidals, the maximal contributions from the dwarf's unmeasured spatial motions can account for the difference between the observed radial velocity and the solar contribution. The answer, evident from Table I, is clearly negative. For example, for Ursa Minor, $|RV_{obs} - RV_{sun}|$ = 87 km/s, whereas $\Delta v_r + \Delta v_\theta$ is only 40 km/s. To achieve reasonably good agreement, it is necessary to assume that the azimuthal velocity of the sun is about 300 km/s (rather than 231).

It is important to understand that this radical change is not so bizarre when seen in the context of a cosinusoidal potential. The conventional value is based on a solar velocity of 6 km/s relative to a local standard of rest (LSR) having a circular velocity of 225 km/s. The concept of a circularly moving LSR of velocity $\Theta(r)$ which varies slowly across the disk is naturally compatible with Newtonian dynamics. But the concept is foreign to this proposal. Here strictly circular motions are permissible for only 1/4 of the available space. At all other positions, including that of the sun, radial motions are a necessity. These are combined with any of whole range of azimuthal velocities to yield stable orbits. There is no single LSR.

The distinction between the two dynamics is most evident in Oort's constants A and B which are generally used to determine both $\Theta(r_o)$ and $d\Theta/dr$ evaluated at r_o (Mihalas & Binney 1981). The classical analysis of Fricke (1967) is intentionally limited to stars farther than 100 pc from the sun and results in $d\Theta/dr = -(A+B) < 0$. This result conflicts with the result in section 3.3.1 above that for stars within 25 pc of the sun the mean azimuthal velocity actually increases with increasing radius.

We return now to the question of whether the observed radial velocities of the dwarf spheroidals are consistent with their being in nearly circular orbits. Using an adopted value of 300 km/s for the azimuthal velocity of the sun we form a somewhat arbitrary figure of merit

$$FM = |RV_{obs} - RV_{sun}|/(\Delta v_r + \Delta v_\theta). \tag{32}$$

For a satellite to be radially confined, $FM \leq 1$. The results, shown in Table I are positive for the faint dwarfs and for the Magellanic clouds, but not for Sculptor, Fornax, and especially Leo I. The last is often regarded as escaping from the Galaxy. A more modest hypothesis could apply to the first two. The two bright dwarfs have both a large self-gravity and a radial velocity which (in distinction to the clouds) is determined by measurements of just a few tens of stars. Further these stars are not representative, but are giants. Imagine that Sculptor, say has stars in two adjacent shells, but giants in only one. The interaction potential between the two shells could give the giants a radial velocity greater than that allowed by the Galactic potential alone. The maximum value of this interaction potential is $4GM_{dwarf}/\lambda_o$. Assuming a M/L ratio of 4, the potential could contribute 30 km/s to the radial velocity of Fornax and 17 to Sculptor.

There is no evidence that the faint dwarfs spread to more than one shell. For Ursa Minor and Sextans there is even evidence that stars are largely confined to a single shell. Star density profiles from the automated APM facility have been published for these two (Irwin & Hatzidimitriou 1993; Hargreaves et al 1994). These profiles show dips at $r \simeq 250$pc, as expected from the cosinusoidal potential.

3.3.5 Globular Clusters

Any gravitational theory must account for the motion of Population II stars as well as the more numerous Population I stars. As compact, massive, and old conglomerations of Population II stars, the globular clusters provide a good test of the cosinusoidal potential. The test is particularly crucial since globular clusters are generally thought to be moving as bees around a hive; ie, in deeply penetrating orbits without much preference for the galactic disk. At the end of this section I offer some evidence against this Newtonian picture. Let us begin, however, by employing the methods of the last section to see that the observed radial velocities of the clusters are consistent with their being in nearly circular orbits.

Unlike the dwarf spheroidals, the galactic clusters are at galactic radii both less than and greater than r_o. They also are not confined to polar orbits in a particular plane. Thus, in constructing a figure of merit FM for the clusters, we shall use different estimates of the maximum possible contributions from radial and azimuthal motions:

$$\Delta v_r = 600\cos(\alpha)[(4/2\pi)(\lambda_o/r)]^{1/2} \tag{33}$$

and

$$\Delta v_\theta = 600\sin(\alpha) \tag{34}$$

. Here 600 km/s is taken as the escape velocity from the galaxy and α is the angle subtended at the globular cluster by the sun and the center of the galaxy. As before, the azimuthal velocity of the sun is taken as 300 km/s and FM is is defined by eq. (32).

The sample of 114 globular clusters is drawn from Hirchfel and Sinnott (1985), updated when possible by compilations of Pryor and Meylan (1993) and Kochanek (1996). The distribution, shown in Fig. 14, has a suggestive break at $FM = 1$. The vast majority has $FM < 1$ and thus can be bound by the cosinusoidal potential. Of the nine clusters having $FM > 1$, only one was discovered before 1900. That cluster, NGC 5694, has already been considered as a cluster which may be escaping from the Galaxy (Harris and Hesser 1976). The remaining eight clusters are so faint that they require a modern observatory such as Palomar for their detection. These eight are all at large radii (20kpc $< r <$ 120kpc) and could very well not be bound by the Galaxy. Their unbound colleagues, at even greater radii, then await 21st century observers.

We see that the cosinusoidal potential requires that the Palomar galactic clusters be of extragalactic origin. This requirement is not part of the Newtonian potential which permits deeply penetrating orbits. Indeed the classic study of Kinman (1959) suggests that the typical globular cluster is in an elliptical orbit of eccentricity $\epsilon = 0.8$. Kinman (and others) observed that the galactic clusters as a whole are rotating with considerably less velocity than the disk. If the typical globular cluster has the properties of the ensemble, then it has far too little angular momentum to be in a circular orbit.

There may be a problem with this model of deeply penetrating globular clusters. Even if one assumes dark matter and the Newtonian potential, the model predicts more globular clusters with high radial velocities than are observed. It is convenient to start with Kinman's paper and consider the orbit of a "typical" globular cluster which at its average radius has a transverse velocity which is $\sqrt{2}\times 69/216$ of that required for a circular orbit. Kinman's elliptical orbit calculation was made for a point galactic mass. Alternatively, we now assume a spherically symmetric distribution of galactic matter producing the standard potential required for a flat rotation curve, $\phi(r) = (225\text{km/s})^2 \ln(r)$. A simple numerical calculation gives the results shown in Fig. 15.

Evidently, 1/4 of the time our typical globular cluster has a speed greater than 270 km/s. Of course it is not necessarily heading directly towards the sun. But if we assume isotropic velocities, there should be six globular clusters (in the sample of 114) which have a radial velocity $|RV| > 270$km/s. In fact there is no globular cluster which has a RV greater than 270 km/s (after correcting the observed RV for the standard solar motion of (9,231,6 km/s). Only seven clusters, NGC 3201, Rup 106, M 68, NGC 5694, M 9, M 70, and NGC 6934, even have radial velocities greater than 225 km/s.

The reader may find an explanation for the paucity of speedy globular clusters. Dead ends that I have pursued include the following. The standard potential is not easily truncated. It begins at $r \simeq 1$ kpc and (to account for an escape velocity much greater than $225\sqrt{2}$ km/s) must extend to at least 60 kpc. Concentrating all the dark matter into the disk is also not likely to be helpful. One then replaces the standard potential with one that depends on latitude, $\phi(r) = (225\text{km/s})^2 \ln[r(1 + |\sin b|)]$. The radial component of the gravitational field g_r is still independent of latitude and (by the virial theorem) the rms speed is 225 km/s independent of orbit. Increasing $v(\text{LSR})$ to 295 km/s also is of no use. Then no cluster has an observed $|RV| > v(\text{LSR})$.

Perhaps the basic problem is that, even with dark matter, the Newtonian potential cannot accommodate two different populations of stars into circular orbits. Having stars with different speeds occupy essentially the same circular orbit is not a difficulty for the cosinusoidal potential.

3.4 Coma Cluster

The greatest need for dark matter is in systems of large size ($R > 100$ kpc) such as clusters of galaxies. Indeed, the application of the virial theorem to the Coma cluster stimulated Zwicky (1933) to introduce dark matter. He found that a mass to light ratio 400 times that of the sun was required to explain the observed 1000 km/s dispersion in the projected velocities v_p of individual galaxies. Subsequent discoveries of a massive halo of X-ray emitting gas (of dispersion $\simeq v_p$) and of a reduced value for the Hubble constant have increased the stock of accountable matter so that it is now about 10% of the dark matter. (See White et al 1993).

This conventional picture of the Coma cluster as a well-relaxed structure bound by gravitational forces has changed recently. Biviano et al (1996) have shown that the mean radial velocity of the galaxies within the cluster varies systematically by 500 km/s over the extent of the cluster. Colless and Dunn (1996) have found two dominant substructures. They conclude, "It is no longer possible to use Coma as the exemplar of a rich, regular, and relaxed galaxy cluster. Studies of the projected distributions of both the galaxies and the X-ray gas show statistically significant substructure on both large and small scales...". Feretti et al (1995) find a magnetic field which is both surprisingly large (8 μG) and surprisingly twisted (on a scale of 2.5" = 1 kpc). This work has been used by Felten (1996) to show that the dominant energy in Coma is tantalizingly close to being magnetic rather than gravitational.

Can this confused situation be helped by using the cosinusoidal rather than the Newtonian potential? The answer is a clear no. But understanding may come from electromagnetism, particularly if, as argued in the Appendix, the photon is not massless, but has a Compton wavelength, $h/m_\gamma \equiv \lambda_1 \simeq \lambda_o$.

Let us begin by considering the energy stored in the gravitational field. As we have seen, the cosinusoidal potential weights source masses inversely with their distance from the center of a spherical distribution. This weighting is disadvantageous for systems with very large cores. The potential at a radius b (or a) of a shell of mass M situated at a (or b) is given by eq. (19), $\phi = -GM[\lambda_o/(2\pi ab)]\sin(2\pi a/\lambda_o)\cos(2\pi b/\lambda_o)$, where $a < b$. The maximum value of ϕ is found by being grossly optimistic and setting the trigonometric factors equal to unity, then $\phi_{max}(a,b) = GM\lambda_o/(2\pi ab)$. Approximate the Coma cluster as a homogeneous sphere of radius $R = R_{core} = 200$ kpc and density $\rho = 4\times 10^{-3} m_p/\mathrm{cm}^3$. Integration over b yields $\phi_{max}(a) = G\rho\lambda_o R^2/a$. Averaging $1/a$ over the sphere gives for the average density of gravitational energy,

$$W_{grav}^{max} =< \rho\phi_{max} >= (3/2)G\rho^2\lambda_o R = 10^{-14.7}\mathrm{ergs/cm}^3. \tag{35}$$

This energy is a factor of 1000 less than the energy stored in the magnetic field,

$$W_{mag} = B^2/8\pi = 10^{-11.7}\mathrm{ergs/cm}^3. \tag{36}$$

The above expression for magnetic energy is augmented if the photon has a mass. As shown in the appendix, there is then an added term for the stored energy which now becomes,

$$W_{mag}^{m_\gamma} = B^2/8\pi - (2\pi/\lambda_1)^2 A^2/8\pi. \tag{37}$$

The vector potential $A \simeq Bl$, where the coherence length l may be taken to be the

1 kpc, observed by Biviano et al (1996). Thus

$$W_{mag}^{m_\gamma} \simeq (2\pi l/\lambda_1)^2 (B^2/8\pi) \simeq 120(B^2/8\pi) = 10^{-9.6}\text{ergs/cm}^3. \tag{38}$$

The magnetic energy density is now comparable to the thermal,

$$W_{therm} = (3/2)\rho v_p^2 = 10^{-9.7}\text{ergs/cm}^3. \tag{39}$$

The thermal energy is explosive. Fortunately, the signs in the expression for $W_{mag}^{m_\gamma}$ are such that magnetic energies are implosive for coherence lengths greater than λ_1. Thus the rough equilibrium in the Coma cluster may be fundamentally a balance of thermal and magnetic forces.

Clusters other than Coma should also show this balance. If gravity is unimportant globally, then all clusters should have extensive magnetic fields. Clusters generally have the radio sources needed to generate such fields. True, most lack Coma's extensive X-ray halo, but, as Tribble (1993) observed for the Perseus cluster, the absence of a halo may indicate "not that there is no magnetic field there, but rather than the relativistic electron population has aged and the synchrotron emission has faded from view."

4. GENERAL RELATIVITY

Lindley (1992) has challenged all proposers of non-Newtonian gravitation. The new theories must be compatible with general relativity. Being compatible is not the same as being identical. General relativity reduces to Newtonian mechanics in the limit of small velocities and weak sources. If the latter theory is changed, the former must be also. Indeed the first modification was suggested by Einstein in 1917 when he added the cosmological term $\Lambda g_{\mu\nu}$ to the left hand side of the fundamental (1915) equation of general relativity, $G_{\mu\nu} = 8\pi T_{\mu\nu}$. Misner, Thorne, & Wheeler (1973) note that $G_{\mu\nu}$ is forced to be the Einstein tensor $G^E_{\mu\nu} = R_{\mu\nu} - (1/2)g_{\mu\nu}R$ by several very reasonable requirements including that $G_{\mu\nu}$ vanish when spacetime is flat. By setting $G_{\mu\nu} = G^E_{\mu\nu} + \Lambda g_{\mu\nu}$, Einstein sacrificed this last requirement to permit a steady state universe.

The present proposal is to add *both* a cosmological term and a Minkowski term to the standard theory. Thus,

$$G_{\mu\nu} \equiv G^E_{\mu\nu} + (g_{\mu\nu} - \eta_{\mu\nu})\Lambda = 8\pi G T_{\mu\nu}, \tag{40}$$

where $\eta_{\mu\nu}$ is the Minkowski metric, $\eta_{oo} = -1, \eta_{ii} = +1$ and $c = 1$. The two added terms cancel when space time is flat, thus permitting $G_{\mu\nu} = 0$ in this limit. The cost of the explicit introduction of $\eta_{\mu\nu}$ in the field equations is the loss of the principle of equivalence. $\eta_{\mu\nu}$ transforms as a tensor only for Lorentz transformations and not for transformations between accelerated frames (Pauli 1958). Thus eq. (40) applies only to frames which are in uniform motion with respect to the rest frame of the universe.[8]

In the non-relativistic limit, the [0,0] element of eq. (40) reduces to the scalar Helmholtz equation,

$$\nabla^2\phi + k_o^2\phi = 4\pi G\rho. \tag{41}$$

To see this, we follow Misner, Thorne, & Wheeler and note that non-relativistically, $G^E_{\mu\nu} \rightarrow \nabla^2\phi + 4\pi\rho(1 - 2\phi)$, $h_{oo} \equiv g_{oo} - \eta_{oo} \rightarrow -2\phi$ and $T_{oo} \rightarrow G\rho(1 - \phi)$. Thus

$\nabla^2\phi - 2\Lambda\phi = 4\pi G\rho$. This equation agrees with the scalar Helmholtz equation, if we identify

$$-2\Lambda = k_o^2 = (2\pi/\lambda_o)^2. \tag{42}$$

The general, singular point source solution of the Helmholtz equation is given by the cosinusoidal potential [eq. (2)]. Thus this proposal may be viewed as usurping the role of the cosmological constant and giving it a tentative value, $\Lambda = -(1/2)(2\pi/\lambda_o)^2 \doteq -0.6 \times 10^{-41}$ cm^{-2}.

4.1 Solar System Tests

Detailed test of gravitational theory are generally made in the parametrized post-Newtonian formalism (PPN) (Nordvedt 1968). Among the many possible gauges, the standard post-Newtonian gauge (Will 1994) is particularly felicitous. This gauge is an inertial one, fixed in the barycenter of the solar system, and thus is a gauge for which this proposal makes definite predictions.[9] We shall need only the primitive (Eddington-Robertson-Schiff) version in which the sun is point-like, spherical and non-rotating.

The effect of this proposal is to add a trigonometric factor to the expression for the potential, leaving for the metric

$$g_{oo} = -1 + 2GM\cos(2\pi r/\lambda_o)/r - 2\beta(GM\cos(2\pi r/\lambda_o)/r)^2$$

$$g_{jk} = 1 + 2\gamma\delta_{jk}GM\cos(2\pi r/\lambda_o)/r; \quad g_{oj} = 0. \tag{43}$$

Here β and γ are the parameters usually used to express departures from general relativity in which $\beta = \gamma = 1$. Observations determine that β and γ are both one to within about 0.1%. In contrast, the trigonometric factors differ from unity by only $(1/2)(2\pi\ \mathrm{AU}/560\ \mathrm{pc})^2 \simeq 10^{-14}$. Thus measurements of the bending of light by the sun or of radar transit times to nearby planets, which measure γ, would have to be improved by 10^{11} to see the effect of λ_o.

The prospects for influencing β are more hopeful. The term in g_{oo} measuring the perihelion shift of mercury is only quadratic in the potential. A non-zero λ_o, however, will alter the linear term. Thus this proposal gives

$$|\beta - 1| \simeq (2\pi r/\lambda_o)^2 (GM/r)^{-1} = (T/\lambda_o)^2 = (0.24\mathrm{yr}/1800\mathrm{yr})^2 = 0.25 \times 10^{-7}. \quad (44)$$

Unfortunately, this value is still about a factor of 3000 smaller than the sensitivity for β which could be achieved by a mercury orbiter (Bender, Ashby, and Wahr 1995).

4.2 Gravitational Radiation

The shortening of the period of the binary pulsar is a well known consequence of the emission of gravitational radiation. Present observations and general relativity agree at the level of 0.8% (Damour & Taylor 1991). As shown below, this agreement is not compromised by my proposal.

Within the framework of linearized gravity and the now obligatory Lorentz gauge,[11] the effect of the added term in eq. (40) is simple. We must add a corresponding term to the gravitational wave equation (Misner, Thorne, and Wheeler 1973) which for propagation in a vacuum now becomes

$$[-\partial^2/\partial^2 t + \nabla^2 + (2\pi/\lambda_o)^2]\bar{h}_{\mu\nu} = 0, \quad (45)$$

where $\bar{h}_{\mu\nu} = h_{\mu\nu} - (1/2)h\eta_{\mu\nu}$ and $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$. The frequency ν and wavelength

λ of the plane wave solutions satisfy the characteristic equation

$$\lambda^{-2} = \nu^2 + \lambda_o^{-2}. \tag{46}$$

Because of the added term, the propagation speed approaches unity only in the high frequency limit. In general, the phase velocity $v_p \equiv \nu\lambda = \nu(\nu^2 + \lambda_o^{-2})^{-1/2}$. The group velocity $v_g = 1/v_p > 1$ at all frequencies.

The departure of v_g from one has only a minor direct effect on the rate of gravitational radiation from the binary pulsar. This rate is inversely proportional to v_g^5 (Weber 1961). For an 8-hour period, $\Delta v_g = (1/2)(8 \text{ h}/1800 \text{ yr})^2 = 10^{-12.5}$. Thus the effect of $\lambda_o = 1800$ lt yr is to decrease the expected rate of decay by roughly a part in a trillion. This change is comparable to the expected fractional effect of λ_o on the precession of the periastron of the binary system.

4.3 The graviton as a tachyon

The departure of v_g from unity is evidently too small to be directly observable for the binary pulsar (or indeed for any source of gravitational radiation of period < 1800 yr). Nonetheless, the fact that $v_g > 1$ would make the graviton the only known tachyon, a particle which travels faster than light. We shall see that this fact may have implications for the stability of the graviton.

A short review of tachyons may be appropriate. At first their existence was deemed impossible. Einstein (1905) argued that to accelerate matter past the speed of light would require infinite energy. Tolman (1917) added a causality paradox: if tachyons existed, it should be possible for you to send them to an accomplice on a receding planet and have him return the tachyonic signal before you sent it. The first objection was removed and the second ameliorated by Bilaniuk, Deshpande, and Sudarshan (1960). They noted that infinite energies would not be required if tachyons *always* moved faster than light. Further any Lorentz transformation that changes the sign of time will change the sign of energy as well so that Tolman's paradox could be avoided if the only tachyons which can be detected are ones with positive energy. The publication of a quantum theory of spinless non- interacting tachyons (Feinberg 1967) encouraged several, unfortunately fruitless, searches (Kreisler 1973; Clay 1988).

The graviton would seem to be an ideal candidate for a tachyon. In the present proposal its velocity is always greater than light. Further it represents a perturbation of space-time itself. Thus the direction of time should be fixed by Hawking's criterion: time increases in the direction of the increasing local area of black holes. As a consequence, the energy of the tachyon in any frame should be positive.

If the graviton is a tachyon, its energy-momentum equation is given by the quantum version of eq. (46): $p^2 = E^2 + \mu^2$, where the tachyon's "mass" parameter $\mu \equiv h/\lambda_o$. For high energies, $p \simeq E$ and $v \simeq 1$; at low energies $E \to 0$, $p \to \mu$, and $v \to \infty$. This peculiar energy-momentum characteristic makes it possible for an energetic tachyon to decay into *itself* and an ordinary particle (bradyon), $t \to t + M$. It is easy to use energy-momentum conservation to determine the threshold for this unique process. Initially one has a tachyon of energy E_t, and momentum $(E_t^2 + \mu^2)^{1/2}$. In the final state the tachyon has transferred all of its energy to the bradyon. The threshold, as first found by Feinberg (1968), is

$$E_t^2 = M^4/4\mu^2 + M^2. \tag{47}$$

The mass parameter μ is so small that enormous energies are required to produce bradyons of ordinary mass. (For instance $E_t = M_{Planck}$ is required for $M \simeq$ 100 eV). But there is an exception if $M = 0$. There is nothing to keep even a low-energy tachyon from decaying into itself and a photon,

$$t \rightarrow t + \gamma. \tag{48}$$

Unfortunately, neither Feinberg (nor anyone else) has developed a quantum theory of *interacting* tachyons. Thus there is presently no way to calculate a transition probability for eq. (48), a process unique to tachyons. We can, however, use dimensional analysis to make an estimate. Since both the tachyon-graviton and the photon couple directly to space-time, the transition rate should be independent of G or e. Further the rate should vanish if either E_t or $\mu = 0$. The simplest choice is that the rate is proportional to the geometric mean of E_t and μ. With this choice, the mean free path for the decay of the graviton is related to its wavelength λ by

$$\lambda_{decay} = const. (\lambda \lambda_o)^{1/2}, \tag{49}$$

where *const.* is a number of order unity.

The prospect of gravitons decaying to photons in flight encourages a speculation. Perhaps this is the mechanism whereby gamma-ray bursters avoid the optical depth problem. Bursters are thought to arise from neutron stars either in the galactic halo or at cosmological distances. Their luminosity requires a very high optical depth for $\gamma + \gamma \rightarrow e^+ + e^-$; yet, the spectra show no evidence for a break at $E_\gamma \simeq 1$ MeV (Harding 1994). I conjecture that the observed photons begin their life as partially coherent gravitons. These could conceivably be produced during the violent superfluid vortex unpinning process that may occur during neutron star glitches (Epstein 1988). The calculated MeV energy scale of the binding of vortices to the nuclear crystal lattice (Link, Epstein, and Baym 1993) is quite reasonable. Yet to be estimated is the amount of coherence which vertex unpinning could generate.

We have seen that a tachyonic graviton is a necessary consequence of the theory proposed here. A related question is whether the photon might be bradyonic. This question is discussed in the Appendix.

4.4 Gravitational Lenses

The deflection of light by galaxies offers tests of the cosinusoidal potential. Since the peculiar velocities of candidate lenses are non-relativistic, the only important term in the stress-energy matrix is T_{oo} and the deflection will be given by the usual formula (Young et al, 1980)

$$\vec{\alpha} = 2 \int \vec{\nabla} \phi dl, \tag{50}$$

where $\vec{\alpha}$ is the vector deflection angle. Now however, the potential is cosinusoidal rather than Newtonian.

Let us compute the expected deflection for a light ray incident on a point mass mass M at an impact parameter b in a flat Euclidean space,

$$\alpha = 4b \int_{b}^{\infty} dr (d\phi/dr)/(r^2 - b^2)^{1/2}, \tag{51}$$

where α is the scalar bending angle, directed towards the galaxy. For the cosinusoidal potential, eq. (3), we have

$$\alpha = 8\pi GM(b/\lambda_o) \int dr(\sin(2\pi r/\lambda_o) + (\lambda_o/2\pi r)\cos(2\pi r/\lambda_o))/(r(r^2 - b^2)^{1/2}). \tag{52}$$

This is not an integral which can be done exactly. However, the singularity at $r = b$ in the denominator suggest an approximation where only the first half cycle of the first term contributes and everything else is ignored yielding

$$\alpha \simeq 8\pi(GM/b)(2b/\lambda_o)^{1/2}\sin(2\pi b/\lambda_o) \tag{53}$$

This approximation has been tested numerically. It is valid for $b > (1/2)\lambda_o$. In particular, maximum bending is achieved when b/λ_o is a little greater than N, an integer. In addition to the sine factor, eq. (53) differs from the Newtonian expression by the factor $(b/\lambda_o)^{1/2}$. This enhances the probability of gravitational lensing by strong, but distant sources, such as clusters of galaxies, and thus may be responsible for the arcs seen in the Abell clusters, for some features of the "venerable one" (0957+561), and for the large separation between images in the "lens without a lens"(2345+007). (The quotations are from Schneider et al (1992)).

To make a quantitative test of the sine factor, we need a nearby lens, one which can be observed with good resolution. We also need to generalize eq. (53) to an extended source. This extension is easily done in the case of a spherically symmetric deflecting galaxy which obeys the "galactic building principle". In that case, all matter contributes to a sinusoidal $(d\phi/dr)$ but with different weights. One then substitutes an M_{eff} for M in eqs. (52) and (53). M_{eff} is the sum of the core contribution, the contribution from matter at $r < b$ diminished by the weighting factor $(\lambda_o/2\pi r)\sin(2\pi r/\lambda_o)$ and the contribution from that matter at $r > b$ whose projected mass is interior to b. The case of a source quasar which is slightly off the axis formed by the viewer and the galaxy is handled by the graphical method of Young et al (1980). The result, shown in Fig. 16, is that, even for off-axis sources, significant magnification occurs only when $b \simeq N\lambda_o$.

The nearby lens is provided by the Einstein Cross (2237+0305) (Huchra et al, 1985). Four images are nearly centered in the middle of a spiral galaxy whose red shift is only 0.0394. The probability of having such a pattern at such a low z is so low that it has been described as "a unique case" by Schneider et al. The four images are at radii of 0.922", 0.970", 0.762", and 0.881" (Rix et al 1992). The average of these radii is 0.884". Compare this average with that calculated assuming $b = \lambda_o$,

$$\theta = (1.08\lambda_o/z_d H_o)(1+z_d)^2/(1+(1/2)z_d), \tag{54}$$

where λ_o =560 pc, $z_d = 0.0394$, the factor 1.08 results from the numerical integration of eq. (52) for a point source, M_{eff}, and Hubble's constant is assumed (as it is throughout this paper) to be 75 km s^{-1} Mpc^{-1}. The small correction factors arise from the assumption, justified later, of an open ($\Omega = 0$) universe. The calculated θ is 0.84", in too good agreement with the measured value. The fact that there are four images, rather than a complete ring and that the images do not all have the same radii probably results from plausible features of the galaxy: a quadrupole moment and a slight offset between the center of mass and center of luminance.

There should be more tests for our principle that a necessary condition for gravitational lensing is that $b = N\lambda_o$. Schneider's book (Schneider Table 2.1,

1992) lists 22 gravitational lenses. But if we skip arcs, which are generally caused by clusters of galaxies, there are only two additional lenses that are both caused by a single galaxy and close enough to be adequately resolved. These are the radio galaxies MG1549+3047 and MG1654+1346. The radio jet which is the source for the former system is evidently too diffuse to be of much use, but the hot spots in the jet behind MG1654+1346 have produced two rings which are most intriguing. The rings (shown in figure 2 of Langston et al, 1990) are quite accurately semi-circles of radii in the ratio 2:1 centered upon the galaxy. Nominally, the rings have radii of $4\lambda_o$ and $8\lambda_o$, (for an assumed λ_o=620 pc), but since the resolution of the VLA $\simeq \lambda_o$ at the galaxy's red shift of 0.25, we cannot use this as a test of our putative value of λ_o. The intriguing feature is that this is a lens where Newtonian mechanics has evidently produced a poor fit. The calculated images of the hot spots shown in figure 3 of Langston are not semi-circular, nor do they have radii in the observed ratio of 2:1.

Measurement of the time delay between these two rings should offer a clear test of the cosinusoidal potential. This is because the contribution of the potential to the time delay

$$\delta t_{pot} = \int \phi dl = \pi(GM)(2b/\lambda_o)^{1/2}\cos(2\pi b/\lambda_o) \qquad (55)$$

is vanishingly small. The time delay is reduced from the Newtonian because the integrand contains the oscillating potential itself, not its gradient.

We are left with the geometric contribution to the time delay,

$$\Delta t = H_o^{-1}(1+z_d)(D_d D_s/2D_{ds})(\vec{\theta}-\vec{\beta})^2 \qquad (56).$$

Since the images are semicircles, the angle between the optic axis and the source β is much smaller that the angle between optic axis and image θ. Thus the difference between the time delays for the larger and smaller rings is simply

$$\Delta t = (3/4)H_o^{-1}(1+z_d)(D_d D_s/2D_{ds})(\theta_B^2) = 25 \text{ days} \tag{57}$$

To evaluate this expression, we take the redshifts of the galaxy and lens to be $z_d = 0.254$ and $z_s = 1.75$. The distances D are then calculated in an open universe using the standard formulae (eg. Young 1980). The diameter of the larger ring θ_B is measured from fig 2 of Langston to be 1.44". The predicted time delay has not yet been measured.

4.5 Cosmology

Unlike the Yukawa or Newtonian potentials, the sign of the cosinusoidal potential oscillates. This alternation leads to the curious property that a potential which is attractive at short distances is, on the whole, repulsive at large. The repulsion is far too small to be relevant for the present universe, but can have cosmological consequences.

To see the effect of the alternation, consider the potential at the center of a homogeneous sphere of radius R,

$$\phi = -4\pi G\rho \int_o^R \cos(2\pi r/\lambda_o) r dr$$

$$= -(G\rho\lambda_o^2/\pi)[-1 + \cos(2\pi R/\lambda_o) + (2\pi/\lambda_o)R\sin(2\pi R/\lambda_o)] \tag{58}$$

The last two terms oscillate and tend $\to 0$ as $R \to \infty$ if ρ is subject to a soft cut-off (such as $\rho \to \rho e^{-br}, 1 << b\lambda_o$). The first term, however, is constant and opposite in sign to that expected from the Newtonian potential. The result, $\phi = (G\rho\lambda_o^2/\pi)$, is clearly consistent with the scalar Helmholtz equation for constant ρ.

Since ϕ is of the same sign as ρ, the effect is inflationary; the universe will tend to expand to lessen the stored gravitational energy. A quantitative measure is given by the energy stored per unit volume,

$$W = \rho\phi = +G\rho^2\lambda_o^2/\pi \tag{59}$$

Substituting the present value of the density of baryons for ρ; namely, 1.4×10^{-7}nucleons/cm^3 (Peebles 1993; Walker et al 1991), we find $W_o = 3\times10^{-27}$ergs/cm^3. This value is very small compared to the mass-energy stored in the baryons directly, 2×10^{-10}ergs/cm^3 or to the energy stored in the 2.7 K radiation, 4×10^{-13}ergs/cm^3.

The relation between the gravitational, thermal, and mass densities is altered dramatically at earlier times. This is because the gravitational density varies as ρ^2 and thus will increase with red shift as $(1+z)^6$ until $z \simeq 1000$ at which point radiation dominates over matter and the increase in gravitational energy goes as $(1+z)^8$. The result, shown in Fig. 17, is that at $(1+z) \simeq 10^5$, the gravitational energy dominates.

The spacing between the two circled cross-overs in Fig. 17 should provide enough time for inhomogeneities in n_γ to relax to the smooth Cosmic Background Radiation seen today. Alternatively, the time for nucleosynthesis (point f) is *earlier* than the time when the whole universe has expanded to a horizon of λ_o. Thus the gravitational fields at the critical time of nucleosynthesis should be unaffected by this proposal.

This domination is expected in fluctuations of gravitational energy over a scale length of λ_o as well as in the mean. (Clumping of mass should occur because only *implosive* gravitational forces act on all matter within an isolated, homogeneous sphere of radius $R < \lambda_o/4$. See eq. (58). There is some evidence that these primordial fluctuations have left observable relics. Fluctuations in present galactic densities have been reliably observed at a continuum of repetition lengths up to 30 Mpc (Peebles, 1993) and even longer (Da Costa et al 1994). Particularly intriguing is the specific repetition length of 128 h_o^{-1} Mpc. discovered by Broadhurst et al (1990). Assume this length expands with the universe. Then at a lookback redshift of $z = 10^{5.5}$, a little earlier than the end of a gravitation dominated universe, the present structure would have had a length of 560 pc, the putative value of λ_o. (See point B in Fig. 17).

The newly discovered "chain galaxies" also may evidence a repetition length, this time of λ_o itself. Cowie, Hu, and Songaila (1995) have discovered a new class of objects at redshift $z \simeq 1$. The objects appear as chains of length 2" - 3" and "blob separations of 0.5" or several times larger." For an open universe, a transverse length of 560 pc asymptotically approaches 0.58" as $z \to \infty$. As Cowie et al note, a chain is not a stable structure (assuming Newtonian gravity). For the cosinusoidal potential, a chain (in distinction to a two or three dimensional lattice) is stable. Consider a linear chain with $2N+1$ identical masses m each spaced a distance λ_o from its neighbor. The potential at the center is $(-2Gm/\lambda_o)(\ln(\mathrm{N}) + 0.577..)$. Similar expressions apply at the site of each mass and even for the case where some sites are vacant. Since the potential varies only logarithmically with N, the binding is weak.

Finally, note that the cosinusoidal potential is incompatible with a critically closed universe. The oscillations reduce the desired $\Omega = 1$ by a factor $\simeq -(\lambda_o H_o^{-1})^2$; the avoidance of dark matter gives a further 10^{-2} reduction leaving $\Omega \simeq -10^{-16}$. (The assumption $\Omega = 0$ has already been used here in the confirmation of the bending of light by the Einstein Cross and, more significantly, in the predicted time delay between the arcs in MG 1654+1346 and in the discussion of chain galaxies).

4.6 Real Sinusoidal Potential

The Helmholtz equation for a point source M also has a non-singular, sinusoidal solution, of the same wavelength as the cosinusoidal one,

$$m\phi(r)_s = -(G_s m M/r)\sin(2\pi r/\lambda_o). \tag{60}$$

Present observations are consistent with $G_s = 0$. Unfortunately, a tight limit cannot be given. This is because only the gravitational *field* is measured directly. For sensitive solar system measurements, the effect of G_s on the field is reduced by the ratio $(R/\lambda_o)^3$ relative to that of the ordinary G. In galaxies, measurements of rotational velocities are presently poor enough that all one can say is that $G_s \lesssim G$. The deflection of light in the Einstein cross gives evidence against the sinusoidal potential, allowing a somewhat tighter limit, $G_s \lesssim 0.1G$.

The present proposal is that $G_s = 0$. There is, however, a possible roll for an imaginary sinusoidal potential. The possibility that such a potential distinguishes matter and antimatter is discussed in the following paper. Ironically, the imaginary potential helps the present proposal as well. It provides an explicit manifestation of the violation of the equivalence principle in eq. (40).

5. CONCLUSION

Several astronomical observations support the concept of a cosinusoidal potential with a universal length λ_o. Three, the shells of NGC 3923, the distribution of lens diameters, and the Einstein Cross give a tentative value for λ_o. Hopefully this proposal will encourage further observations and analysis, particularly of shells and of gravitational lenses. To understand the implications for singularities, such as black holes, it will be necessary to extend the relativistic theory beyond the simple linear gravitation offered here.

There are several ways to falsify this proposal. One could find shells separated by distances of $1/2\ \lambda_o$ or $3/2\ \lambda_o$. Equivalently, it is sufficient to find a rapidly rotating homogeneous disk which ignores the beginning of the first forbidden zone at 250 pc. The globular clusters could be proven really to be on deeply penetrating orbits. (To do so would require that proper motions be measured directly with respect to external galaxies). Finally, the time delay in a gravitational lens (with impact parameters $\gg \lambda_o$) could by shown to require a contribution from the potential as well as from the geometry.

If this proposal is correct, the current extensive search for MACHOS might change its focus. Looking towards the Magellanic clouds is, at present, vital to the discovery of the nature of Galactic dark matter. This outward search is irrelevant to the cosinusoidal potential which weights interior sources more than exterior. Here the search towards the Galactic bulge may find the central baryonic dark matter which allows the Galaxy to have such a high escape velocity.

6. HISTORICAL NOTE

The idea of an oscillating potential perhaps originated with the Milano Croatian Jesuit philosopher Ruder Boskovic (1711-1787). Boskovic attempted to find a single force that could unify atomic, terrestrial, and astronomical phenomena. Martinovic (1987) summarizes Boskovic's thesis, "The number and properties of the flow of the curve of forces depend on the distance between the particles. The changes are more numerous and more significant at *imperceptibly small* distances. At distances corresponding *to the distances between the planets* the form of the curve approaches the hyperbola of the second degree $-1/xx$ and at the greatest distances, like those between fixed stars or their distance from us, the curve can cut the axis at any number of points...."

7. ACKNOWLEDGMENTS

I am grateful to Neil Ashby, Uriel Nauenberg for reading this manuscript and giving comments and encouragement. Catharine Garmany, Michael Shull, Andrew Hamilton, and J. Mc Kim Malville have helped with the answers to specific questions about astronomy. Stephen Leeds gave information about history. A chance comment by Richard Isaacson stimulated my thinking about gravitational theory. I appreciate the many significant suggestions of the referees. These have led to the writing of completely new sections. Finally I thank the Astronomical Data Center and National Space Science Center for providing the data on nearby stars.

8. APPENDIX - MASSIVE PHOTON

A colleague, William O'Sullivan, has observed that it is difficult for scientists to accept a new universal length, particularly if it affects only one of the four fundamental forces. The roll of electromagnetism in the possible decay of gravitational waves has already been discussed. Here I suggest that the photon itself may have a (real) mass and that recent evidence allows its Compton wavelength, $\lambda_1 \equiv h/m_\gamma$ to be approximately the same as the gravitational wavelength, $\lambda_o = 560$ pc.

This suggestion apparently conflicts with the existing *lower* limit, $\lambda_1 > 14$ kpc (Montanet et al 1994). Chibisov (1976) established this value from the observed stability of the Small Magellanic Cloud. He argued that the known forces which act on the SMC are all explosive. These are thermal and magnetic pressures as well as centrifugal forces. The only available implosive force is provided by a massive photon of reduced Compton wavelength comparable to the 3 kpc extent of SMC. Not considered was a roll for gravity. But this roll could be provided by the enhanced implosive gravitational forces considered in the present proposal.

Thus it is prudent to consider that Chibisov's limit might not be valid. The general approach of looking for a balance of forces is, however, very valuable. Let us first look at this balance in a situation where non-electromagnetic forces provide the basic equilibrium, and the magnetic field configuration has merely to be stable. This is the case for galactic magnetic fields.

The condition for stability is readily found from the magnetic part of Maxwell's stress tensor, a tensor which must be changed if the photon is massive. To the usual terms in the magnetic field B, we add terms in the vector potential A (Chibisov 1976) to arrive at

$$4\pi T_{ij}^{M} = B_i B_j - (2\pi/\lambda_1)^2 A_i A_j - (\delta_{ij}/2)(B^2 - (2\pi/\lambda_1)^2 A^2). \tag{61}$$

For stability $T_{ij}^{M} = 0$. Equilibrium can be achieved if $B = \nabla \times \mathbf{A} = \pm(\boldsymbol{2\pi/\lambda_1})\mathbf{A}$. To satisfy the stability condition, B must be *parallel* to A, everywhere. Additionally, the wavelength of the spatial variation of B should equal the Compton wavelength of the photon. The first condition is the familiar one of "force free fields", $\mathbf{J} \times \mathbf{B} = \mathbf{0}$. These are found in solar flares, but there the spatial variation in B is so marked that the terms in A are inconsequential. The lines of B are in tension, but this tension is overwhelmed by the expansive pressure of the B^2 term (Parker 1979). We shall be interested rather in galactic magnetic fields where the second condition may also be satisfied.

The planar solution for force free fields of wavelength λ_1 is simply $A_x = A_o \sin(2\pi z/\lambda_1); A_y = A_o \cos(2\pi z/\lambda_1)$. This solution has constant A_x and A_y over any plane $z = const.$, and has positive helicity $H \equiv \int \mathbf{B}.\mathbf{A} dv$. (The complementary negative helicity solution is $A_x = A_o \cos(2\pi z/\lambda_1); A_y = A_o \sin(2\pi z/\lambda_1)$). To obtain a force free field having components in all three directions we add two planar fields of different directions, arbitrary amplitudes, but the same wavelength and helicity. For example, adding a planar (x) solution yields,

$$A_x = A_o \sin(2\pi z/\lambda_1); A_y = A_o \cos(2\pi z/\lambda_1) + A_1 \sin(2\pi x/\lambda_1); A_z = A_1 \cos(2\pi x/\lambda_1). \tag{62}$$

In practice A_o and A_1 can vary slowly with $\mathbf{r}$, $(|\nabla A_i| << 2\pi A_i/\lambda_1)$.

Recent observations indicate that the dominant magnetic fields both in M31 (Urbanik, Otmianowska-Mazur, and Beck 1994) and in the Galaxy (Rand & Lyne 1994) are in equatorial rings rather than along spiral arms. To see the form of the predicted stable configuration, we simply substitute cylindrical coordinates for Cartesian in eq. (62). Thus $x \to r - r_o$, $y \to r\phi - r\phi_o$ and $z \to z - z_o$. (Since $2\pi r >> \lambda_1$, the evaluation of the curl is unaffected by using curvilinear coordinates).

It is intriguing that Urbanik et al offer evidence that the field in M31 is dominantly helical with a pitch angle of about 30 degrees and a spacing between helices of a few hundred pc. Both features are consistent with eq. (62). Measurements in our own galaxy are not so far advanced, but indicate a reversal in the direction of the azimuthal field at a radius 400 pc closer to the center of the Galaxy than the sun. Perhaps both galaxies are approaching stable configurations.

The influence of a massive photon on the relaxation time of galactic magnetic fields was investigated by Williams and Park (1971). They set a lower limit ($\lambda_1 >$ 6 lt yr) from the observation that the local galactic field is at least 10^6 years old. They assumed the field to be solenoidal (and directed along a spiral arm). We may turn their analysis around to state that if λ_1 really is 560 pc, an initial field having $B \perp A$ will decay to the stable configuration in about the lifetime of the universe. The fact that a field with $B \parallel A$ is more stable that one with $B \perp A$ was first discussed in a classic review of massive photons (Goldhaber and Nieto 1971).

Finally, the same generalized Maxwell stress tensor (eq. (61)), though not the force-free plane wave solutions (eq. (62)) may be used in more complicated situations, such as the Coma cluster.

REFERENCES

1. Bouwkamp(1947) was evidently the first to show that the Yukawa family of potentials (Yukawa, Newton, and cosinusoidal) permits one to represent the potential outside a homogeneous sphere as the product of a unit point potential and an effective mass. Sneddon and Thornhill (1947) proved that, of all possible central potentials, members of the Yukawa family are unique in having this desirable property. The extension to multipoles is, I believe, new to this paper. This extension can apply to the Yukawa potential itself if modified spherical Bessel functions are used and thus, may be helpful in revivals of finite length anti-gravity (FLAG) (Sanders 1984) or the fifth force (Fischbach et al 1986).

2. Neighboring choices for ϕ_o also give a minimum near 5.34: viz, for $\phi_o =$ 1/2 cycle, $\chi^2 = 26$ at $r_o = 5.38$, for $\phi_o = 3/4$ cycle, $\chi^2 = 27$ at $r_o = 5.25$

3. Our previous concern that some disk galaxies are expanding rapidly (Bartlett & Pike 1990) has been retracted (Bartlett & Pike 1993).

4. The reader may find evidence of this clumping by viewing the bound volume obliquely from the side.

5. As Sanders (1990) observed, alternative theories of Newtonian gravity which add a Yukawa term to the usual Newtonian gravity (Sanders 1984) cannot explain why M_N/M is roughly independent of the size of the galaxy. Also see Kenmoku, Okamoto, & Shigemoto (1993).

6. Finding a truly face-on galaxy is difficult because the ratio of the minor and major axes gives one only the cosine of a small angle. The problem is

illustrated in Tully's Catalogue (1988). Of the approximately 1500 tabulated disk galaxies, none has a *measured* inclination $< 15^o$; only 13 have inclinations $< 20^o$.

7. The reader is cautioned that there is some arbitrariness in the drawing of these curves. There is the critical question of the outliers. What fraction of stars can be expected to lie outside the dynamic limits? The answer is zero if there are no measurement errors or binaries and the galaxy is like the solar system, enduring unaltered for many revolutions. But the galaxy is dynamic; blue stars evolve to supernova in a fraction of a revolution. The resulting hydrogen gas collides inelastically with stars, possibly altering the radial distribution of matter slightly so as to change the effective monopole M_{eff}. Further study is clearly needed, particularly since recent observations have shown that old galaxies look very different from new. [A. Dressler (1993)].

8. Measurements can still be made in an accelerated frame (such as an earth-bound laboratory) if the internal accelerations within the local clock are large compared to the acceleration of the laboratory with respect to the comoving inertial frame. This is the case for nuclear and atomic clocks (Isaak 1970).

9. This proposal makes use only of Lorentz gauges, both here and in the Appendix, where the possibility of a massive photon is considered. This curtailment of the usual freedom is a direct consequence of the fact that, even in the non-relativistic Helmholtz equation, the potential itself is directly observable. One cannot add an arbitrary constant as can be done in Newtonian mechanics. Since $\mathbf{g} = -\nabla\phi$, the Helmholtz equation may be written as $k_o^2\phi = 4\pi G\rho + \nabla \cdot \mathbf{g}$, thus permitting the determination of an absolute ϕ

from measurements of the local ρ and g.

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FIGURE CAPTIONS

FIG 1. Galaxy NGC 3923. Prieur's measurement[a] of maximum of shell radii vs fitted N of cycle using $r_N = (N + \phi_o)r_o$; $\phi_o = 5/8$ cycle; $r_o = 5.34$"

[a] Column 5 of Table 3 of Prieur 1988.

FIG 2. NGC 3923. χ^2 vs r_o for a fixed ϕ_o=5/8 cycle.

FIG 3. Periodicity of Lens Radii. N_{wt} vs λ_o75. The scale for the abscissa has been multiplied by 2/3 from that obtained directly from Kormendy's data. This corrects for the fact that he assumed a Hubble constant of 50 km/s/Mpc rather than the 75 assumed here. The cross-hatched interval is the value of λ_o as determined by the elliptical NGC 3923 using either Prieur's or Tully's estimate of the distance to that galaxy. The arrows at smaller wavelengths mark where expected harmonics of 560 pc should occur.

FIG 4. Number of stars in 10 km/s intervals for 1946 stars in CNS3R. Radial, azimuthal, and z-velocities wrt galactic center are -u, v, and w respectively.

FIG 5. Normalized effective one-dimensional potential ϕ_u vs radius in cycles r/λ_o. Curves given for normalized angular momenta $L_u = 1.25, 2.2514.25$. Also shown are possible positions for the sun assuming for $(v_{escape}, v_\odot)$: Top dot - (460,300); center dot (680,300) or (510,231); bottom dot - (1200,300)(Non-spherical potential; escape velocity out of plane of disk assumed to 600 km/s).

FIG 6. Envelopes of permissible azimuthal velocities v_ϕ vs radial velocity v_r for seven r/λ_o between 14 5/8 and 15 3/8. Permissible velocities are under envelopes. Solid Curves have vertices at inner turning points; dashed at outer. Bold curve corresponds to possible radius for the sun. All velocities normalized to global escape velocity $v_\phi(max)$.

FIG 7. Scatter diagram of u and v velocities of stars in CNS3R. Curves are envelopes expected for different combinations of $(v_{escape}, v_\odot)$: Solid (460,300), Dash (510,231), Dot (680, 300), Dot-Dash(1200,300) (Noncentral potential).

FIG 8. Differential histograms of azimuthal velocities v. (a) N_+ stars more than 6 pc further away from center of galaxy than sun. (b) N_- stars more than 6 pc closer to center of galaxy than sun. (c) $N_+ - N_-$.

FIG 9. Differential histograms, $N_+ - N_-$. (a) For stars $|r_{star} - r_{sun}| > 12$ pc; (b) 6 pc $< |r_{star} - r_{sun}| < 12$ pc and (c) $|r_{star} - r_{sun}| < 6$ pc.

FIG 10. Potential close to a point source mass. Circular orbits allowed only in regions marked $\odot$. No bound orbits allowed in regions marked "No". Unmarked intervals can have radially confined, but non-circular orbits.

Fig 11. Circular velocity vs radius for M31 (from Table in Rubin & Ford). Markers a (250 pc) and b (380 pc) show limits of first forbidden region. Dashed curve: $1/r$ falloff expected by angular momentum conservation for matter which escapes from radius a.

FIG 12. Potential close to an extended source. Bound stars having radial amplitudes $> \lambda_o$ confined to $r < r_B$. Orbits more likely to stop at $r = r_A$.

FIG 13. Magellanic Plane shown as a projection of a celestial sphere viewed along the axis of the central bar.

FIG 14. Globular Clusters. Histogram of Number vs Figure of Merit, FM. Clusters having $FM > 1$ in ascending order: Pal 15, AM-1, Rup 106, Pal 2, Pal 3, NGC 5694, Pal 1, Pal 14, Eridanus.

FIG 15. Typical globular cluster in a field $g_r \propto 1/r$. Average $v = 225$ km/s. Solid curve: speed, Dotted: v_r, Dashed: v_θ.

FIG 16. Deflection angle α vs. Image displacement angle θ for light from a quasar Q not quite on axis of intervening galaxy. Image occurs whenever line intersects solid curve. Since magnification is inversely proportional to angle between curve and line, only detectable images are a points of tangency, as illustrated for positive θ.

Fig. 17. Mean energy density of universe W vs radiation temperature $T =$ 2.76K $(1+z)$. $0 =$ present. Gravitational energy dominates before the early circle; after late circle mass dominates. Characteristic temperatures indicated. f: fusion of deuterium and light elements, H: $\lambda_o H_o = 2.76\text{K}\ /T$, B: 128 Mpc $\times 2.76\text{K}\ /T = \lambda_o$.