# HW3 - Phys 7810-001

due 03/11/21

## Problem 1

[40 pts]

Two Higgs doublet models: Suppose the Higgs doublet of the standard model is supplemented by a second complex doublet, $\psi$ , transforming as $(1; \mathbf{2}; -1/2)$ under $SU_c(3) \times SU_L(2) \times U_Y(1)$.

a) [5 pts] If $\psi = \begin{pmatrix} \chi \\ \xi \end{pmatrix}$, what are the electric charges of the component fields $\chi, \xi$?

b) [5 pts] Write out the covariant derivative $D_\mu \psi$ explicitly in terms of the gauge fields $G^\alpha_\mu, W^a_\mu$ and $B_\mu$.

c) [10 pts] Assuming the potential must be a function of the invariants $a = \phi^\dagger \phi$, $b = \psi^\dagger \psi$, and $c = \phi^{\mathrm{T}} \epsilon \psi$ , where $\phi$ is the usual Higgs doublet, what is the most general renormalizable form? How many independent real parameters does it contain? Need the parameters appearing in the potential be real? Is the combination $d = \phi^\dagger \psi$ $SU_L(2) \times U_Y(1)$ invariant?

d) [20 pts] Suppose the parameters of the potential are such that it is minimized when

$$\begin{aligned} \phi &= \phi_{\min} = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \\ \psi &= \psi_{\min} = \begin{pmatrix} (u + iw)/\sqrt{2} \\ 0 \end{pmatrix}, \end{aligned}$$

with $u, v, w$ all real. Do these values break the electromagnetic group $U_{em}(1)$ generated by the electric charge $Q = T_3 + Y$ ? Identify the terms in the

Lagrangian that are quadratic in the gauge fields and find their masses in terms of u, v, and w. Call the mass eigenstates $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_{\mu 1} \mp i W_{\mu 2})$, $Z = W_3 \cos\theta - B_\mu \sin\theta$, and $A_\mu = B_\mu \cos\theta + W_{3\mu} \sin\theta$. Express $\cos\theta$ in terms of the gauge couplings $SU_L(2) \times U_Y(1)$ gauge couplings. Is the standard model mass relation $M_W = M_Z \cos\theta$ also true for this model?

## Problem 2

[40 pts]

Decay of the top quark: Consider the top quark, with a mass of $m_t = 173$ GeV. a) [15pts] Identify the only interaction term in the Lagrangian which is linear in the top quark. Can a single insertion of this interaction term cause the top quark to decay? What are the decay products? b) [15 pts] Write an expression for the matrix element for the dominant top quark decay process. Find a compact expression for the square of the matrix element, summing over final-state spin or helicity states and averaging over the initial top-quark helicity state. [10 pts] Compute the width of the top quark. Neglect the masses of any other fermions in comparison to the top-quark mass, but treat the masses of W and Z bosons as comparable to the top-quark mass. You should be able to find an analytic expression for the decay rate. Then, substitute in physical values and express the answer in GeV.

## Problem 3

[20 pts] Consider the Abelian Higgs theory. After rewriting $A_\mu = Z_\mu$ and $(\mu, h) \to \frac{1}{\sqrt{2}}(\mu, h)$ you would have got after spontaneous symmetry breaking, the Lagrangian

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{4!}\mu^2 h^2 - \frac{\lambda}{4!}\mu h^3 \\ &- \frac{\lambda}{4 \times 4!}h^4 + \frac{1}{2}e^2\mu^2 Z_\mu Z^\mu (1 + \frac{h}{\mu})^2 \end{aligned}$$

a) [10 pts] Write down the Feyman rules for this theory of Z-bosons and Higgs particles.

b) [10 pts] Calculate to lowest order the decay rate of Higgs particles into $Z$ bosons. What inequality must $\lambda, e$ satisfy in order for this process to occur.

(The differential decay rate in the rest frame of a decaying particle of mass $M$ is given by $d\Gamma = \frac{1}{32\pi^2}|\mathcal{M}|^2\frac{|\mathbf{p}|}{M^2}d\Omega$ where $\mathcal{M}$ is the invariant amplitude for decay into two particles and $\mathbf{p}$ is the three momentum of either of the decay particles.)

Sol 3

which uses the properties $\overline{\psi}_1\gamma^\mu\psi_2 = -\overline{\psi}_2\gamma^\mu\psi_1$ and $\overline{\psi}_1\gamma^\mu\gamma_5\psi_2 = +\overline{\psi}_2\gamma^\mu\gamma_5\psi_1$, that hold for any two majorana spinors, to conclude $\bar{\nu}\gamma^\mu\nu = 0$ and so $\bar{\nu}\gamma^\mu P_L\nu = \frac{1}{2}\,\bar{\nu}\gamma^\mu\gamma_5\nu$. In particular, notice that this does not involve any dangerous flavor-changing neutral currents. The coupling to the photon remains zero, since neither $\nu$ nor $N$ have an electric charge.

Note that in the limit $M \gg m$, the angle $\theta$ is very small and so $\psi_1$ is light and couples with full strength to $W_\mu$ and $Z_\mu$, while $\psi_2$ is very massive and approximately decouples. In this limit, at energies too low to produce the $\psi_2$ particle, the theory looks like the normal electroweak theory but with a tiny mass added for the $\nu$ field; a mass which is quadratic in the Higgs field. Chapter 10 describes another way of understanding this in terms of higher dimension interactions in effective field theories.

Similarly, using eq. (2.20) as well as

$$N = i\gamma_5\psi_1 \sin\theta + \psi_2 \cos\theta\,, \tag{2.23}$$

in the Higgs interactions gives the following Yukawa couplings

$$-\frac{kH}{2}\,(\bar{\nu}N + \bar{N}\nu) = \frac{kH}{2}\left[(\overline{\psi}_1\psi_1 + \overline{\psi}_2\psi_2)\sin 2\theta - 2i(\overline{\psi}_2\gamma_5\psi_1)\cos 2\theta\right]. \tag{2.24}$$

This does have flavor-changing couplings, and these are the dominant interactions in the $M \gg m$ limit, where $\theta \ll 1$. Because the small $\psi_1$ mass is due to $M \gg m$ in this limit, rather than due to $k \ll 1$, this off-diagonal neutrino-Higgs coupling need not be small. By contrast, for small $\theta$ the diagonal coupling is proportional to $k\,\theta \simeq (m/v)(m/M) \simeq m_1/v$, ensuring that both diagonal couplings are suppressed by the light neutrino mass.

Problem I

## 2.4 Two Higgs doublet models

### 2.4.1 Electric charges

Note that this second Higgs field has the same charge assignments as $\tilde{\phi}$. The electric charge assignments of the components are therefore

$$\psi = \begin{bmatrix} \chi \\ \xi \end{bmatrix} \quad \text{have charges} \quad \begin{bmatrix} 0 \\ -1 \end{bmatrix}. \tag{2.25}$$

### 2.4.2 Covariant derivative

The full covariant derivative acting on $\psi$ then is

$$D_\mu\psi \;=\; \left(\partial_\mu - \frac{ig_2}{2}W^a_\mu\tau_a + \frac{ig_1}{2}B_\mu\right)\psi$$

$$= \begin{bmatrix} \partial_\mu - ig_2 W^3_\mu/2 + ig_1 B_\mu/2 & -ig_2(W^1_\mu - iW^2_\mu)/2 \\ -ig_2(W^1_\mu + iW^2_\mu)/2 & \partial_\mu + ig_2 W^3_\mu/2 + ig_1 B_\mu/2 \end{bmatrix} \begin{bmatrix} \chi \\ \xi \end{bmatrix}. \quad (2.26)$$

### 2.4.3 Scalar potential

Notice that the combination $\psi^\dagger\phi$ has net hypercharge +1 and so is not $SU_L(2) \times U_Y(1)$ covariant. However the combination

$$\psi^T \epsilon \phi = (\psi^\dagger \epsilon \phi^*)^* = (\psi^\dagger \tilde{\phi})^* \quad (2.27)$$

(with $\epsilon = i\tau_2$) *is* $SU_L(2) \times U_Y(1)$ invariant since $\tilde{\phi}$ and $\psi$ have the same charges. It is also complex, which means many terms will be allowed complex coefficients. In terms of the quantities

$$a \equiv \phi^\dagger\phi, \quad b \equiv \psi^\dagger\psi, \quad c \equiv \phi^T e \psi = \tilde{\phi}^\dagger\psi. \quad (2.28)$$

The most general available quadratic potential is

$$V_2 = m_1^2 a + m_2^2 b + m_3^2(c + c^*) - i m_4^2 (c - c^*), \quad (2.29)$$

which has 4 real parameters. We could rewrite the last two terms as $Mc$ + c.c. with $M$ a complex parameter. (In fact it is possible to re-define fields in terms of a new linear combination of $\psi$ and $\tilde{\phi}$ to eliminate the cross-term but we do not pursue this here.)

At the quartic level, one might think that the combination $(\psi^\dagger\phi)(\psi^T\phi^*)$, which is $SU_L(2)\times U_Y(1)$ invariant, would be an extra possible term in the potential, besides those quadratic in $a$, $b$, and $c$. However, it is not independent, since

$$(\psi^\dagger\phi)(\psi^T\phi^*) = ab - c^*c, \quad (2.30)$$

as can be shown by explicit calculation. Similarly, $\sum_a |\psi^\dagger\tau^a\tilde{\phi}|^2 = |\psi^\dagger\tilde{\phi}|^2 = c^*c$ is not independent. The most general quartic terms for the two Higgs fields are

$$\begin{aligned} V_4 &= \lambda_1 a^2 + \lambda_2 b^2 + \lambda_3 ab + \lambda_4 c^*c + \lambda_5 [c^2 + (c^*)^2] - i\lambda_6 [c^2 - (c^*)^2] \\ &\quad + \lambda_7 a(c + c^*) - i\lambda_8 a(c - c^*) + \lambda_9 b(c + c^*) - i\lambda_{10} b(c - c^*) \\ &= \lambda_1 a^2 + \lambda_2 b^2 + \lambda_3 ab + \lambda_4 c^*c + [(\lambda_5 - i\lambda_6) c^2 \\ &\quad + (\lambda_7 - i\lambda_8) ac + (\lambda_9 - i\lambda_{10}) bc + \text{c.c.}]. \end{aligned} \quad (2.31)$$

As written the $\lambda_i$ are all real. The second form writes the potential in terms of complex couplings. Note that many of these terms can be eliminated by suitable choices of additional symmetries.

### 2.4.4 Symmetry breaking

To see if the electromagnetic group is broken by the VEV's, we have to see how the VEV's rotate under an electromagnetic transformation. These transformation properties are given by the electromagnetic charges; the lower component of $\phi$ and the upper component of $\psi$ are charge zero, so they are not rotated and the VEV's are unchanged. Therefore, electromagnetism will still be unbroken. Breaking electromagnetism would require that part of the VEV of $\chi$ lie in the lower entry. Whether this happens depends on the details of the effective potential, which means that it can be used to constrain the form of the effective potential (specifically, the terms involving $c$ above).

Observe that, dropping all fluctuations from $\psi$ and $\phi$,

$$D_\mu\psi \to \frac{i}{2\sqrt{2}}\begin{bmatrix} g_2W^3_\mu - g_1B_\mu & g_2(W^1_\mu - iW^2_\mu) \\ g_2(W^1_\mu + iW^2_\mu) & -g_2W^3_\mu - g_1B_\mu \end{bmatrix}\begin{bmatrix} u+iw \\ 0 \end{bmatrix}, \tag{2.32}$$

and so

$$\begin{aligned}(D_\mu\psi)^\dagger D^\mu\psi &= \frac{u^2+w^2}{8}\left[g_2W^3_\mu - g_1B_\mu \;\; g_2(W^1_\mu - iW^2_\mu)\right]\begin{bmatrix} g_2W^\mu_3 - g_1B^\mu \\ g_2(W^\mu_1 + iW^\mu_2) \end{bmatrix} \\ &= \frac{u^2+w^2}{8}\left[g_2^2(W^1_\mu W^\mu_1 + W^2_\mu W^\mu_2) + (g_2^2+g_1^2)Z_\mu Z^\mu\right].\end{aligned} \tag{2.33}$$

Combining this with the kinetic term for $\phi$ gives the masses

$$m_W^2 = \frac{g_2^2}{4}(v^2+u^2+w^2) \quad \text{and} \quad m_Z^2 = \frac{g_1^2+g_2^2}{4}(v^2+u^2+w^2), \tag{2.34}$$

so both the $W$ and $Z$ boson masses depend on the same combination of VEV's.

The mass eigenstates $A_\mu$ and $Z_\mu$ are related to $W^3_\mu$ and $B_\mu$ as in the standard model, with mixing angle $\tan\theta_W = g_1/g_2$. It also remains true that the ratio of the $W$ and $Z$ masses are related as they were in the standard model: $m_Z = m_W\cos\theta_W$.

In this model there are 5 physical spinless particles that remain after the Higgs mechanism: three that are electrically neutral and two charged.

### 2.4.5 Yukawa couplings

The field $\psi$ is in general allowed Yukawa coupling matrices which are identical in form to those allowed for Higgs fields with the substitution $\phi \to \tilde{\psi}$ and $\tilde{\phi} \to \psi$. That is, we can write down

$$p_{mn}\bar{L}_mP_RE_n\tilde{\psi} + q_{mn}\bar{Q}_mP_RD_n\tilde{\psi} + r_{mn}\bar{Q}_mP_RU_n\psi + \text{h.c.}, \tag{2.35}$$

with $p, q, r$ arbitrary 3 by 3 matrices. There is no *a priori* reason why these matrices should be in any way related to the matrices $f, g, h$ of the $\phi$ Higgs field. In general, once these are expressed in terms of the fermion mass eigenstates, this allows flavor changing neutral currents which are grossly in conflict with observations.

The conclusion changes given the global $U(1)$ symmetry proposed, for which all right-handed singlets, $P_R E_m$, $P_R U_m$, $P_R D_m$, and the conjugate scalars, $\tilde{\phi}$ and $\tilde{\psi}$ all transform identically — *e.g.* $\tilde{\psi} \rightarrow e^{i\theta}\tilde{\psi}$. In this case all of the Yukawa interactions involving the tilde quantities are forbidden, since they get rotated by a phase of $2\theta$ under the transformation, leaving only

$$f_{mn}\bar{L}_m P_R E_n \phi + h_{mn}\bar{Q}_m P_R D_n \phi + r_{mn}\bar{Q}_m P_R U_n \psi + \text{h.c.}\,. \tag{2.36}$$

This ensures that up- and down-type quarks each couple exclusively to only one of the scalars, and so ensures that the neutral scalar Yukawa couplings are automatically diagonalized by the same transformations that diagonalize the fermion mass matrices. (The field $\phi$ is called the 'down-type' Higgs and $\psi$ is the 'up-type' Higgs.) Note that the relation between a Yukawa coupling and a fermionic mass only involves the VEV of the particular participating scalar, so the relation between Yukawa couplings and couplings to the Higgs particle is no longer universal.

The scalar potential also simplifies if it is required to be invariant under this symmetry, since only the scalar interaction terms involving the couplings $m_1^2$, $m_2^2$, and $\lambda_1$ through $\lambda_4$ survive.

This kind of 2 Higgs doublet model is a commonly proposed alternative to the standard model Higgs, and in particular arises in supersymmetric extensions to the standard model.

## 2.5 Adjoint Higgs fields

### 2.5.1 Electric charges

The $t_i$ are the $J_{x,y,z}$ operators for spin 1 particles in the $z$ spin state basis, so they surely satisfy the Lie algebra of the group SU(2). Explicit verification is straightforward. Since the hypercharge of the triplet scalar is zero, the electric charges can be read off from the action of $T_3$ on the field:

$$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix} \quad \text{have charges} \quad \begin{bmatrix} +1 \\ 0 \\ -1 \end{bmatrix}. \tag{2.37}$$

and so the integrated rate is

$$
\begin{aligned}
\Gamma(W \to e\bar{\nu}) &= \frac{e_W^2 p}{6\pi m_W^2}\left(g_V^2 + g_A^2\right)\left[3m_W p^0 - m_e^2 - 2(p^0)^2\right] \\
&= \frac{e_W^2 m_W}{12\pi}\left(g_V^2 + g_A^2\right)\left(1 + \frac{m_e^2}{2m_W^2}\right)\left(1 - \frac{m_e^2}{m_W^2}\right)^2 , \qquad (4.26)
\end{aligned}
$$

as before.

## 4.2 Decay of the top quark

### 4.2.1 Interaction Hamiltonian

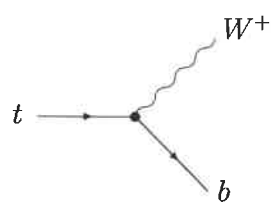
The top quark decays into a quark (almost always a $b$ quark) and a $W$ boson. To leading order it takes place due to one insertion of the following term in the interaction Hamiltonian,

$$\mathcal{H}_{\text{int}} = ie_W\, V_{mn}^\dagger\, W_\mu \left[\bar{d}_m \gamma^\mu (1+\gamma_5) u_n\right] + \text{h.c.}\,. \qquad (4.27)$$

The relevant term is when $n = 3$ since the top quark is in the 3rd generation; that is the terms involving $V_{mt}^\dagger$. Of these we see that the $b$ quark term dominates, with the width to go to some other quark suppressed by $(V_{ts}/V_{tb})^2 < 0.005$. We take $V_{tb} = 1$ for current purposes.

### 4.2.2 Matrix element

The relevant Feynman diagram is




and so the matrix element is therefore

$$\mathcal{M} = e_W \bar{u}(p_b, \sigma_b) \not{\epsilon} (1+\gamma_5) u(p_t, \sigma_t)\,. \qquad (4.28)$$

### 4.2.3 Spin-summed squared matrix element

Squaring and summing (averaging) over final (initial) spins gives (similarly to the steps in problem 4.1)

$$\overline{|\mathcal{M}|^2} = \frac{1}{2}\sum_{\lambda,\sigma_b,\sigma_t} |\mathcal{M}|^2 \;=\; -\frac{e_W^2}{2}\sum_{\sigma_b,\sigma_t,\lambda_W} \text{tr}\left[u\bar{u}(b)\not{\epsilon}(W)(1+\gamma_5)u\bar{u}(t)\not{\epsilon}^*(W)(1+\gamma_5)\right] \qquad (4.29)$$

$$= -\frac{e_W^2}{2}\left(\eta^{\mu\nu} + \frac{p_W^\mu p_W^\nu}{m_W^2}\right) \mathrm{tr}\left[(-i\not{p}_b+m_b)\gamma_\mu(1+\gamma_5)(-i\not{p}_t+m_t)\gamma_\nu(1+\gamma_5)\right].$$

Neglecting $m_b \ll m_t$ simplifies the trace, since it means that the explicit term proportional to $m_t$ also does not contribute (since it appears only in terms with an odd number of gamma matrices). Now we do the trace. The two factors of $(1+\gamma_5)$ can be combined into $2(1+\gamma_5)$, and the $\gamma_5$ does not contribute as only terms symmetric in $\mu \leftrightarrow \nu$ survive the contraction with the first factor. The trace gives,

$$\begin{aligned}
\overline{|\mathcal{M}|^2} &= 4e_W^2\left(\eta_{\mu\nu} + \frac{p_{W\mu}p_{W\nu}}{m_W^2}\right)(p_b^\mu p_t^\nu + p_b^\nu p_t^\mu - p_b\cdot p_t\,\eta^{\mu\nu}), \\
&= 4e_W^2\left[2p_b\cdot p_t + \frac{2(p_b\cdot p_W)(p_t\cdot p_W)}{m_W^2} - 3p_b\cdot p_t\right] \\
&= \frac{g_2^2}{2}\left[-p_b\cdot p_t + \frac{2(p_b\cdot p_W)(p_t\cdot p_W)}{m_W^2}\right]. \qquad (4.30)
\end{aligned}$$

Both terms are positive. Now work in the center of mass frame, and use that $\mathbf{p}_W = -\mathbf{p}_b$, $p_t^0 = m_t$ and $|\mathbf{p}_b| = p_b^0 \equiv p_b$; the expression becomes

$$\overline{|\mathcal{M}|^2} = \frac{g_2^2}{2}\left[p_b m_t + \frac{2m_t E_W(p_b E_W + p_b^2)}{m_W^2}\right] = \frac{g_2^2}{2}p_b m_t\left[1 + \frac{2E_W(E_W+p_b)}{m_W^2}\right]. \qquad (4.31)$$

### 4.2.4 Final integration

The total width for $t \to Wb$ then is

$$\begin{aligned}
\Gamma_t &= \int \frac{\mathrm{d}^3p_W\,\mathrm{d}^3p_b}{(2\pi)^6 2p_t^0 2p_W^0 2p_b^0}(2\pi)^4\delta^4(p_t - p_W - p_b)\,\overline{|\mathcal{M}|^2} \\
&= \frac{1}{8\pi}\int \frac{p_b^2\mathrm{d}p_b}{p_b E_W m_t}\,\delta(p_b + E_W - m_t)\overline{|\mathcal{M}|^2}, \qquad (4.32)
\end{aligned}$$

with $E_W \equiv \sqrt{p_b^2 + m_W^2}$. In the $t$-quark rest frame energy conservation implies $E_W + E_b = m_t$, and so if $E_b \simeq p_b$ these together imply

$$E_b = p_b = \frac{m_t^2 - m_W^2}{2m_t} \quad \text{and} \quad E_W = \frac{m_t^2 + m_W^2}{2m_t}. \qquad (4.33)$$

To integrate the delta function we need to know the $p_b$ dependence of its argument;

$$\frac{\mathrm{d}}{\mathrm{d}p_b}(p_b + E_W - m_t) = 1 + \frac{p_b}{E_W} = \frac{m_t}{E_W}, \qquad (4.34)$$

and so

$$\int \frac{p_b\mathrm{d}p_b}{E_W m_t}\,\delta(p_b + E_W - m_t)\overline{|\mathcal{M}|^2} = \frac{p_b}{m_t^2}\,\overline{|\mathcal{M}|^2}. \qquad (4.35)$$

Therefore the result of the $p_b$ integration is

$$\begin{aligned}
\Gamma = \frac{1}{8\pi}\frac{p_b}{m_t^2}|\mathcal{M}|^2 &= \frac{g_2^2}{16\pi}\frac{p_b^2}{m_t}\left(1+\frac{2m_tE_W}{m_W^2}\right) \\
&= \frac{\alpha_2(m_t^2-m_W^2)^2(2m_W^2+m_t^2)}{16m_t^3m_W^2} \\
&= \frac{\alpha_2 m_t}{16}\left(\frac{m_t}{m_W}\right)^2\left(1+\frac{2m_W^2}{m_t^2}\right)\left(1-\frac{m_W^2}{m_t^2}\right). \qquad (4.36)
\end{aligned}$$

Inserting numerical values gives

$$\Gamma = 1.5 \text{ GeV} \qquad (4.37)$$

which converts into a lifetime of

$$\tau \simeq 4.4 \times 10^{-25} \text{ sec}. \qquad (4.38)$$

Even at quite relativistic speeds, since $c = 3 \times 10^8$m/s, a top quark can propagate less than $10^{-15}$ meters, which is obviously not detectable (about 100 microns is the actual limit).

## 4.3 Gamma matrix identities

First,

$$\not{k}\not{k} = k_\mu k_\nu \gamma^\mu \gamma^\nu . \qquad (4.39)$$

But $k_\mu k_\nu$ is obviously symmetric, so this can be rewritten as

$$\not{k}\not{k} = k_\mu k_\nu \frac{1}{2}\left(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu\right) = k_\mu k_\nu \frac{1}{2}(2\eta^{\mu\nu}) = k^2 . \qquad (4.40)$$

Second,

$$\begin{aligned}
\not{k}\not{p}\not{k} &= \left(\not{k}\not{p}+\not{p}\not{k}\right)\not{k} - \not{p}\not{k}\not{k}, \\
&= 2k_\mu p_\nu\left(\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu\right)\not{k} - \not{p}k^2, \\
&= 2k_\mu p_\nu \eta^{\mu\nu}\not{k} - k^2\not{p}, \\
&= 2p\cdot k\,\not{k} - k^2\,\not{p}, \qquad (4.41)
\end{aligned}$$

as desired. Third,

$$\gamma^\mu\gamma_\mu \equiv \gamma^\mu\gamma^\nu\eta_{\mu\nu} , \qquad (4.42)$$

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$$\mathcal{L} = \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{24}\lambda\mu^2 h^2$$

$$- \frac{1}{3!}\frac{\lambda\mu}{4} h^3 - \frac{\lambda}{4!}\frac{h^4}{4} + \frac{1}{2}e^2\mu^2 Z_\mu Z^\mu \left(1 + \frac{h}{\mu}\right)^2$$

$$\left(1 + \frac{2h}{\mu} + \frac{h^2}{\mu^2}\right)$$

$$F_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$$

Free Lagrangian

$$h: \quad -\frac{1}{2} h \Box h - \frac{1}{24}\lambda\mu^2 h^2 = -h\left[\Box + m_h^2\right] h$$

$$m_h^2 = +\frac{1}{12}\lambda\mu^2 . \qquad \overline{\quad h \quad} \Rightarrow \frac{i}{p^2 - m_h^2}$$

$$-\frac{1}{4}\left(\partial_\mu Z_\nu - \partial_\nu Z_\mu\right)\left(\partial^\mu Z^\nu - \partial^\nu Z^\mu\right) + \frac{1}{2}e^2\mu^2 Z_\mu Z^\mu .$$

$m_Z^2 = e^2\mu^2$

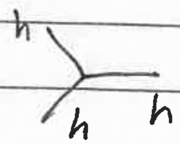
$$= -\frac{1}{2}\left(\partial_\mu Z_\nu - \partial_\nu Z_\mu\right)\partial^\mu Z^\nu + \frac{1}{2} m_Z^2 Z_\mu^2 .$$

$$= \frac{1}{2} Z^\nu \left[\left(\Box \eta_{\mu\nu} - \partial_\nu\partial_\mu\right) + \eta_{\mu\nu} m_Z^2\right] Z^\mu$$

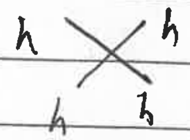
gauge – $\partial_\mu Z^\mu = 0$.


$$\rightarrow \frac{-i\,\eta_{\mu\nu}}{p^2 - m_Z^2 + i\epsilon} .$$

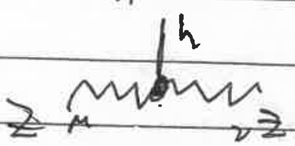
Interaction



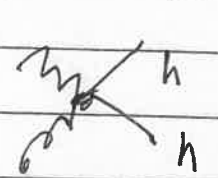

$$\rightarrow -i\frac{\lambda\mu}{4} .$$




$$\rightarrow -i\frac{\lambda}{4} .$$

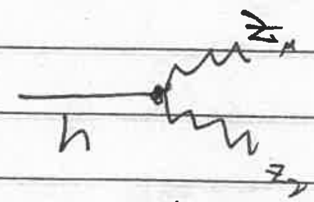



$$\rightarrow i\left(e^2\mu^2 \cdot \frac{2}{\mu}\right)\eta_{\mu\nu} = i\,2\mu e^2\,\eta_{\mu\nu}$$




$$\rightarrow 2ie^2$$

Lowest order - one diagram.




$M_{\mu\nu} = i\,2\mu e^2 \eta_{\mu\nu}$.

Pol$^n$ not observed.

$M(\sigma,\sigma') = \epsilon^\mu(\sigma)\, M_{\mu\nu}\, \epsilon^\nu(\sigma')$

polarization

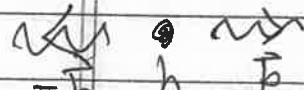
$\sum_{\sigma\sigma'} \epsilon^\mu(\sigma) M_{\mu\nu} \epsilon^\nu(\sigma')\, \epsilon^\lambda(\sigma) M^*_{\lambda\delta} \epsilon^\delta(\sigma')$

$= \sum_\sigma \epsilon^\mu(\sigma)\epsilon^\lambda(\sigma) \sum_{\sigma'} \epsilon^\nu(\sigma')\epsilon^\delta(\sigma') M_{\mu\nu} M^*_{\lambda\delta}$

$= (-\eta^{\mu\lambda})(\eta^{\nu\delta})(i\,2\mu e^2 \eta_{\mu\nu})(-i\,2\mu e^2 \eta_{\lambda\delta})$

$= 4 \times 4 \mu^2 e^4 = 16\mu^2 e^4$

$\Rightarrow d\Gamma = \frac{1}{32\pi^2} \frac{|p|}{m_h^2} \cdot 16\mu^2 e^4\, d\Omega$.




$2p_0 = m_h \qquad 2\sqrt{\vec{p}^2 + m_Z^2} = m_h$.

$|\vec{p}| = \sqrt{\left(\frac{m_h^2}{4} - m_Z^2\right)}$

Decay can proceed if

$m_h > 2 m_Z$.

i.e. $\frac{1}{12}\lambda\mu^2 > 4e^2\mu^2$ i.e. if $\lambda > 48e^2$.